

# Online Deep Learning from Doubly-Streaming Data

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## ABSTRACT

This paper investigates a new online learning problem with doubly-streaming data, where the data streams are described by feature spaces that constantly evolve, with new features emerging and old features fading away. A plausible idea to deal with such data streams is to establish a relationship between the old and new feature spaces, so that an online learner can leverage the knowledge learned from the old features to better the learning performance on the new features. Unfortunately, this idea does not scale up to high-dimensional multimedia data with complex feature interplay, which suffers a tradeoff between *onlineness*, which biases shallow learners, and *expressiveness*, which requires deep models. Motivated by this, we propose a novel OLD<sup>3</sup>S paradigm, where a shared latent subspace is discovered to summarize information from the old and new feature spaces, building an intermediate feature mapping relationship. A key trait of OLD<sup>3</sup>S is to treat the *model capacity* as a learnable semantics, aiming to yield optimal model depth and parameters *jointly* in accordance with the complexity and non-linearity of the input data streams in an online fashion. Empirical studies substantiate the viability and effectiveness of our proposed approach. The code is available online at <https://github.com/X1aoLian/OLD3S>.

## CCS CONCEPTS

• **Computing methodologies** → **Online learning settings**; *Neural networks*; • **Information systems** → Online analytical processing engines; • **Theory of computation** → Streaming models.

## KEYWORDS

Online Learning, Streaming Features, Hedge Backpropagation

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## 1 INTRODUCTION

Machine learning has become a fundamental building block in many cyber infrastructures, provides an automated hence scalable apparatus to analyze the high-dimensional data streams (e.g., images, texts, videos) pervading all corners of the Internet [22, 23, 38]. Examples include multimedia retrieval [63, 64], online speech analytics [16, 18], recommender systems [11, 13, 66, 68, 69], to just name a few. Generally speaking, wherever it is infeasible to inspect and process the data growing in an increasingly unmanageable volume with manpower, machine learning prevails.

Despite their fashionability, a prominent drawback shared by most existing machine learning methods is their limited *generalization capability* [50]. As a matter of fact, machine learning models usually do well in practice only if the data arriving in future tend to follow a nearly identical distribution as the data they were trained on [8, 39]. This so-called *i.i.d.* assumption inevitably limits the model expressiveness to our society that constantly evolves.

To aid the situation, a new learning paradigm termed *online learning from doubly-streaming data* has emerged with both algorithmic designs [4, 25–32, 34, 77] and domain applications [9, 45, 53, 71, 74]. Its key idea is to generalize learning models in two spaces. First, the *sample space*, where the data instances are generated ceaselessly, requiring to train learners on-the-fly, making real-time predictions as the data arrive. As such, if the patterns underlying data changed, an online learner can be updated instantly to adapt to the shift, thereby retaining its accuracy performance over time [21, 47, 73].

Second, the *feature space*, where sets of features describing the arriving data samples evolve, with new features emerge and old features stop to be generated. To wit, a smart manufacturing pipeline may employ a set of sensing techniques to detect unqualified products [36], where each sensor coheres to a feature. The feature space evolves, when the old sensors wear out and a batch of new sensors are deployed [31]. Tangibly, as the new and old sensors (i.e., features) often differ in terms of amount, version, metric, and positions, a new classifier needs to be initialized. Yet, this new classifier

may stay *weak* and error-prone before the training samples carrying these new features grows to a sufficiently large volume. Meanwhile, the old classifier becomes unusable with the unobserved features, leading to substantial waste of the data collection and training effort. A relationship between the pre-and-post evolving feature spaces must be established, so that the old features can be *reconstructed* from the new ones. Online learners can thus harvest the information embedded in the old classifier to aid the weak new classifier, enjoying a boosted learning performance [26, 28–30].

Unfortunately, all existing studies suffer from a tradeoff between *onlineness* and *expressiveness*. Specifically, on the one hand, shallow learners (e.g., generalized linear models [81], Hoeffding trees [59]) possess a faster online convergence rate, thanks to their simple model structures with a small number of trainable parameters [61]. However, due to their limited learning capacity, they usually end up with inferior performance when dealing with high-dimensional media streams, of which the feature interplay is often complex.

On the other hand, deep learners (e.g., neural networks [43, 57], deep forests [56, 80]) enjoy a low-dimensional hidden representation to build accurate predictive models on complex raw inputs. Yet, their large number of parameters residing in the entangled model structures invites stochastic updates, leading to a very slow convergence rate. In an online learning context, more error predictions tend to be made before the learners converge to an equilibrium. These additional errors are recognized as *regrets*, where the slower the convergence rate, the larger the learner regrets in a hindsight.

Motivated by this tradeoff, this paper mainly explores one question: *How can we build an online learner that joins the two merits, namely, 1) converges as fast as shallow models to minimize the online regrets and 2) learns latent representations as expressive as deep models from high-dimensional inputs with complex feature relationships.*

Our affirmative answer provides a novel learning paradigm, termed *Online Learning Deep models from Data of Double Streams* (OLD<sup>3</sup>S). Our key idea is to train an online learner that *automatically* adjusts its learning capacity in accordance with the complexities and temporal variation patterns of input data stream. Specifically, OLD<sup>3</sup>S is with an over-complete neural architecture [44, 57, 65] and starts from using its shallow layers, approximating a simple classifier to attain fast convergence at initial rounds. Over time, the deeper layers are gradually mobilized, as more samples streaming in requires 1) a highly capable classifier that can learn expressive latent representations and 2) a precise delineation of complex feature interplay. Knowledge reuse is enabled in both i) the shallow-to-deep model switch via representations sharing and ii) the pre-and-post evolving feature spaces via reconstructive mapping and ensemble learning [79]. This benefits our approach by expediting the convergence in a temporal continuum, so as to maximize its online efficiency and efficacy when learning from doubly-streaming data. **Specific contributions** of this paper are summarized as follows:

- i) This is the first study to explore the doubly-streaming data mining problem in an online deep learning context, where the high-dimensional data streams with feature space evolution tend to incur a tradeoff between convergence rate and learning capacity. The technical challenges are manifested from empirical evidence in Section 3.
- ii) A novel OLD<sup>3</sup>S approach is proposed to tackle the problem, where a modeling architecture with its depth *learned* from data is devised to adapt to minimize the online classification regrets and precisely approximate the feature-wise relationship on-the-fly. Details are in Section 4.
- iii) Real-world high-dimensional datasets covering domains of machine translation and image classification are employed to benchmark our approach. Results suggest the viability and effectiveness of our proposal, documented in Section 5.

