THE LAGS OF MONETARY POLICY

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Abstract

The length of the transmission lags from monetary policy to output has been the subject of much research over the years, but there are serious problems in isolating the lags with any precision. This paper uses a simple model of Australian output to estimate the length of the lags, and then examines how attempts to grapple with the estimation problems might change the results.

We estimate that output growth falls by about one-third of one per cent in both the first and second years after a one percentage point rise in the short-term real interest rate, and by about one-sixth of one per cent in the third year. This implies an average lag of about five or six quarters in monetary policy’s impact on output growth. Each of these estimates is, however, subject to considerable uncertainty. We discuss the implications for policy of these relatively long and uncertain lags. Finally, we find no evidence that the average lag from monetary policy to output growth has become any shorter in the 1990s.

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1. Introduction

Good policy-making requires an appreciation of the dynamic relationship between the monetary policy instrument – the overnight cash rate – and the final objectives of policy – inflation and output. A thorough analysis of this relationship between instrument and objectives is a large task, however, because of the many transmission channels through which monetary policy influences the economy. Rather than examining each of these channels, this paper has the more modest aim of estimating the aggregate impact on Australian economic output of changes in the domestic short-term real interest rate.

The task of isolating the impact of monetary policy on output would be made much easier if we had a good explanation for the underlying business cycle. While this is obviously complex, we do have good empirical evidence over the past 15 years that Australian output is strongly influenced by economic activity in the United States. US output is clearly ‘exogenous’ to the Australian economy since it is not affected by either Australian output or Australian monetary policy. But the presence of this powerful exogenous influence on Australian output makes it easier to identify econometrically the dynamic effect of monetary policy on Australian output.

Despite this econometric benefit delivered by the exogenous influence of the US, estimating the lags of monetary policy in Australia is not without its difficulties. As a rule, short-term real interest rates change only gradually, so that the current real interest rate is quite strongly correlated with interest rates in the recent past. As a consequence, it is hard to separate the effect on output of the current real interest rate from the delayed effects of the real rate in earlier periods. This problem leads to fairly wide margins of error in our estimates of the dynamic effect of monetary policy on output. Nevertheless, despite these wide margins of error, there is still strong evidence of an impact on output growth in the first, second and third years after a change in the domestic short-term real interest rate.
Another difficulty in isolating the dynamic impact of monetary policy on output arises from the forward-looking nature of policy. As well as responding to data about the past, policy-makers also act on information about current and future economic developments that is not part of any simple aggregate analysis of the relationship between monetary policy and output. This paper will show that this implies that standard estimation techniques underestimate the strength of monetary policy’s impact on output, and overestimate the length of the lags of monetary policy.

The next section of the paper begins with an analytical discussion of the sources of the lags of monetary policy. It then turns to single equation models of Australian output which provide good empirical descriptions of the domestic business cycle over the past 15 years. The main focus of the section is to estimate the effect of a one percentage point change in the short-term real interest rate on output growth over the subsequent three years.

The following section, Section 3, discusses the implications of policy-makers responding to information that is not available to the econometrician estimating the relationship between monetary policy and output. Under plausible assumptions, this is likely to result in an underestimation, using standard techniques, of the impact of monetary policy changes on output growth in the short run.

Making some allowance for this bias, we conclude that output growth falls by about one-third of one per cent in both the first and second years after a one percentage point rise in the short-term real interest rate and by about one-sixth of one per cent in the third year. Section 3 also finds that there is no evidence that the lags of monetary policy have become any shorter over the course of the 1990s. Finally, Section 3 discusses the policy implications of the relatively long and uncertain lags of monetary policy.

The paper ends with a brief summary of the main results.
2. The Lags of Monetary Policy

2.1 The Sources of Monetary Policy Lags

There are six main channels through which changes in interest rates affect economic activity: intertemporal substitution (since interest rates represent the price of expenditure in the present relative to the future), the effect of induced changes in the exchange rate on the tradeable sector, interest rate effects on other asset prices, cash-flow effects on liquidity constrained borrowers, credit supply effects, and the direct effect of changes in monetary policy on expectations of growth (Grenville 1996). Each of these channels – and the interaction between them – makes a contribution to the lags of monetary policy.

To begin at the beginning, however, the first source of monetary policy lags is the delay in pass-through of changes in the overnight cash rate to other interest rates. While the response of short-term money market interest rates is rapid and complete, pass-through to other interest rates such as the deposit and lending rates of financial intermediaries appears to be slower (Lowe 1995). Since intermediaries’ interest rates are important determinants of cash-flow, asset prices, and the incentive to postpone expenditure, slow pass-through contributes to the transmission lag from the real cash rate to activity.

