Functional Programming — Scheme

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Nov. 10, 2003

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Functional Programming

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1 Scheme

Scheme is a dialect of LISP (*LISt Processing*)

1. Data Types
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1.1 Data Types

Data in Scheme is divided into

1. atoms
2. lists

1.1.1 atoms

Atoms can be

- integers: 3, 0
- real numbers: 3.14
- symbols: ’x, ’abc
  - unquoted, these are variables

Special symbols:

- #t, #f are booleans

1.1.2 lists

General form of compound data is the *list*:

$$\langle list \rangle ::= \texttt{(} \{ \langle item \rangle \} \texttt{)}$$

$$\langle item \rangle ::= \langle atom \rangle \mid \langle list \rangle$$

Examples:

```
()    (1)    ('a 'b 'c)    ('a ('a 'b 'c) 'c)
(((('a))) 'b 'c)
```

- Although called "lists", these are actually trees.
- Also called s-expressions

Scheme is weakly typed. A list can mix different types of data:

```
('a 123 ('b 2.05))
```

1.2 Expressions

1. General Form
2. Quoting
3. List Ops
4. Conditionals

1.2.1 General Form

General form of code is the parenthesized prefix expression:

$$\langle expr \rangle ::= \texttt{(}\ \langle operator \rangle\ \{\langle item \rangle\}\ \texttt{)}$$

$$\langle operator \rangle ::= \langle item \rangle$$

Examples:

```
(+ 5 3)
(* 2 (+ 5 3))
```

Note that the forms for data and code are actually the same.

- Easy to write Scheme programs that build and execute other scheme programs.
- Simple syntax
- Only one kind of compound data

1.2.2 Quoting

Suppose we want the data "$5 + 3$" instead of the value 8.

To indicate that we want the $+$ treated as an atom rather than as an operator, quote the list:

```
(quote (+ 5 3))
```

or

```
'(+ 5 3)
```

1.2.3 List Ops

Lists are manipulated with three basic operators:

- `cons`
- `car`
- `cdr`

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cons

`cons` prepends an item onto a list

- `(cons 1 (2 3))` produces `(1 2 3)`
- `(cons (1) (2 3))` produces `((1) 2 3)`

---

car

`car` extracts the first element from a list

- `(car (1 2 3)` is `1`
- `(car ((1 2) (3 4))` is `(1 2)`

---

cdr

`cdr` returns the list of all except the first element

- `(cdr (1 2 3))` is `(2 3)`
- `(cdr ((1 2) (3 4))` is `((3 4))`

`cons`, `car`, and `cdr` correspond to SML's `::`, `hd`, and `tl`.

Because expressions like `(car (cdr (car ...)))` are common, they can be abbreviated:

- `(cadr L)` stands for `(car (cdr (L)))`
- `(cddr L)` stands for `(cdr (cdr (L)))`
- `(caddr L)` stands for `(car (cdr (cdr (L))))`

etc.

---

null?

`null?` tests a list to see if it is empty.

- `(null? ())` is `#t`
- `(null? (1 2))` is `#f`
- `(null? ())` is `#f`

1.2.4 Conditionals

`(if P E1 E2)` is the Scheme equivalent to SML's `if-then-else`

- `(if (null? L) () (cdr L))`

A more general form of conditional is
`(cond (P1 E1) (P2 E2) ...(else En))`

- The predicates P_i are evaluated, one after another, until one is not `#f`.
- Then the corresponding E_i is returned.

Test Operators

- `(null? L)` Is L empty?
- `(pair? X)` Is X a list (a cons pair)?
- `(atom? X)` Is X an atom?
- `(number? X)` Is X a number?
- `(symbol? X)` Is X a symbol?
- `(equal? L M)` Are L and M equal? (deep)
- `(eq? L M)` Are L and M equal? (shallow)
- `(< X Y)` Is number X < number Y?

1.3 Functions

Functions are declared via `define`:
`(define (name ⟨formals⟩) ⟨expr⟩)`

```
(define (abs x)
  (if (> x 0) x (- x)))
```

An alternate, and perhaps more interesting form, is:
`(define name ⟨function-value⟩)`
`(define pi 3.14159)`

Functions are 1st Class Objects

More generally, function values are written as **lambda expressions**:

- `(lambda (⟨formals⟩) ⟨expr⟩)`

```
(define abs
  (lambda (x) (if (> x 0) x (- x))))
```

1.4 Lexical Scope

Like SML, we can bind names to constant values in a limited scope:
`(let ((x1 E1) (x2 E2) ...) E)`

```
(define a 2)
(define b 3)
(let ((a 4) (d 2)) (+ a b d))
(let ((a 4) (c (+ a b))) c)
```

What are the values of the let expressions?