## 2 RELATED WORK

**Online Learning with Doubly-Streaming Data.** Online learning algorithms were devised for data stream processing [2, 60], where the reality of learning is in an on-the-fly setting hence lifts the memory constraint for data analysis at scale. In addition to allowing data to grow in terms of *volume*, in an orthogonal setting, hoping the *features* describing input data to stay strictly unchanging is unrealistic over long time spans. As a response, the pioneering studies [5, 33, 67, 75, 76] explored a setting of *incremental* feature learning, allow the arriving data instances to carry different sets of features yet later instances are assumed to include monotonically more features than the earlier ones. Subsequent works that strive to learn *evolving* feature spaces [4, 27–32, 34, 77] further relaxed the monotonicity constraint on the feature dynamics, enable effective learning when later instances stop carrying old features that appeared theretofore. A key technique shared by these methods is to establish a mapping relationship between the old and new feature spaces. As such, once the old features fade away, their information can be reconstructed via the mapping, aiding the weak learner trained on insufficiently few instances carrying new features, join to make highly accurate predictions.

Despite their effectiveness in various settings, these methods all prescribe a *linear* model to fit the mapping, which is unfortunately not capable to deal with complex real data, e.g., images in an evolving spectrum domain, documents written in different languages. We are aware of a very recent work [26] that does not use linear but copula model to fit a non-linear mapping with statistical guarantees. However, this work requires to deem each feature as a copula component, and hence cannot scale up to a high-dimensional space (e.g., images or natural languages). Our proposed OLD<sup>3</sup>S approach does not suffer this restriction by discovering a latent feature space in which the original data dimension is largely condensed, thereby being generalizable to a wider range of real applications.

**Deep Learning with Adaptive Capacity.** Neural networks have emerged for several decades to approximate underlying functions with arbitrary complexity [12, 48, 49]. However, their universal approximation capability is grounded on an assumption of an infinitely wide hidden layer, which cannot be satisfied in practical modeling. The advent of Deep Neural Networks sidestepped this issue by imposing a *hierarchical* representation learning procedure [3, 46, 52], trading in width for depth, so as to fit complex decision functions underlying data. However, this hierarchical design introduces *over-parameterization*, where the large number of learnable parameters request massive rounds of training iterations over huge datasets to converge. Online decision-making using deep learning thus becomes seemingly impossible.

A key question to solve the challenge is how to choose the network depth (representing the entire model capacity) in accordance with the underlying function in an adaptive, automated, and data-agnostic fashion. Huang et al. [35] firstly theorized and implemented the concept of stochastic depth, a training procedure that trains shallow networks and tests with deep networks, randomly dropping a subset of layers to quickly identify key layers. A method of deducing which layers can be trimmed is therefore needed. Larsson et al. [42] later identified a strategy to construct deep networks structured as fractals. This confers the ability to regularize co-adaptation of subpaths, effectively allowing for the isolation of high performing layers within a larger architecture. We can now judge values of groups of layers, making a delineation of value more concrete. Sahoo et al. [57] and He et al. [25] demonstrated a *Hedge Backpropagation* mechanism for online/lifelong deep learning, where the model depth is deemed as a trainable semantic metric, jointly with the layer parameters to decide the function complexity learned from data streams in a dynamic way.

Unfortunately, all these deep methods fail to take the feature space evolution into account, a factor that can largely affect the non-linearity of the resultant learning function. As a result, they cannot be adapted to learn the doubly-streaming data. To fill the gap, we propose to bring together the two fragmented subfields of online deep learning and doubly-streaming data mining. In particular, we respect that the mapping relationship between the pre-and-post evolving feature spaces can be massively more complex than the previously explored linear models, and must be gauged by a neural approximator that grows its capacity autonomously and adaptively.

### 3 PRELIMINARIES

We formulate the problem in Section 3.1, present the challenges in Section 3.2, and outline the key design ideas in Section 3.3.

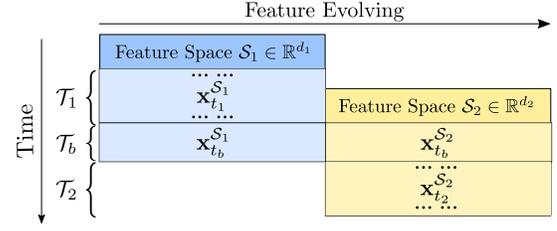
#### 3.1 Problem Statement

Let  $\{(x_t, y_t) \mid t = 1, 2, \dots, T\}$  denote an input sequence, where  $x_t$  is the data instance observed at the  $t$ -th round, accompanied with a ground truth label  $y_t \in \{1, 2, \dots, C\}$ . It is worth noting that our online classification problem is formulated in a multi-class regime with in total  $C$  class options, which excels our competitors [27, 29, 31, 76] that focus on binary classification only.

In the context of doubly-streaming data, we follow the pioneer [31], consider the set of features describing  $x_t$  to evolve with the following regularity, illustrated in Figure 1. Specifically,

- In the span  $t_1 \in \mathcal{T}_1 := \{1, \dots, T_1\}$ , the classifier observes the instances described by the feature space  $\mathcal{S}_1$ , i.e.,  $x_{t_1} \in \mathcal{S}_1 \subseteq \mathbb{R}^{d_1}$ , each of which is a  $d_1$ -dimensional vector.
- In the span  $t_b \in \mathcal{T}_b := \{T_1 + 1, \dots, T_b\}$ , the feature space evolves, and the classifier observes the two feature spaces  $\mathcal{S}_1$  and  $\mathcal{S}_2$  simultaneously, with each data instance being  $x_{t_b} = [x_{t_b}^{\mathcal{S}_1}, x_{t_b}^{\mathcal{S}_2}]^\top \in \mathcal{S}_1 \times \mathcal{S}_2 \subseteq \mathbb{R}^{d_1+d_2}$ .
- In the span  $t_2 \in \mathcal{T}_2 := \{T_b + 1, \dots, T_2\}$ , the old space  $\mathcal{S}_1$  opts out, and the classifier observes the evolved  $\mathcal{S}_2$  only. Each data instance is  $x_{t_2} \in \mathcal{S}_2 \subseteq \mathbb{R}^{d_2}$ , a  $d_2$ -dimensional vector.