Beyond pass-through, an important source of lags arises from the gradual response of investment – both business investment and consumer investment in durables and dwellings – to changes in monetary policy. Adjustment costs associated with changing the level of the relevant capital stock are partly responsible. However, changes in interest rates also affect the incentive to postpone investment when returns are uncertain. The largely irreversible nature of many investments means that there is an option value to waiting to invest in a world of uncertainty (Dixit and Pindyck 1994). When a firm or individual makes an irreversible investment, this option is exercised, eliminating the possibility of waiting for the arrival of new information that might have affected the timing or the desirability of the investment. A change in interest rates affects this option value, and will therefore affect the timing of the investment.
Empirical estimates for the US suggest quite long lags in the adjustment of investment to shocks. For example, Jorgenson and Stephenson (1967) report a mean lag of seven quarters between changes in the rental price of capital and investment in US manufacturing, while Shapiro (1986) estimates that, in response to a shock to the required rate of return on capital, more than half the adjustment in the manufacturing capital stock occurs in the first year, but it takes over four years to be complete.

Turning to asset markets, economic theory would lead one to expect the full implications of a change in monetary policy to be incorporated into asset prices as soon as the change became apparent. In the important case of the exchange rate, however, this does not appear to occur. Thus, for example, Eichenbaum and Evans (1995) find, for the US, that contractionary monetary policy leads to a prolonged gradual appreciation of the domestic currency with the maximal appreciation occurring after two to three-and-a-half years. As a consequence, the exchange rate effects on the tradeable sector of the economy are also gradual and prolonged.

Finally, developments in one sector of the economy are gradually transmitted to other parts of the economy as agents who were initially unaffected by the monetary policy change respond to the altered behaviour of their suppliers and customers. These transmission channels to the wider economy also contribute to the aggregate lags of monetary policy.

2.2 Single Equation Models

Turning to empirical analysis, we begin with single equation models for Australian output. We use a general-to-specific modelling strategy in which insignificant lags of the variables are sequentially eliminated, leading eventually to parsimonious specifications. The models are variants of an earlier model estimated by Gruen and Shuetrim (1994).
We present two models which differ only in their treatment of inflationary expectations. After eliminating insignificant lags, both models take the form,

$$
\Delta y_t = \alpha + \sum_{j=0}^{6} \beta_j r_{t-j} + \left[ \gamma_2 \Delta f_{t-2} + \gamma_4 \Delta f_{t-4} \right] + \delta y_{t-1} + \chi w_{t-1} + \sum_{j=0}^{1} \phi_j \Delta w_{t-j} + \varepsilon_t, \quad (1)
$$

where $\Delta y_t$ is quarterly growth of Australian non-farm output, $r_t$ is the short-term real interest rate, $\Delta f_t$ is growth of Australian farm output, $y_{t-1}$ and $w_{t-1}$ are the lagged log levels of Australian non-farm output and US output, and $\varepsilon_t$ is a mean-zero error term. Summary results for the two models, estimated by ordinary least squares, are shown in Table 1.¹

The first two sets of independent variables model the influence of domestic variables on output. To control for domestic monetary policy, we use current and lagged values of the short-term real interest rate. With our focus on the length of the lags of monetary policy, we want to allow considerable flexibility in the estimated pattern of influence of monetary policy on output. We therefore use lags 0 to 6 of the short-term real interest rate, rather than eliminating all insignificant lags as we do for other variables.

We assume inflationary expectations are backward-looking. For the underlying CPI model, we use the overnight cash rate set by the Reserve Bank minus underlying consumer price inflation over the past year to measure the short-term real interest rate, while for the headline CPI model, we subtract headline consumer price

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¹ We are faced with the common difficulty in econometrics that we require a timespan long enough to generate meaningful results but not so long that the underlying economic relationships change substantially during the estimation period. With this in mind, we omit the more financially regulated 1970s, and estimate from the financial year 1980/81 to the present, that is, 1980:Q3 to 1996:Q1, giving 63 quarterly observations. For our purposes, the float of the Australian dollar in December 1983 was not an important regime change because, from 1980 to 1983, the exchange rate was adjusted daily via a crawling peg with the US$ and was therefore fairly flexible. For both models, the general specification from which we begin includes contemporaneous and four lags of farm output growth and US GDP growth as well as lags one to four of the dependent variable. A trend term is insignificant when added to either regression.
inflation over the past year. For both models, the coefficients on individual lags of the real interest rate are estimated imprecisely, but the mean of the real interest rate coefficients is negative, as expected, and highly significant (Table 1).