1.5 Programming in Scheme

Start with a simple list manipulator:
`append` should join two lists.
`(append (1 2) (3 (4)))` should return
`(1 2 3 (4))`

- Note that `cons` joins an item and a list:
  - `(cons (1 2) (3 (4)))` returns
    `((1 2) 3 (4))`

```
(define (append x y)
  (cond ((null? x) y)
        (else (cons
                (car x)
                (append (cdr x) y)
              )
        )
  )
)
```

Functors

As in SML, much of the power of the language comes from the use of higher-order functions.

- `(map f L)` applies `f` to each element of `L`, collecting the result into a list.

  `(map abs '(2 -4 -7))` returns `(2 4 7)`

Implementing map

Could be defined as

```
(define (map f L)
  (if (null? L)
      '()
     (cons (f (car L))
           (map f (cdr L)))
  ))
```

- but `map` is actually predefined in Scheme
- Predefined `map` can apply to functions of different arity

  `(map + '(2 -4 -7) (1 2 3))` returns

  `(3 -2 -4)`

Another interesting h.o.f. is `reduce`

- `(reduce f x (`v_1 v_2 `...`v_n`))` computes

  `(f x (f `v_1` (...(f `v_{n-1} v_n`)...)))`

- For example, we can define a summation function Σ_i as

```
(define (sumAll x)
   (reduce + 0 x))
```

`reduce` is implemented as

```
(define (reduceX f v)
  (cond ((null? (cdr v)) (car v))
        (else (f (car v)
                 (reduce f (cdr v))
              ))
  ))
(define (reduce f x v)
  (cond ((null? v) x)
  (else (f x (reduceX f v)))))
```

A vector dot product is defined as

$$\bar{x} \cdot \bar{y} = \Sigma_i x_i * y_i$$

Can you use `sumAll`, `reduce`, and/or `map` to produce a dot produce function?

Association Lists

A common idiom in Scheme is the **association list**, or **a-list**

- a list of pairs, which map keys to values
- first element of each pair is usually a symbol

```
(define people
  (let ((edv '((name "Ed") (id 123)))
        (suev '((name "Sue") (id 278)))
        (billv '((name "Bill") (id 380)))
       ))
   `((ed, edv) (sue, suev)
     (bill, billv))))

(define project1 '((manager ed)
                   (staff (sue bill))))
```

`(assoc x A)` extracts the (first) pair keyed by `x` in the a-list `A`.

```
(define (manager project)
   (cadr (assoc 'manager project)))
```

```
(define (managerName project)
   (cadr
    (assoc 'name
          (cadr
           (assoc (manager project)
                  people
                  )
           )
          )))
```

---

2 Implementing LISP

LISP was originally envisioned as a LLL to implement a a List processing HLL.

It offers some interesting insights into implementation of FP.

1. Implementing Lists
2. Garbage Collection

2.1 Implementing Lists

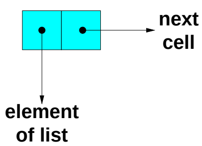
In LISP/Scheme, typically two separate memory pools

- storage for atoms

  - contains no pointers to other objects
  - may be subdivided by kind/size of atom
- storage for lists

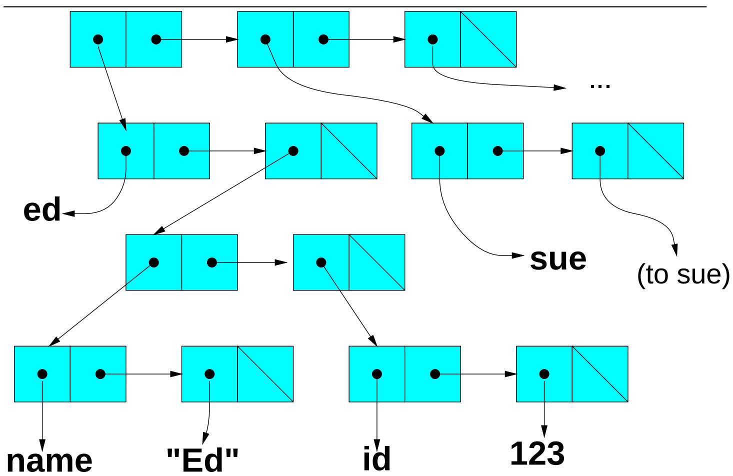
2.1.1 List Cells

A list is represented as a collection of cells:



- `(car L)` retrieves the pointer from the first part of the cell.
- `(cdr L)` retrieves the pointer from the second part of the cell.
- `(cons H L)` allocates a new cell, placing `H` and `L` in the two parts of the cell.

```
(let ((edv '((name "Ed") (id 123)))
      (suev '((name "Sue") (id 278)))
      (billv '((name "Bill") (id 380)))
     )
  `((ed ,edv) (sue ,suev) (bill ,billv))
```



Association lists are used heavily in the implementation of LISP/Scheme.

- A special a-list, called the **environment**, contains the current list of bound variable names, associated with their values.
- Whenever the intepreter encounters a variable name, it evaluates it as `(assoc name Environment)`.

- A binding statement like `(define ...` or `let ((name1 val1) ...(nameN valN) exp)` simply adds (name,value) pairs to the front of the envionment.

2.2 Garbage Collection

- FPL's use rely on shared data structures to make constructive manipulation efficient.
- Their implementations therefore make heavy use of pointers.
- Automatic storage management (garbage collection) is essential.

Some non-FP languages (e.g., Java, Modula 3) use automatic garbage collection as well.

- Often these languages feature reference semantics.
- Such languages usually do not have a `delete` command, so both garbage and dangling ponters are eliminated.

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3 Functional Programming Influences on C++

The influence of the functional style can be seen in the new standard C++ library, which is filled with higher-order functions:

```
list<Student> cs355Roster;
  ⋮
void printName(const Student& s) {
  cout << s.name() << endl;
}
  ⋮
void printRoster() {
  list<Student>::iterator start
     = cs355Roster.begin();
  list<Student>::iterator stop
     = cs355Roster.end();
  for_each (start, stop, printName);
}
```

```
list<Student> univ;
  ⋮
Student updateGPA(Student s) {
  Grades g = thisSemester.grades(s);
  s.gpa = computeGPA(s.gpa, s.hours, g);
  return s;
}
```

```
void reportCards() {
  list<Student>::iterator start
     = univ.begin();
  list<Student>::iterator stop
     = univ.end();
  transform (start, stop,
             start, updateGPA);
  ⋮
}
```

```
bool honors(const Student& s) {
  return s.gpa() > 3.5;
}
  ⋮
void selectHonors() {
  list<Student>::iterator start
     = univ.begin();
  list<Student>::iterator stop
     = univ.end();
  list<Student>::iterator toBeRemoved;
  toBeRemoved =
     remove_if (start, stop, honors);
  univ.erase (toBeRemoved, stop);
}
```

Unfortunately this removes the honors students instead of selecting them.

```
bool honors(const Student& s) {
  return s.gpa() > 3.5;
}
  ⋮
void selectHonors() {
  list<Student>::iterator start
     = univ.begin();
  list<Student>::iterator stop
     = univ.end();
  list<Student>::iterator toBeRemoved;
  toBeRemoved =
     remove_if (start, stop,
                not1(honors));
  univ.erase (toBeRemoved, stop);
}
```

Note how `not1` is used to generate a new function from an old one.

Objects can also simulate functions, and have the advantage of being fully 1st-class.

```
struct GPASelector
  public unary_function<Student, bool>
{
  typedef double argument_type;
  double limit;
  GPASelector (double lim)
    {limit = lim;}

  bool operator() (const Student& s) {
     return s.gpa() > limit;
  }
};
```

```
void selectHonors(double gpa) {
  list<Student>::iterator start
     = univ.begin();
  list<Student>::iterator stop
     = univ.end();
  list<Student>::iterator toBeRemoved;

  GPASelector honors (gpa);
  toBeRemoved =
    remove_if (start, stop,
               not1(honors));
  univ.erase (toBeRemoved, stop);
}
```

`not1` is a true h.o.f. It takes a function as a parameter and produces a new function (actually a simulating object).

```
template <class Predicate>
class unary_negate
 : public unary_function<
    Predicate::argument_name, bool>
{
  Predicate pred;
public:
  unary_negate (Predicate p): pred(p)
  {}
  bool operator()
    (typename Predicate::argument_name x)
  { return !pred(x); }
};

template <class Predicate>
```

```
UnaryFunction not1 (Predicate p) {
  return unary_negate<Predicate>(p);
};
```