Note, such feature space evolving from  $\mathcal{S}_1$  to  $\mathcal{S}_2$  can be easily generalized to infinitely more spaces (e.g.,  $\mathcal{S}_2$  to  $\mathcal{S}_3$ , then  $\mathcal{S}_3$  to



**Figure 1: Illustration of doubly-streaming data. Only in a very short timespan  $|\mathcal{T}_b| \ll |\mathcal{T}_1|$  or  $|\mathcal{T}_2|$ , the samples are described by the two feature spaces concurrently.**

$\mathcal{S}_4$ ), wherein all spaces can have disparate properties and semantic meanings and the mapping relationship between any two spaces can be arbitrarily complex. Such dynamism in the doubly-streaming data makes a prefix of learner capacity close to impossible.

At any time instant  $t = \{t_1, t_b, t_2\}$ , the learner  $f_t$  observes  $x_t$  and makes a prediction  $\hat{y}_t = f_t(x_t)$ . The true label  $y_t$  is revealed thereafter, and an instantaneous loss indicating the discrepancy between  $y_t$  and  $\hat{y}_t$  is suffered. Based on the loss information, the learner updates to  $f_{t+1}$  using first-order [15, 51, 54, 62] or second-order [1, 24, 58, 72] oracles, getting prepared for the next round. Our goal is to find a sequence of classifiers  $\{f_1, \dots, f_T\}$  that minimize the *empirical risk* [10] over  $T$  rounds:  $\min_{f_1, \dots, f_T} \frac{1}{T} \sum_{t=1}^T \ell(y_t, f_t(x_t))$ , where  $\ell(\cdot, \cdot)$  denotes the loss metric and often is prescribed as convex in its argument such as square loss or logistic loss.

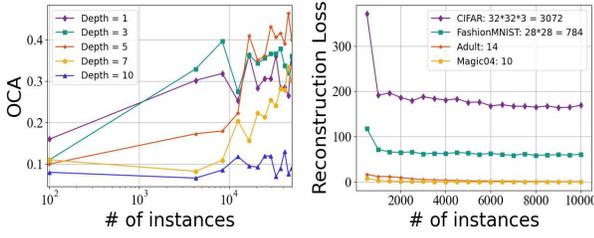
#### 3.2 Opportunities and Challenges

A common practice to enable online learning with doubly-streaming data is to leverage the overlapping timespan  $\mathcal{T}_b$  to learn a *reconstructive mapping*  $\phi: \mathcal{S}_2 \mapsto \mathcal{S}_1$ , such that once the features of  $\mathcal{S}_1$  are not observed during  $\mathcal{T}_2$ , their information can be reproduced, allowing the learner to harvest the old information learned during the  $\mathcal{T}_1$  time period for better performance [26, 27, 29, 31].

Let  $f_t = \{f_t^{\mathcal{S}_1}, f_t^{\mathcal{S}_2}\}$  denote the learner with  $f_t^{\mathcal{S}_1}$  and  $f_t^{\mathcal{S}_2}$  being the two classifiers corresponding to the  $\mathcal{S}_1$  and  $\mathcal{S}_2$  feature spaces, respectively. During  $\mathcal{T}_2$ , instead of predicting the observed instance as  $f_t^{\mathcal{S}_2}(x_t)$ , the learner exploits the unobserved information from  $\mathcal{S}_1$  to make prediction as:  $f_t(x_t) = \lambda_1 \cdot f_t^{\mathcal{S}_1}(\tilde{x}_t) + \lambda_2 \cdot f_t^{\mathcal{S}_2}(x_t)$ , with  $\tilde{x}_t = \phi(x_t) \in \mathcal{S}_1$  being the reconstructed data vector in the  $\mathcal{S}_1$  space. With delicately tailored ensemble parameters  $\lambda_1$  and  $\lambda_2$ , this reconstruction-based learning method enjoys a provably better prediction performance than using the classifier  $f_t^{\mathcal{S}_2}$  only.

Unfortunately, this method does not scale up to cope with real-world media data streams because of two challenges as follows.

**Challenge I – Train Deep Models On-The-Fly.** The real-world media data carrying non-linear patterns often request deep learners (e.g., neural network models) for effective processing. However, the large number of trainable parameters and complex model architectures tend to make deep learners data-hungry and converge slowly. In an online learning context, as each instance requiring immediate prediction is presented only once, the deep learners tend to *regret* [10], making substantial errors before converging to equilibria. To verify this, a simple example reduced from the CIFAR



**Figure 2: Two challenges underlie the OLD<sup>3</sup>S problem. Left: The deeper the learning model, the slower the convergence rate. Right: The higher the data dimensionality, the more inferior the feature relationship captured by linear mappings.**

experiment is illustrated in the left panel of Figure 2, where neural networks with various depths are trained in one-pass.

This example suggests that, as the model depth goes deeper, the learner suffers from a flatter convergence rate. Although such deep learners can end up with high online classification accuracy (OCA), they constantly underperform shallower models before given sufficient instances, thereby regretting largely. Notably, a learner with an improperly ultra-deep architecture (*cf.* depth = 10) may even fail to converge in an online setting. The reason can be possibly attributed to the diminishing feature reuse [35, 42] where the semantic meanings of raw inputs tend to be *washed out* by the layer-by-layer feedforward with massive randomly initialized parameters; No expressive representations can be learned online.

**Challenge II – Learn Complex Reconstructive Mapping in Short Overlapping Timespans.** In practice, an overlapping phase  $\mathcal{T}_b$  in which the two feature spaces  $\mathcal{S}_1$  and  $\mathcal{S}_2$  coexist is very short. Revisit the smart manufacturing example, where we can construct  $\mathcal{T}_b$  by pre-deploying a batch of new sensors before the old sensors expiring their lifespans – a too long  $\mathcal{T}_b$  is economically not affordable. This constraint blocks several seemingly plausible methods, *e.g.*, online transfer learning [70, 78], domain adaptation [37, 55], to work well, as they all require a sufficiently long overlapping phase to *align* the features pre and post evolution.

Prior studies [27, 29, 31] have advocated deducing *linear* functions to approximate the mapping relationship  $\phi$  between the old and new features in a short  $\mathcal{T}_b$ , with the objective formulated as:  $\min_{\phi} \sum_{t_b=T_1+1}^{T_b} \left\| \phi(x_{t_b}^{\mathcal{S}_2}) - x_{t_b}^{\mathcal{S}_1} \right\|_2^2$  where  $\phi(\cdot) = W^T \cdot$ . Unfortunately, this linear reconstructive mapping  $\phi$  cannot work for media data streams with nonlinear feature interplay. An empirical evidence is presented in the right panel of Figure 2, in which we observe that, the higher the data dimensionality, the more complex the mapping relationship between two feature spaces, and hence the larger the reconstruction loss that a linear mapping suffers.