The second set of domestic variables controls for the influence of farm output on the rest of the Australian domestic economy. Although the farm sector accounts for only about 4 per cent of the Australian economy, widespread droughts, and the subsequent breaking of those droughts, lead to large changes in farm output which have multiplier effects on the wider economy.

The rest of the independent variables in the equation control for both the short-run and longer-run effects of US output growth on Australian output. Including lagged log levels in the regression allows for a possible long-run (cointegrating)

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2 We also generated estimates of the short-term real interest rate using a survey-based measure of consumers’ inflation expectations from the Melbourne Institute survey. The estimation results were qualitatively similar, though the explanatory power of the regression was reduced. Of the two measures of the past inflation used in our estimation, it is unclear which is a better measure of inflationary expectations in the economy. The headline measure is more widely reported but is directly affected by changes in the overnight cash rate (via their effect on variable-rate housing mortgage interest rates); by contrast, the underlying measure, which excludes this direct effect, is a better measure of core consumer price inflation.

3 As a check of robustness, we also estimated the regression using the yield gap (the cash rate minus the 10-year bond rate) instead of the real cash rate to control for the influence of monetary policy. The results are qualitatively similar, although both the explanatory power of the regression and the significance of this measure of monetary policy are much reduced.
Table 1: Australian Non-farm GDP Growth Regressions\(^{(a)}\)

\[
\Delta y_t = \alpha + \sum_{j=0}^{6} \beta_j r_{t-j} + \left[ \gamma_2 \Delta y_{t-2} + \gamma_4 \Delta y_{t-4} \right] + \delta y_{t-1} + \chi w_{t-1} + \sum_{j=0}^{1} \phi_j \Delta w_{t-j} + \varepsilon_t
\]

<table>
<thead>
<tr>
<th>Variables</th>
<th>Underlying CPI Model(^{(b)})</th>
<th>Headline CPI Model(^{(b)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>24.64** (2.82)</td>
<td>23.46** (2.64)</td>
</tr>
<tr>
<td>Real cash rate(^{(c)})</td>
<td>-0.035 (0.00)</td>
<td>-0.036 (0.00)</td>
</tr>
<tr>
<td>Farm output % change (lag 2)</td>
<td>0.020* (2.39)</td>
<td>0.020* (2.26)</td>
</tr>
<tr>
<td>(lag 4)</td>
<td>-0.020* (-2.23)</td>
<td>-0.020* (-2.04)</td>
</tr>
<tr>
<td>Lagged Australian GDP log level</td>
<td>-0.31** (-5.78)</td>
<td>-0.34** (-5.96)</td>
</tr>
<tr>
<td>Lagged US GDP log level</td>
<td>0.38** (6.02)</td>
<td>0.42** (6.21)</td>
</tr>
<tr>
<td>US GDP % change(^{(c)})</td>
<td>0.047 (0.00)</td>
<td>0.061 (0.00)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Underlying CPI Model</th>
<th>Headline CPI Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>R(^2)</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>Adjusted R(^2)</td>
<td>0.60</td>
<td>0.59</td>
</tr>
<tr>
<td>Standard error of residuals</td>
<td>0.56</td>
<td>0.56</td>
</tr>
<tr>
<td>F-test for joint significance of Australian and US GDP levels</td>
<td>20.0 (0.00)</td>
<td>21.3 (0.00)</td>
</tr>
<tr>
<td>LM test for autocorrelation of residuals:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First order</td>
<td>1.49 (0.22)</td>
<td>0.083 (0.77)</td>
</tr>
<tr>
<td>First fourth order</td>
<td>6.58 (0.16)</td>
<td>4.99 (0.29)</td>
</tr>
<tr>
<td>Breusch-Pagan test for heteroscedasticity</td>
<td>20.27 (0.09)</td>
<td>16.83 (0.21)</td>
</tr>
</tbody>
</table>

Notes:  
(a) The models are estimated by ordinary least squares using quarterly data over the period 1980:Q3 to 1996:Q1. Numbers in parentheses () are t-statistics. Numbers in braces {} are p-values. Individual coefficients marked with *(**) are significantly different from zero at the 5%(1%) level. All variables in log levels and their differences are multiplied by 100 (so growth rates are in percentages).

(b) To derive the real interest rate, inflation expectations are based on the underlying CPI or on the headline CPI.

(c) The mean coefficient is reported for the real cash rate and US GDP % change to summarise the coefficients on these variables. The p-values are derived from F-tests of the joint significance of the lags.

