The definition of a full binary tree is:

Basis step:
The simplest full binary tree is a single node, r.

Recursive step:
If T₁ and T₂ are full binary trees, another full binary tree can be constructed from
T₁, T₂, and a new root node r by adding an edge from r to the root of T₁ and
adding an edge from r to the root of T₂.

Example:
The binary tree shown on the left has root node r.
It has 5 nodes and a height of 2.

4. Use structural induction and the definition of a full binary tree given above to prove that
n(T) ≥ 2h(T) + 1, where T is a full binary tree, n(T) equals the number of nodes in T, and h(T)
equals the height of T.

3 pts.

Basis step: T = one node
\[ h(T) = 0 \]
\[ n(T) = 1 \]
\[ 1 \geq 2(0) + 1 = 1 \]
\[ \therefore n(T) = 2^h(T) + 1 \]

Inductive (recursive) step:
Assume T₁ and T₂ are full binary trees, and that
\[ n(T₁) = 2^h(T₁) + 1 \] and \[ n(T₂) = 2^h(T₂) + 1 \]

Prove that
\[ n(T) = 2^h(T) + 1 \] where T is a new full
binary tree formed by a new root node r,
with edges to T₁ and T₂.