### 3.3 Our Thoughts

To overcome the two challenges, our key idea is to discover a set of *shared latent* features that summarize information from the pre- and post-evolving feature spaces  $\mathcal{S}_1$  and  $\mathcal{S}_2$ . Compared with learning the mapping  $\phi : \mathcal{S}_2 \mapsto \mathcal{S}_1$  directly in the short  $\mathcal{T}_b$ , our idea can exploit the long  $\mathcal{T}_1$  timespan to learn a latent feature subspace from  $\mathcal{S}_1$  independently at first, and then align it with that from  $\mathcal{S}_2$  to expedite learning efficiency. Specifically, we employ variational

inference [6] to model the underlying distribution of  $\mathcal{S}_1$  stream as:

$$Q(z_{t_1}^{\mathcal{S}_1} | x_{t_1}, t_1 = 1, \dots, T_1) = \prod_{i=1}^z \mathcal{N}(z_i^{\mathcal{S}_1} | \mu_i^{\mathcal{S}_1}, (\sigma_i^{\mathcal{S}_1})^2), \quad (1)$$

where a variational code  $z^{\mathcal{S}_1} \in \mathbb{R}^z$  is drawn from a multivariate Gaussian that surrogates the data instances streaming from the original feature space  $\mathcal{S}_1$  [40]. Later in the overlapping  $\mathcal{T}_b$  phase, a new variational code  $z^{\mathcal{S}_2} \in \mathbb{R}^z$  is extracted from the  $\mathcal{S}_2$  stream, similar as Eq. (1) and omitted for simplicity. The two surrogate Gaussians that approximate the  $\mathcal{S}_1$  and  $\mathcal{S}_2$  distributions (from which  $z^{\mathcal{S}_1}$  and  $z^{\mathcal{S}_2}$  were drawn) are enforced to be identical, such that they can be deemed as the shared latent subspace that connects the old and new feature spaces. As such, we intermediately reconstruct the  $\mathcal{S}_1$  data representations from the shared surrogate statistics.

To make this process online, we propose a neural architectural design which *learns* the optimal model depth from data streams autonomously, starting from shallow and gradually turning to deep if more complex variational feature mapping relationships are required to be approximated. The more accurate this reconstructive mapping is approximated, the better the learner can leverage the old classifier trained on the  $\mathcal{S}_1$  stream, and hence the higher the online classification accuracy can be obtained by ensembling the old and new classifiers. The details are presented in the next Section 4.

## 4 OUR APPROACH

**Overview.** In a nutshell, our proposed OLD<sup>3</sup>S approach can be conceptually framed in a learning objective as follows.

$$\min_{f_t, \phi} \sum_{\text{HBP}} \left[ \sum_{t_1, t_b} \left( \mathcal{L}_{\text{VI}}(\phi) + \mathcal{L}_{\text{REC}}(\phi) \right) + \sum_{t_b, t_2} \mathcal{L}_{\text{CLF}}(f_t, \phi) \right].$$

In this section, we scrutinize this learning objective in sequence. The variational inference loss  $\mathcal{L}_{\text{VI}}$  and the reconstruction loss  $\mathcal{L}_{\text{REC}}$  together determine how the shared latent subspace is learned, presented in Section 4.1. The classification loss  $\mathcal{L}_{\text{CLF}}$  synthesizes how the old and new classifiers are ensemble to expedite convergence for better prediction performance in Section 4.2. We end this section by elaborating how this minimization problem is realized by an *elastic* neural network model that automatically adjusts its depth in an online, data-driven fashion in Section 4.3.

### 4.1 Variational Latent Subspace Discovery

To discover the latent subspace  $\mathcal{Z}$ , we employ the Variational Auto-Encoder (VAE) [7, 20, 40] to summarize the observed data instances into latent variational codes. As illustrated in Figure 3, two independent VAEs are established, trained by minimizing the loss term:

$$\mathcal{L}_{\text{VI}}^{\{\mathcal{S}_1, \mathcal{S}_2\}} = -\mathbb{E}_{Q(z_t | x_t)} \left[ \log P(x_t | z_t) \right] + \text{KL} \left( Q(z_t | x_t) \parallel P(z_t) \right), \quad (2)$$

where  $t \in \mathcal{T}_1 \cup \mathcal{T}_b$  and  $t \in \mathcal{T}_b$  for the VAEs on  $\mathcal{S}_1$  and  $\mathcal{S}_2$ , respectively.

**Intuition 1.** The physical meanings of minimizing Eq. (2) are as follows. i) Minimizing the first term equates to maximizing the data generation quality, namely, the likelihood that the original data observations can be decoded from the extracted latent codes. Let the tuple (Enc, Dec) denote the encoder and the decoder networks in a VAE, the first term encourages  $x_t \approx \text{Dec}(z_t)$  where  $z_t = \text{Enc}(x_t)$ . ii) The second term gauges the Kullback-Leibler

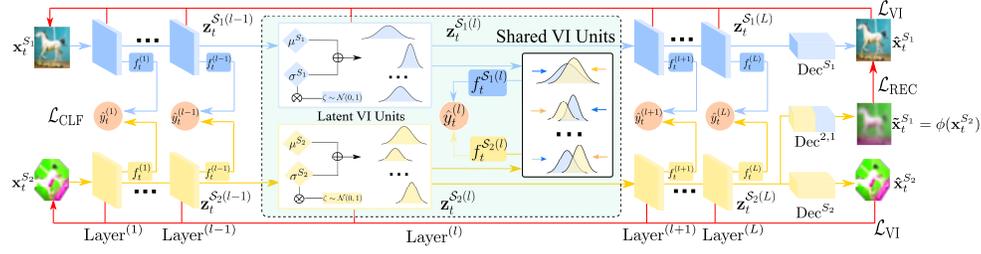


Figure 3: An architectural illustration of our OLD<sup>3</sup>S computational network during the overlapping  $\mathcal{T}_b$  timespan.

(KL)-divergence [7, 41] between the underlying posterior  $Q(z_t | x_t)$  and the latent marginal  $P(z_t) = \mathcal{N}(0, I)$ . With the posterior calculated by Eq. (1), for the extracted latent code  $z_t$ , we denote its  $i$ -th entry with  $z_i$  and is drawn from a Gaussian with mean  $\mu_i$  and variance  $\sigma_i^2$ . To make the variational inference differentiable, reparameterization is employed as  $z_i = \mu_i + \sigma_i \cdot \zeta$  with  $\zeta \sim \mathcal{N}(0, 1)$  being normal noises.