\(^{(b)}\) To derive the real interest rate, inflation expectations are based on the underlying CPI or on the headline CPI.

\(^{(c)}\) The mean coefficient is reported for the real cash rate and US GDP % change to summarise the coefficients on these variables. The p-values are derived from F-tests of the joint significance of the lags.
relationship between the log levels of Australian and US output, with the results providing strong evidence of the existence of this relationship.\textsuperscript{4}

The importance of US output for the Australian business cycle, recently highlighted by McTaggart and Hall (1993), appears to arise for several reasons. In the shorter run, links between financial markets (Gruen and Shuetrim 1994; de Roos and Russell 1996; and Kortian and O’Regan 1996), effects on Australian business confidence (Debelle and Preston 1995) and a disproportionately large response of Australian exports to the US business cycle (de Roos and Russell 1996) all play a role. In the longer run, technology transfer from the US seems to be important (de Brouwer and Romalis 1996).

Both model regressions have appealing statistical properties, with no evidence of first-order or first to fourth-order serial correlation and no strong signs of heteroscedasticity. Despite their simplicity, the equations explain a substantial part of the variation in Australian quarterly non-farm GDP growth, with adjusted $R^2$ s of 0.60 and 0.59. Both models explain the major features of the Australian business cycle since 1980. Figure 1 shows the results from the underlying CPI model.

\textsuperscript{4} Augmented Dickey-Fuller tests do not reject the hypothesis that the log levels of GDP are stochastically non-stationary I(1) variables (Gruen and Shuetrim 1994). In the regressions in Table 1, the F-statistic for the joint significance of the lagged log levels of Australian and US GDP can be used to test for the existence of a long-run relationship between these variables whether they are I(0) or I(1) variables. It does, however, have a non-standard distribution. For both regressions, we reject the null of no long-run relationship at the 1 per cent level based on critical values tabulated in Pesaran, Shin and Smith (1996).


2.3  Quantifying the Lags of Monetary Policy

We now return to the lags of monetary policy, and examine the impact on domestic output of a sustained one percentage point rise in the domestic short-term real interest rate. Of course, in conducting this exercise, we should not lose sight of the fact that the domestic real rate is determined in the longer run by the world real rate rather than by domestic monetary policy.\textsuperscript{5}

For both models, Figure 2 shows the effect of the rise in the domestic real interest rate on the level of non-farm output, while Figure 3 shows the effect on the

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\textsuperscript{5} Since Australia is small in the world capital market, the Australian short-term real interest rate is determined in the long run by the world short-term real interest rate plus or minus a risk premium (assuming no long-run trend in the Australian real exchange rate). Nevertheless, domestic monetary policy determines the Australian short-term real interest rate for long enough to have an important influence on the Australian macroeconomy.
year-ended growth of non-farm output. Both figures show point estimates and 90 per cent confidence intervals.\(^6\)

**Figure 2: Impact of Monetary Policy on the Level of Output**

<table>
<thead>
<tr>
<th>%</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.2</td>
<td>-0.2</td>
</tr>
<tr>
<td>-0.4</td>
<td>-0.4</td>
</tr>
<tr>
<td>-0.6</td>
<td>-0.6</td>
</tr>
<tr>
<td>-0.8</td>
<td>-0.8</td>
</tr>
<tr>
<td>-1.0</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

Underlying CPI model

<table>
<thead>
<tr>
<th>%</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.2</td>
<td>-0.2</td>
</tr>
<tr>
<td>-0.4</td>
<td>-0.4</td>
</tr>
<tr>
<td>-0.6</td>
<td>-0.6</td>
</tr>
<tr>
<td>-0.8</td>
<td>-0.8</td>
</tr>
<tr>
<td>-1.0</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

Headline CPI model

Quarters after interest rate rise

Note: The figure shows point estimates and 90 per cent confidence intervals for the impact on the level of non-farm output of a one percentage point rise in the short-term real interest rate at the beginning of quarter 1.

The level of output in either model falls slightly for the first few quarters after a rise in the short-term real interest rate, but the fall is statistically insignificant. Over time, however, the contractionary effect on output gets stronger and becomes increasingly significant. Almost all the effect on the level of output occurs within three years.

