1. Which of the following statements are true and which are false? [5 points each]
   (a) \((L^*)^+ = L^*\) for an arbitrary language \(L\).
   (b) \(|L_1L_2| = |L_1| |L_2|\) for languages \(L_1\) and \(L_2\).
   (c) \((0^*1 + 1^*0^* + (10)^*1)^* = (0^*1^*)^*\)
   (d) The string 101010 is not in the language represented by \((1 + 01^*0)^*\).
   (e) Every string of \(L^+\) is can be expressed as the concatenation of some strings of \(L\).
   (f) \(LL^* = L^*L\)

2. Prove by General (Structural) Induction that for arbitrary languages \(L_1\) and \(L_2\), if \(L_1 \subseteq L_2\), then \(L_1^* \subseteq L_2^*\) [20]
3 (a) Find a string of minimum length in \{0, 1\} that is NOT in the language corresponding to the regular expression \((1^* + 01^*)^*01^*\). [4]

(b) Find a string of minimum length in \{0, 1\} that is IN the language corresponding to the regular expression of (a). [4]

4. Define the language **RECURSIVELY** which is represented by each of the following regular expressions:

(a) \(1^*0\) [7]

(b) \((1 + 010)^*01^*\) [7]
5. Simplify the following regular expressions:
   
   (a) \((0^*0 + 1^*0 + 1^*)^*\)  

   (b) \(0(0^*0 + 0^*) + 0^*\)

6. Find a regular expression for each of the following languages over the alphabet \(\{0, 1\}\):

   (a) The set of strings with an even number of 0’s.

   (b) The language \(L\) defined recursively as follows:

   Basis Clause: \(0 \in L\)
   
   Inductive Clause: If \(x \in L\) then \(1x, 010x, x1 \in L\)
   
   Extremal Clause: Nothing is in \(L\) unless it is obtained from the above two clauses.