

CS 600 Final Exam

Fall 2002

1. Answer whether or not the following statements are true. You DO NOT need to give your reasons. [25]

- (a) $n! = O(n^n)$
- (b) $2^{2 \lg n} = O(n^2)$
- (c) $n \ln n = O(n^{21/20})$
- (d) $2^{2n} = O(3^n)$
- (e) If a problem is NP-complete, it can be solved by an algorithm which backtracks a polynomial number of times.
- (f) A certificate is a string that shows that the corresponding problem instance is a "yes" instance.
- (g) We can never solve NP-complete problems in polynomial time in the worst case.
- (h) A verification algorithm always halts.
- (i) The concatenation of two languages in class NP is also in class NP.
- (j) If a language is in class NP then its complement is also in class NP.

2. Let L be an array of size n , let $L[i]$ denote the i -th key of L , let x be the key being searched for in L , and let $p(i)$ be the probability for $x = L[i]$. Suppose that x is always found in L with the following probability:

$$\begin{aligned} p(i) &= ci && \text{if } i \text{ is even,} \\ &= cn - ci && \text{if } i \text{ is odd.} \end{aligned}$$

where c is a constant.

Formulate the equation for computing the average time of the Sequential Search with the probability distribution given above in terms of c and n . Do not compute. [??]

3. Answer the questions below for the following linear programming problem:

$$\begin{aligned} \text{Maximize} & \quad 3x_1 + 2x_2 \\ \text{Subject to} & \\ & -x_1 + x_2 \leq 3 \\ & x_1 - x_2 \leq 2 \\ & x_1 + 2x_2 \leq 10 \\ & 3x_1 + x_2 \leq 9 \\ & x_1, x_2 \geq 0. \end{aligned}$$

- (a) Draw a graph of the feasible region.
- (b) Show how the graph of the objective function moves as the simplex method is applied. Assume that x_1 and x_2 are the initial non-basic variables.

4. Formulate the separable convex programming problem to find an approximate solution for the following problem. You do not have to solve it.

$$\text{Maximize} \quad 4 - x_1^2 + 4x_2 - x_2^2$$

Subject to:

$$2x_1 + x_2 \leq 6$$

$$x_1 - x_2 \leq 2$$

$$0 \leq x_1 \leq 2$$

$$0 \leq x_2 \leq 2$$

5. Given the following problem, answer the questions (a) and (b) below:

$$\text{Max} \quad 4x_1 + 4x_2 + 5x_3 + 2x_4$$

$$\text{Subject to: } 10x_1 + 11x_2 + 11x_3 + 6x_4 \leq 38$$

$$x_1, x_2, x_3, x_4 \geq 0 \text{ and they are integers.}$$

(a) Give a strategy for branching and bounding to solve this problem by Branch-and-Bound. [??]

(b) Solve it using the strategy of (a). [??]

6. A steamship company must deliver goods between several different origin-destination pairs. These goods must reach the destinations by specific dates. The steamship company wants to determine the minimum number of ships needed to meet the requirements. Given the following origin-destination pairs, delivery dates, shipment transit times and return times, find a schedule of shipments that uses the smallest number of ships.

Origin-Destination and Delivery Date

Shipment	Origin	Destination	Delivery Date
1	Port A	Port C	3
1	Port A	Port C	8
1	Port B	Port D	3
1	Port B	Port C	6

Shipment Transit Time

	C	D
A	3	2
B	2	3

Return Time

	A	B
C	2	1
D	1	2