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\(^6\) The rise in the real interest rate occurs at the beginning of quarter 1. Therefore, for quarters 1, 2 and 3, the effect on year-ended growth shown in Figure 3 is small partly because the real interest rate has been raised for less than a year. The first year is defined to be from quarter 0 to 4, and the second year from quarter 4 to 8. Since the effect on output of a rise in the real interest rate is a non-linear function of the model parameters, the confidence intervals are estimated using a Monte Carlo procedure described in Appendix A. The point estimates shown in the figures are median outcomes from these simulations, and so they differ very slightly from results derived from the OLS regressions.
Figure 3: Impact of Monetary Policy on Four-quarter-ended Growth

Note: The figure shows point estimates and 90 per cent confidence intervals for the impact on four-quarter-ended growth of non-farm output of a one percentage point rise in the short-term real interest rate at the beginning of quarter 1.

It is also clear, however, that the confidence intervals are quite wide. The current real interest rate and its lags are quite strongly correlated, leading to unavoidable problems of multicollinearity in the regressions. As a consequence, it is hard to disentangle the effect on output of the current real interest rate from the delayed effects of the real rate in earlier quarters. In other words, it is hard to estimate accurately the length of the lags of monetary policy.

Another way to highlight this problem is to compare results from the two models on the estimated effect on output growth in the first and second years after a rise in the real interest rate. Assuming inflationary expectations respond to underlying inflation, the fall in output growth is smaller in the first year than in the second (the point

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7 For example, the correlation coefficient between the current real interest rate, defined using underlying inflation, and its lags falls from 0.88 for the first lag to 0.74, 0.63, 0.50, 0.34 and 0.22 for the sixth lag.
estimates are falls in growth of 0.20 per cent and 0.34 per cent), suggesting that the lags in the transmission of monetary policy to output are quite long. Alternatively, assuming inflationary expectations respond to headline inflation, the fall in output growth in the two years is almost the same (the point estimates are 0.26 per cent and 0.28 per cent), suggesting rather shorter lags. The point of this comparison is clear enough: subtle changes in assumptions about inflationary expectations can lead to somewhat different estimated results.

As might be expected, while the point estimates from the two models are different, these differences are not statistically significant. Although the underlying CPI model suggests that the contractionary impact is stronger in the second year, at conventional levels of significance we cannot reject the alternative hypothesis that the impact is in fact stronger in the first year.

An alternative way to summarise the length of the lags of monetary policy is to calculate the average lag length, defined by

\[
\text{average lag length (in quarters)} = \frac{\sum_{i=1}^{\infty} (i - 1) \Delta m_i}{\sum_{i=1}^{\infty} \Delta m_i},
\]

where \(\Delta m_i\) is the effect on non-farm output growth in quarter \(i\) of the one percentage point rise in the real interest rate. Using this formula for the underlying CPI model, the average length of the monetary policy lag is 6.4 quarters; for the headline model, it is a slightly smaller 5.8 quarters. For the reasons explained above, these numbers are again estimated imprecisely, with the 90 per cent confidence interval from 5.1 to 7.7 quarters for the underlying model, and 4.5 to 7.0 quarters for the headline model.

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8 In about 12 per cent of the Monte Carlo simulations of the underlying CPI model, the contractionary impact on output is stronger in the first year.

9 To give an example, if the interest rate rise led to an immediate once-off fall in the level of output (and hence a once-off fall in output growth in quarter 1) the average lag length as calculated by Equation (2) would be zero, as required.
3. Taking Account of the Policy Response

3.1 The Problem with Ordinary Least Squares Estimation

Estimating the lags of monetary policy using the approach adopted in the last section has an important drawback. In setting monetary policy, policy-makers do not rely solely on data about the past, but base their decisions also on information about current and expected future developments in inflation and output. From an econometric viewpoint, this renders the real interest rate an endogenous variable, that is likely to be correlated with the current and future residuals in an output equation.

The nature of the problem can be explained using a simple model. Assume that output depends negatively on the real interest rate, while the real interest rate is set on the basis of incoming information about output. In symbols,

\[ y_t = \alpha r_t + u_t \]
\[ r_t = I_t + \nu_t, \]

where \( y_t \) is output, \( r_t \) is the real interest rate, \( u_t \) and \( \nu_t \) are independent error terms and \( I_t \) is information about current output, and is therefore correlated with current output (\( E_t(I_t, u_t) = \gamma \)). The correlation \( \gamma \) is positive because monetary policy will be tighter when output is expected to be above average.

If the output equation, \( y_t = \alpha r_t + u_t \), is now estimated by ordinary least squares (OLS) the resulting estimate of \( \alpha \) is, on average, smaller in magnitude than the true value, \( \alpha = -1 \). If the variables \( u_t \), \( \nu_t \), and \( I_t \) have zero means and unit variances, the ratio \( R_\alpha \) of the OLS estimate of \( \alpha \) to its true value is \( R_\alpha = 1 - \gamma / 2 \), which is less than one.\(^{10}\) Since monetary policy is not set solely on the basis of past information, the strength of its impact on output is underestimated by OLS – at least in this simple framework.