A reconstruction loss is then imposed to regularize the two independently learned latent spaces, from which a shared latent feature subspace is discovered during the overlapping timespan  $\mathcal{T}_b$ :

$$\mathcal{L}_{\text{REC}} = \ell \left[ x_{t_b}^{S_1}, \text{Dec}^{2,1}(z_{t_b}^{S_2}) \right] + \text{KL} \left( Q(z_{t_b}^{S_1} | x_{t_b}^{S_1}) \parallel Q(z_{t_b}^{S_2} | x_{t_b}^{S_2}) \right). \quad (3)$$

**Intuition 2.** In the first term of Eq. (3), a new decoder network  $\text{Dec}^{2,1}(\cdot)$  which takes in the latent code from  $\mathcal{S}_2$  to reconstruct the data of  $\mathcal{S}_1$  approximates our desired reconstructive mapping  $\phi$ . The second term gauges the KL-divergence between the posteriors that were independently drawn from different variational distributions – the surrogate Gaussians to have similar probability densities, as conceptually illustrated in the middle panel of Figure 3. We note that this term is asymmetric, where the variational density of  $\mathcal{S}_2$  is required to resemble that of  $\mathcal{S}_1$  but not the opposite. This makes an intuitive sense as the variational distributions of  $\mathcal{S}_1$  have been learned from  $\mathcal{T}_1$  over a long time horizon, which is more likely to yield an accurate approximation of the underlying data distribution than that from a much shorter  $\mathcal{T}_b$  only.

The two losses in Eqs.(2) and (3) together discover the shared latent feature subspace  $\mathcal{Z}$ . In the subsequent  $\mathcal{T}_2$  timespan in which only the  $\mathcal{S}_2$  space can be observed, an arriving instance  $x_{t_2}$  is embedded into  $\mathcal{Z}$  by its corresponding VAE as  $z_{t_2} = \text{Enc}(x_{t_2})$ , from which a reconstructed data representation of the  $\mathcal{S}_1$  space is decoded, i.e.,  $\tilde{x}_{t_2}^{S_1} := \phi(x_{t_2}) = \text{Dec}^{2,1}(z_{t_2})$ . This VAE architecture lends to learn a complex mapping relationship between  $\mathcal{S}_1$  and  $\mathcal{S}_2$ , hence better suits the high-dimensional media streams in the wild.

## 4.2 Online Prediction with Ensembled Learners

Once the old features of  $\mathcal{S}_1$  vanish, the learner  $f_t$  is not likely to make accurate predictions on the arriving instances by relying on  $\mathcal{S}_2$  solely. Let  $f_t = \{f_t^{S_1}, f_t^{S_2}\}$  denote the learner at the beginning of  $\mathcal{T}_b$  when  $\mathcal{S}_2$  just emerges. As  $\mathcal{T}_b$  is short, the  $f_t^{S_2}$  part of the learner corresponding to the new features of  $\mathcal{S}_2$  have been trained with very few instances hence is not likely to converge. Relying on  $f_t^{S_2}$  to predict the instances in  $\mathcal{T}_2$  would incur substantial regrets.

To aid, we leverage the old  $f_t^{S_1}$  part that has been trained with a much larger number of instances during  $\mathcal{T}_1$ . Thanks to the reconstructive mapping  $\phi$  approximated by the VAEs in Section 4.1, we can realize an online ensemble classification to yield accurate predictions when  $f_t^{S_2}$  is not ready, defined as follows.

$$\mathcal{L}_{\text{CLF}} := \ell(y_t, \hat{y}_t) = - \sum_{c=1}^C y_{t,c} \log(\hat{y}_{t,c}), \quad \forall t \in \mathcal{T}_b \cup \mathcal{T}_2, \quad (4)$$

$$\hat{y}_t = p \cdot f_t^{S_1}(\tilde{x}_t^{S_1}) + (1-p) \cdot f_t^{S_2}(x_t), \quad x_t \in \mathcal{S}_2, \quad (5)$$

where Eq. (4) employs cross-entropy [22] to gauge the multi-class learning loss, with  $y_{t,c}$  and  $\hat{y}_{t,c}$  being the true and predicted probability that  $x_t$  belongs to the  $c$ -th class, respectively.

**Intuition 3.** The idea behind Eq. (5) is to let the ensemble coefficient  $p \in (0, 1)$  decide the impacts of the observed  $x_t$  and its reconstructed version  $\tilde{x}_t^{S_1}$  in making predictions. At the beginning of  $\mathcal{T}_2$  when the feature space just evolved, the old classifier  $f_t^{S_1}$  should be largely helpful with large  $p$ . Over time, the value of  $p$  decays because of two reasons 1) the new classifier  $f_t^{S_2}$  becomes stronger and 2) the old classifier  $f_t^{S_1}$  can be less useful due to the distribution drift. An updating strategy needs to be designed to echo this intuitive process, where the new classifier takes over gradually as the old classifier conveys less discriminative power.

In this work, we update the ensemble coefficient with exponential experts [10], where the empirical risks of using the old and new classifiers to make independent predictions are accumulated as:

$$R_T^{S_1} = \sum_{t=\mathcal{T}_1+1}^{\mathcal{T}_2} \ell(y_t, f_t^{S_1}(\tilde{x}_t^{S_1})), \quad R_T^{S_2} = \sum_{t=\mathcal{T}_1+1}^{\mathcal{T}_2} \ell(y_t, f_t^{S_2}(x_t)). \quad (6)$$

The smaller the cumulative empirical risk is suffered, the better the classifier is, and hence the higher its corresponding coefficient is uplifted exponentially. The updating rule is defined as  $p = e^{-\eta R_T^{S_1}} / (e^{-\eta R_T^{S_1}} + e^{-\eta R_T^{S_2}})$ , where  $\eta$  is a tuned parameter.

## 4.3 Adaptive Model Depth Learning with HBP

With the reconstructive mapping and the ensemble prediction, the information conveyed by the unobserved  $\mathcal{S}_1$  can be reaped to better the learning performance. The remaining problem is how to realize the mapping and the classifiers with models of appropriate depths that are most likely to produce the optimal solutions. Unfortunately, fixing such depths beforehand is impossible without prior knowledge of how the data streams evolve in the sample space (e.g., distribution drift that may require classifiers with various discriminant power to avoid overfitting) and the feature space

(e.g., a diversity of feature mapping relationships requires VAEs with disparate architectures). As it is unrealistic to rely on human experts to provide such knowledge constantly over long timespans, this problem boils down to the desire of a model architecture that can *learn* the best depth from data autonomously.