Unfortunately, the problem is more serious than simply underestimating the strength of monetary policy. The ordinary least squares approach also tends to overestimate

\(^{10}\) The wording in the text is heuristic rather than rigorous. The actual definition of \( R_\alpha \) is \( R_\alpha = \text{plim} \hat{\alpha} / \alpha \) where \( \hat{\alpha} \) is the OLS estimate of \( \alpha \), and plim is the probability limit.
the length of the lags of monetary policy. To illustrate this point, extend the above model to one in which output depends negatively on both the current and first lag of the real interest rate, while the real interest rate responds to information about output in both the current and next periods. In symbols,

\[ y_t = -r_t - r_{t-1} + u_t \]

\[ r_t = I_t + J_t + v_t, \]

where we have introduced the variable \( J_t \) which is information available in period \( t \) about output in period \( t+1 \), and is therefore correlated with output in period \( t+1 \) (\( E_t(J_t u_{t+1}) = \delta \)). Estimating the output equation, \( y_t = \alpha r_t + \beta r_{t-1} + u_t \), by OLS leads to estimates of \( \alpha \) and \( \beta \) which are, on average, smaller in magnitude than their true values, \( \alpha = \beta = -1 \). If the two errors, \( u_t \) and \( v_t \), and the two information variables, \( I_t \) and \( J_t \), have zero means and unit variances, the ratios \( R_\alpha \) and \( R_\beta \) of the OLS estimates of \( \alpha \) and \( \beta \) to their true values are \( R_\alpha = 1 - \gamma / 3 \) and \( R_\beta = 1 - \delta / 3 \).\(^{11}\)

Since information about the current period should be more reliable than information about the future, \( \gamma \) should be greater than \( \delta \). Then, while both coefficients are biased towards zero, the bias is likely to be more serious for \( \alpha \), implying that OLS overestimates the length of the lags of monetary policy.

### 3.2 How Serious is the Bias?

We can extend this analysis to examine the bias in the length of the transmission lags from monetary policy to output estimated in Section 2. To derive empirical estimates of the bias, we need to make assumptions about the correlation between the current real interest rate and the current and future unexplained residuals in the non-farm output equation (Equation (1)).

We assume a correlation coefficient \( \rho_k \) between the current real interest rate, \( r_t \), and the unexplained residual in Equation (1) in \( k \)-periods time, \( \epsilon_{t+k} \), of \( \rho_k = \gamma^{k+1} \), \( 0 < \gamma < 1, \ k = 0,...,6 \). This formula embodies the idea used in the simple models above that the correlation is strongest when the output shock and the real interest

\(^{11}\) Again, the wording in the text is heuristic rather than rigorous. See the previous footnote. The bias we refer to in the text is really asymptotic bias.
rate are contemporaneous and falls as the distance in time between the two variables rises. We derive results for values of $\gamma$ in the range from zero to 0.75.\textsuperscript{12}

Assuming a small value for $\gamma$ implies that the policy-maker raises real interest rates only slightly when information arrives that current growth will be stronger than is implicit in Equation (1), and has virtually no reliable information about future growth other than that predicted by Equation (1). By contrast, assuming $\gamma = 0.75$ implies the policy-maker is in possession of good information about both current and future growth shocks, and uses that information to make significant changes to real interest rates.\textsuperscript{13}

Making these assumptions about the correlations between the real interest rate and growth shocks, we can adjust for the bias in the ordinary least squares estimates generated in Section 2. The details of the calculation are presented in Appendix B, and Figure 4 shows results for the underlying CPI model for four values of $\gamma$: 0.1, 0.25, 0.5 and 0.75 (both the size and pattern of the bias are very similar in the two models).

As for Figure 3, Figure 4 shows the estimated impact on year-ended non-farm output growth of a one per cent sustained rise in the domestic real interest rate. The figure shows point estimates for the OLS regression summarised in Table 1, and for OLS adjusted using the correlations between growth shocks and real interest rates defined above.

\textsuperscript{12} In general, the correlation between the real interest rate and the unexplained residual depends both on the quality of information available about the unexplained residual and the extent to which policy reacts to that information. The assumption of a falling correlation as the distance in time between the two variables rises could arise either because information is poorer about future residuals, or because policy reacts less to that information.