To this end, we leverage the Hedge Backpropagation (HBP) [25, 57] mechanism to incorporate the model depth as a learnable semantic that shall be determined in a data-driven manner through optimization. Instead of evaluating the loss based on the output from the last network layer only (as most deep learning models do), the main idea of HBP is to evaluate the losses on *all* the intermediate hidden representations yielded from the network layers from shallow to deep. Specifically, given an overcomplete network with  $L$  hidden layers in total, the output of the  $l$ -th encoder layer of the VAE is recursively denoted as  $z_t^{(l)} = \text{Enc}^{(l)}(z_t^{(l-1)})$ , with  $z_t^{(0)} = x_t$ , where  $t \in \mathcal{T}_1 \cup \mathcal{T}_b$  and  $t \in \mathcal{T}_b \cup \mathcal{T}_2$  for the VAEs corresponds to  $\mathcal{S}_1$  and  $\mathcal{S}_2$ , respectively. The objective of HBP is defined as follows.

$$\min_{\{\alpha^{(l)}\}_{l=1}^L} \sum_{l=1}^L \alpha^{(l)} \left[ \sum_{t_1, t_b} (\mathcal{L}_{\text{VI}}^{(l)} + \mathcal{L}_{\text{REC}}^{(l)}) + \sum_{t_b, t_2} \mathcal{L}_{\text{CLF}}^{(l)} \right], \quad (7)$$

where the loss terms  $\mathcal{L}_{\text{VI}}^{(l)}$ ,  $\mathcal{L}_{\text{REC}}^{(l)}$ , and  $\mathcal{L}_{\text{CLF}}^{(l)}$  are evaluated on  $z_t^{(l)}$  at the  $l$ -th layer as shown in Figure 3. In particular, 1) Evaluated by  $\mathcal{L}_{\text{VI}}^{(l)}$  is how well the latent code  $z_t^{(l)}$  can summarize the raw inputs with a surrogate Gaussian via using Eq. (2); For instances of  $\mathcal{S}_1$ , it is evaluated over  $\mathcal{T}_1$  and  $\mathcal{T}_b$  timespans, and for instances of  $\mathcal{S}_2$ , it is evaluated over  $\mathcal{T}_b$  only. 2) Evaluated by  $\mathcal{L}_{\text{REC}}^{(l)}$  is how precisely the reconstructive mapping is learned so that the  $\mathcal{S}_1$  feature space can be reconstructed from the data instances of  $\mathcal{S}_2$  via using Eq. (3); It is only evaluated during the overlapping phase  $\mathcal{T}_b$  where  $\mathcal{S}_1$  and  $\mathcal{S}_2$  coexist. 3) Evaluated by  $\mathcal{L}_{\text{CLF}}^{(l)}$  is how accurately the ensemble of both old and new classifiers can make online predictions via using Eq. (4); It is evaluated during  $\mathcal{T}_b$  and  $\mathcal{T}_2$  as the ensemble prediction is used only if the features of  $\mathcal{S}_2$  become observed.

**Intuition 4.** The crux of HBP lies in finding the equilibrium that minimizes the three loss terms in Eq. (7) into a Pareto optimum. To do this, we update the hedge weight  $\alpha^{(l)}$  that determines the impact of the  $l$ -th layer in a boosting fashion [19]:  $\alpha_{t+1}^{(l)} \leftarrow \text{Norm}(\alpha_t^{(l)} \beta^{\mathcal{L}_{\text{VI+REC+CLF}}^{(l),t}})$ , where  $\beta \in (0, 1)$  is a discounting rate and  $\mathcal{L}_{\text{VI+REC+CLF}}^{(l),t}$  accumulates the three losses in Eq. (7) suffered at the  $t$ -th round. Denoted by  $\text{Norm}(\cdot)$  is a normalization function that reweighs each  $\alpha^{(l)}$  by the sum of all  $L$  layers, ensuring  $\alpha^{(l)} \in (0, 1)$ . The idea is straightforward: the layer of which the output incurs large losses should be penalized and takes a discounted weight in the next round. Otherwise, if a layer is in an *optimal* depth, it approaches the minimizer of Eq. (7) with the incurred losses very small, such that the remaining layers (*i.e.*, those deeper than this hidden layer) cannot identify and learn meaningful gradient directions. Their hedge weights would stay in small values.

## 5 EXPERIMENTS

Empirical results are presented to verify the viability and effectiveness of our OLD<sup>3</sup>S approach. We elaborate the experimental setups in Section 5.1 and extrapolate the results and findings in Section 5.2.

**Table 1: Statistics of the 10 datasets.  $|\mathcal{S}_1|$  and  $|\mathcal{S}_2|$  are the dimensions of the old and new feature spaces, respectively.**

No.	Dataset	# Samples	$ \mathcal{S}_1 $	$ \mathcal{S}_2 $	# Classes
1	magic04	36,119	10	30	2
2	adult	61,559	14	30	2
3	EN-FR	34,758	21,531	24,892	6
4	EN-IT	34,758	21,531	15,506	6
5	EN-SP	34,758	21,531	11,547	6
6	FR-IT	49,648	24,893	15,503	6
7	FR-SP	49,648	24,893	11,547	6
8	CIFAR	95,000	3072	3072	10
9	Fashion	114,000	784	784	10
10	SVHN	139,257	3072	3072	10

### 5.1 Evaluation Setup

**5.1.1 Dataset Preparation.** We benchmark our OLD<sup>3</sup>S approach on 10 real-world datasets covering three domains to verify its versatility. Statistics of the studied datasets are summarized in Table 1.

- **UCI Data Science (No. 1-2):** The two datasets have one feature space  $\mathcal{S}_1$  at first, and we artificially create a new feature space  $\mathcal{S}_2 = \text{sigmoid}(W^\top \mathcal{S}_1)$  with a random Gaussian  $W$  and a nonlinear sigmoid function. The two feature spaces are concatenated as the shape in Figure 1 to simulate the doubly streaming data.

- **Multilingual Text Categorization (No. 3-7):** A set of documents are described by four languages including English (EN), French (FR), Italian (IT), and Spanish (SP). By treating each document as a bag of words (features), the vocabulary of each language can be deemed as a feature space. At each time, a document is presented and our model aims to classify it into one of the six categories. To simulate doubly-streaming, the language describing the documents shifts over time, e.g., EN-FR, where the model learned to classify English documents is soon presented with French documents after a short overlapping  $\mathcal{T}_b$  timespan, requiring to approximate the translation relationship between languages. To exacerbate the non-linearity of the mapping between two languages, we apply the sigmoid function on the  $\mathcal{S}_2$  feature space.

- **Online Image Classification (No. 8-10):** Images are typical media data of high dimensionality and low information density. To simulate doubly-streaming data, we follow the preprocessing steps suggested by [17, 25] to create an evolved space by transforming the original images with various spectral-mapping, shearing, rescaling, and rotating. Images are presented one at a step, and the model needs to learn the complex pixel transformation online.

**5.1.2 Compared Methods.** Three state-of-the-art competitors tailored for processing double-streaming data are employed for comparative study, with their main ideas presented as follows.

- **FOBOS** [14] is a canonic online learning baseline that operates over first-order oracles with a projected subgradient that encourages sparse solutions. To make it work for doubly-streaming data, zeros are padded to the new features and vanished old features.

- **OLSF** [75] is the first study to tackle an incremental feature space, where new features constantly emerging are carried in all subsequent data instances. OLSF updates the online learners in a passive-aggressive fashion, where the learning coefficients of old features are re-weighted to new features only if these new features

convey significant information that changes the decision boundary.

• **FESL** [31] is the pioneer work to deal with doubly-streaming data, which nevertheless employed linear functions to learn classifiers and to approximate a mapping relationship between feature spaces. A comparison with FESL rationalizes our design of adaptive deep learner and variational feature mapping approximator.

**5.1.3 Ablation Variants.** For the ablation study, two variants of our OLD<sup>3</sup>S approach are proposed, named OLD-Linear and OLD-FD. They differ from our original OLD<sup>3</sup>S design by: 1) **OLD-Linear** employed linear mapping to approximate the feature mapping relationship and 2) **OLD-FD** trains a deep neural network with a fixed depth. We craft the two variants to necessitate the designs of a non-linear, VI-based feature mapping approximator and the HBP that allows model depth to be learned from data autonomously.

**5.1.4 Evaluation Metric.** As the traditional classification accuracy is ill-conditioned in online learning, we employ the Online Classification Accuracy (OCA) and Averaged Cumulative Regret (ACR) to measure the performance. Specifically, they are defined:

$$\text{OCA}(f_t) = 1 - \frac{1}{B} \sum_{i=t-B}^t \mathbb{1}[y_i \neq f_t(x_i)], \quad T = |\mathcal{T}_1 \cup \mathcal{T}_b \cup \mathcal{T}_2|$$

$$\text{ACR} = \frac{1}{T} \sum_{t=1}^T \left[ \max_{f^*} \text{OCA}(f^*) - \text{OCA}(f_t) \right].$$

Intuitively, OCA dynamically measures the accuracy of a classifier  $f_t$  the  $t$ -th round, evaluated at the *most recent*  $B$  instances. ACR evaluates how large the online learner regrets comparing to a hindsight optimum  $f^*$  by accumulating the OCA differences between  $f_t$  and  $f^*$  over  $T$  rounds. The smaller the value of ACR, and the better the online classification was performed.

## 5.2 Results and Findings

We present the experimental results in Table 2 and Figure 4, aiming to answer three research questions (Q1 – Q3) as follows.

**Q1.** *How does our OLD<sup>3</sup>S approach compare to the state-of-the-arts?*

From the comparative results presented in Table 2, we make three observations as follows. *First*, our OLD<sup>3</sup>S achieves the best ACR performance. This result rationalizes our proposal of learning deep learners with complex feature relationships, as the competitors mainly relying on linear models manifest inferior performances. *Second*, our OLD<sup>3</sup>S outperforms FOBOS by 69% on average. In addition, FOBOS suffers the largest performance drop in terms of OCA when the old features become unobserved, as shown in Figures 4a, 4b, and 4c. This is because that FOBOS does not correlate the old and new feature spaces thus can be equated to initializing a new learner for the newly emerged features. Our approach excels as we learned the feature correlation to boost the learning performance on the new features, and then enjoys a much smoother learning curve as soon as the feature space evolves.

*Third*, compared to OLSF, our approach wins by 77% on average. The reason can be attributed to that OLSF is tailored for dealing with an incrementally increasing feature space only, and does not possess the mechanism to handle the fading away features. The learned knowledge of the old feature space is hence wasted. Our approach aids the situation by learning a reconstructive mapping

between the two feature spaces, letting the learner enjoy the information conveyed by the old and unobservable features, thereby attaining better ACR and sharper OCA curves along the time horizon.

**Q2.** *How helpful is the deep learner enabled by the VI mapping?*

The comparison among FOBOS, FESL, our OLD<sup>3</sup>S approach and its OLD-Linear variant amounts to the answer. *First*, our OLD<sup>3</sup>S outperforms FESL and OLD-Linear by ratios of 69% and 44% on average, respectively. This performance gap indicates the non-linear mapping relationship between feature spaces must be respected, as FESL and OLD-Linear both employed linear functions to approximate the reconstructive mapping. *Second*, more significant OCA drops are observed from OLD-Linear in Figures 4b, 4c, and 4e. This result suggests that the low-dimensional latent space resulted from the variational encoding does not suffice to simplify the complex feature reconstruction relationships to an extent that they can be approximated by linear functions.

*Third*, we observe that FESL may even underperformed FOBOS in terms of ACR, despite that FESL suffers a smaller performance drop of OCA overtime. This observation advocates that FESL learned the feature relationship at a certain level, but the linearity of the mapping function does suffice to fully capture the complex feature interactions, such that the linear reconstruction of old features is helpful at the beginning of  $\mathcal{T}_2$  (smaller OCA drop) but soon becomes less useful overtime (slower learning rate), and eventually becomes *noises* which negatively affect the prediction accuracy, ending up with inferiority to FOBOS. In other words, it is better to initialize a new learner than trying to reconstruct old features inaccurately with an insufficiently capable linear mapping.

**Q3.** *In which cases does an adaptive learning capacity excel?*

A comparison between our OLD<sup>3</sup>S with the OLD-FD variant answers this question. We observe that 1) OLD<sup>3</sup>S excels and significantly outperforms OLD-FD in six settings 2) OLD<sup>3</sup>S converges faster with steeper OCA curves in all settings. These two observations validate the *tightness* of HBP in the sense that, although OLD-FD may end up with higher OCA with increasingly more arriving data instances (e.g., Figures 4a and 4d), its slower convergence rate incurs larger online prediction errors before the network parameters are readily trained. This necessitates the usage of HBP to expedite the online learning efficiency.