\textsuperscript{13} Assume that the information available to the policy-maker enables him/her to make unbiased forecasts of the current and future shocks to the output equation. Then, for our parameter values, $\gamma = 0.75$ means that when the policy-maker has information implying that the level of output will be half a per cent higher than forecast by the equation in each of the next two years, the short-term real interest rate is immediately raised by two-thirds of one per cent. We judge this to be a significant policy reaction.
The figure displays both of the features revealed by the simple models described earlier. First, OLS underestimates the average strength of the impact of monetary policy on output. And, second, almost all this underestimation occurs for the impact of monetary policy in the first year, implying that OLS overestimates the average length of the lags of monetary policy.

**Figure 4: Impact of Monetary Policy on Four-quarter-ended Growth**

![Figure 4: Impact of Monetary Policy on Four-quarter-ended Growth](image)

**Note:** The figure shows, for the underlying CPI model, the estimated impact on four-quarter-ended growth of non-farm output of a one percentage point rise in the short-term real interest rate adjusting for the bias from a correlation coefficient $\rho_k$ between the current real interest rate and the error in the output equation in $k$-quarters time of $\rho_k = \gamma^{k+1}$, $k = 0, \ldots, 6$, for the values of $\gamma$ shown.

The results in Figure 4 suggest that a moderate amount of correlation between real interest rates and output growth shocks implies that the estimated impact on output growth in the first and second years after a one percentage point rise in the real interest rate is roughly the same, at about one-third of one per cent, falling to about one-sixth of one per cent in the third year. Given the error bands around the original OLS estimates, these numbers are again subject to considerable uncertainty.
We can also calculate the average monetary policy lag as the parameter $\gamma$ rises from zero to 0.75. Figure 5 shows the results, which confirm that the average lag length falls with rising correlation between the real interest rate and the unexplained residuals in the output equation. It is of interest to note, however, that the effect is not a very powerful one: plausible values of the correlation coefficient imply only small falls in the estimated average monetary policy lag length.

**Figure 5: Average Lag of Monetary Policy**

Adjusting for possible bias in the ordinary least squares regression

Note: The figure shows the average lag in monetary policy’s effect on output growth for the underlying CPI model, adjusting for the bias from a correlation coefficient $\rho_k$ between the current real interest rate and the error in the output equation in $k$-quarters time of $\rho_k = \gamma^{k+1}$, $k = 0,...,6$, for values of $\gamma$ ranging from zero to 0.75.

### 3.3 Controlling for the Policy Response using Instrumental Variables

Rather than hypothesising about the correlation between real interest rates and output growth shocks, an alternative approach is to use instrumental variables to control for the forward-looking nature of monetary policy.
Choosing appropriate instruments for the short-term real interest rate is a difficult task, however, made more so by a change in focus of Australian monetary policy in the 1990s with the introduction of a medium-term inflation target. To allow for this change, we assume that there are two monetary policy regimes, old and new, with the real interest rate responding differently in the two regimes.

Rather than conducting an extensive search for instruments, we confine the list to those variables which should have a strong influence on the short-term real interest rate: inflation, $\pi$, and the output gap, $\text{gap}$. In the old regime, we also include the current account deficit to GDP ratio, $\text{cad}$, as an instrument because we find that this variable has considerable explanatory power. The variables are lagged to reduce the possibility of correlation between them and the error term in the output equation (1).

For brevity, we again show only the results using underlying inflation (although the results are similar for headline inflation). We use the fitted values from the regression,

$$ r_t = a^o d_{t-1}^o + b^o \pi_{t-1}^o + c^o \text{gap}_{t-1}^o + e^o \text{cad}_{t-1}^o + a^n d_{t-1}^n + b^n \pi_{t-1}^n + c^n \text{gap}_{t-1}^n + u_t, \quad (3) $$

to instrument for the contemporaneous real interest rate in the non-farm output equation (1). The notation implies, for example, that $\pi_{t-1}^o$ is the (four-quarter-ended) underlying inflation rate in period $t-1$ when $t$ is in old regime and 0 when $t$ is in the new regime. $d_{t-1}^o$ and $d_{t-1}^n$ are dummy variables for the two regimes and $\text{gap}$ is defined as the deviation of non-farm output from a linear trend. The start of the new regime is the first quarter of 1990, chosen to maximise the explanatory power of the equation. The instruments together explain 78 per cent of

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14 This does not mean that monetary policy ‘targeted’ the current account, as is sometimes suggested, only that policy responded to changes in demand conditions which were reflected in changes in the current account deficit.
the variance of the short-term real interest rate over the estimation period, 1980:Q3 to 1996:Q1.\textsuperscript{15}

From this instrumental variable (IV) regression, we derive the impact of a sustained one per cent rise in the short-term real interest rate on the year-ended growth of non-farm output. The results are shown in Figure 6. As before, the

\textbf{Figure 6: Impact of Monetary Policy on Four-quarter-ended Growth}

IV regression simulations

Note: The figure shows, for the underlying CPI model, point estimates and 90 per cent confidence intervals for the impact on four-quarter-ended growth of non-farm output of a one percentage point rise in the short-term real interest rate at the beginning of quarter 1.

confidence intervals are estimated using a Monte Carlo procedure described in Appendix A.