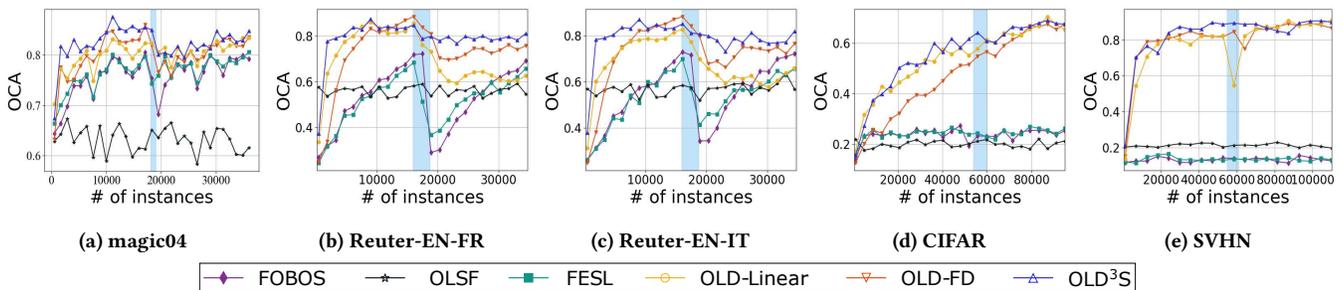
In addition, from Figures 4d and 4e, we observe that OLD-FD learns slower as the learning task becomes more difficult. (The objects in CIFAR impose more complex visual concepts than the street-view numbers in SVHN, where the hindsight optimal OCAs in CIFAR and SVHN are 72.7% and 93.3%, respectively). Our OLD<sup>3</sup>S is invariant to the inherent complexity of the datasets and manifests a fast online learning rate. This finding advocates the adaptive model capacity of our OLD<sup>3</sup>S is generalizable to more learning tasks, without requiring prior knowledge of the underlying distribution or learning complexity of the doubly-streaming data of interest.

## 6 CONCLUSION

This paper proposed a new online learning paradigm, named OLD<sup>3</sup>S, which enables a deep learner to make on-the-fly decisions on data streams with a constantly evolving feature space. The key idea

**Table 2: Comparative results of averaged cumulative regret (ACR  $\pm$  mean variance) benchmarked on 10 datasets, where the lower the value, the better the method performs. The best results are bold. The bullet  $\bullet$  indicates that our OLD<sup>3</sup>S approach outperforms the competitors with a statistical significance supported by the *paired t-tests* at 95% confidence level.**

Dataset	FOBOS	OLSF	FESL	OLD-Linear	OLD-FD	OLD <sup>3</sup> S
magic04	.119 $\pm$ .022 $\bullet$	.335 $\pm$ .021 $\bullet$	.110 $\pm$ .016 $\bullet$	.075 $\pm$ .018	.076 $\pm$ .021	.052 $\pm$ .017
adult	.076 $\pm$ .064	.225 $\pm$ .019 $\bullet$	.067 $\pm$ .044	.055 $\pm$ .017	.068 $\pm$ .018	.049 $\pm$ .019
EN-FR	.326 $\pm$ .064 $\bullet$	.324 $\pm$ .018 $\bullet$	.345 $\pm$ .044 $\bullet$	.168 $\pm$ .030 $\bullet$	.137 $\pm$ .030 $\bullet$	.068 $\pm$ .025
EN-IT	.318 $\pm$ .060 $\bullet$	.314 $\pm$ .019 $\bullet$	.337 $\pm$ .040 $\bullet$	.197 $\pm$ .028 $\bullet$	.143 $\pm$ .033 $\bullet$	.083 $\pm$ .024
EN-SP	.302 $\pm$ .060 $\bullet$	.322 $\pm$ .021 $\bullet$	.335 $\pm$ .037 $\bullet$	.197 $\pm$ .036 $\bullet$	.136 $\pm$ .027 $\bullet$	.077 $\pm$ .024
FR-IT	.278 $\pm$ .047 $\bullet$	.301 $\pm$ .013 $\bullet$	.314 $\pm$ .037 $\bullet$	.195 $\pm$ .031 $\bullet$	.147 $\pm$ .030 $\bullet$	.084 $\pm$ .026
FR-SP	.272 $\pm$ .046 $\bullet$	.310 $\pm$ .014 $\bullet$	.336 $\pm$ .040 $\bullet$	.201 $\pm$ .029 $\bullet$	.155 $\pm$ .026	.102 $\pm$ .027
CIFAR	.468 $\pm$ .017 $\bullet$	.504 $\pm$ .014 $\bullet$	.463 $\pm$ .013 $\bullet$	.166 $\pm$ .032	.232 $\pm$ .038 $\bullet$	.150 $\pm$ .030
Fashion	.305 $\pm$ .033 $\bullet$	.294 $\pm$ .016 $\bullet$	.247 $\pm$ .023 $\bullet$	.160 $\pm$ .033 $\bullet$	.123 $\pm$ .019 $\bullet$	.056 $\pm$ .015
SVHN	.808 $\pm$ .011 $\bullet$	.604 $\pm$ .014 $\bullet$	.806 $\pm$ .011 $\bullet$	.144 $\pm$ .038 $\bullet$	.120 $\pm$ .025	.089 $\pm$ .018
w/t/l	9/1/0	10/0/0	9/1/0	7/3/0	6/4/0	—



**Figure 4: The trends of OCA of six methods on five datasets in the doubly-streaming setting. The blue-shadowed areas indicate the overlapping  $\mathcal{T}_b$  timespans. Due to the space limitation, complete results are deferred to the supplementary file.**

is to establish a mapping relationship between the old and new features, such that once the old features vanish, they are reconstructed from the new features, allowing the learner to harvest both old and new feature information to make accurate online predictions via ensembling. To realize this idea, the crux lies in the harmonization of model onlineness and expressiveness. To respect the high dimensionality and complex feature interplay in the real-world data streams, our OLD<sup>3</sup>S approach discovered a shared latent subspace using variational approximation, which can encode arbitrarily expressive mapping functions for feature reconstruction. Meanwhile, as the real-time nature of data streams biases shallow models, our approach enjoyed an optimal depth *learned* from data, starting from shallow and gradually becoming deep if more complex patterns are required to be captured in an online fashion. Comparative studies evidenced the viability of our approach and its superiority over the state-of-the-art competitors.

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