\textsuperscript{15} Since policy-makers set the short-term real interest rate partly on the basis of information about current output not available to the econometrician estimating Equation (1), we should expect positive correlation between the errors in Equations (1) and (3). The estimated correlation coefficient between the errors, at 0.08, is positive, but small.
The results in Figure 6 are not markedly different from the ordinary least squares results using the underlying CPI model in Figure 3. The estimated contractionary impact in the first year is somewhat stronger in the IV regression than in the OLS one, but it is also less precisely estimated. In common with the OLS results, the estimated impact on output growth in the second year is somewhat stronger than in the first year, but the difference is again statistically insignificant.16

3.4 Have the Lags of Monetary Policy Changed over Time?

To examine whether the lags of monetary policy have changed over time, we conduct recursive estimation on the underlying CPI model, using both ordinary least squares and instrumental variables regressions. The recursive estimation fixes the starting date at the beginning of the full sample, 1980:Q3, and extends the end of the estimation period from 1990:Q3 to 1996:Q1, one quarter at a time. Figure 7 shows how the average monetary policy lag (defined by Equation (2)) changes as the estimation period is lengthened through the 1990s. As before, both point estimates and 90 per cent confidence intervals are shown.

The results do not suggest any shortening of the lags of monetary policy over the 1990s. As the estimation period is lengthened, the ordinary least squares regressions show a very slight rise in the average monetary policy lag length, while the instrumental variables regressions suggest a somewhat larger rise in the lag length. Needless to say, these results are not statistically significant because the confidence intervals are so wide.

16 It would be of interest to instrument not only for the contemporaneous real interest rate but also for some of its lags. Unfortunately, however, given the strong correlation between the real interest rate and its lags and the fact that the instruments are not very good, this exercise did not yield any useful information.
Figure 7: Average Lag of Monetary Policy
Extending the estimation period through the 1990s

Note: The figure shows recursive point estimates and 90 per cent confidence intervals for the average lag in monetary policy’s effect on output growth for the underlying CPI model, using both OLS and IV estimation techniques.

3.5 Comparison with Other Estimates of the Lags of Monetary Policy

Table 2 shows the effect on output growth in the first, second and third years after a rise in the short-term interest rate for a range of models estimated for the Australian and US economies.

There is qualitative agreement between the models that output growth falls in both the first and second years after a rise in the short-term interest rate, although there is no such agreement about the third year. Even for the first two years, however, the quantitative estimates differ quite substantially between models. This is simply

17 In the case of the Murphy model, the stronger contractionary impact in the first year may be partly a consequence of the assumption of uncovered interest parity which implies an initial
another manifestation of the technical difficulties associated with estimating the lags of monetary policy.

Table 2: Model Comparison of the Effect on Output Growth of a One Percentage Point Rise in Short-term Interest Rates

<table>
<thead>
<tr>
<th>Model</th>
<th>First year</th>
<th>Second year</th>
<th>Third year</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Australia</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current model$^{(b)}$</td>
<td>- 0.35</td>
<td>- 0.33</td>
<td>- 0.18</td>
</tr>
<tr>
<td>Murphy$^{(c)}$</td>
<td>- 0.51</td>
<td>- 0.31</td>
<td>+ 0.08</td>
</tr>
<tr>
<td>TRYM$^{(c)}$</td>
<td>- 0.37</td>
<td>- 0.37</td>
<td>- 0.07</td>
</tr>
<tr>
<td><strong>United States</strong>$^{(d)}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPS</td>
<td>- 0.20</td>
<td>- 0.70</td>
<td>- 1.10</td>
</tr>
<tr>
<td>DRI</td>
<td>- 0.47</td>
<td>- 0.53</td>
<td>- 0.13</td>
</tr>
<tr>
<td>FAIR</td>
<td>- 0.24</td>
<td>- 0.25</td>
<td>+ 0.03</td>
</tr>
<tr>
<td>FRBSF</td>
<td>- 0.55</td>
<td>- 0.19</td>
<td>+ 0.04</td>
</tr>
<tr>
<td>VAR</td>
<td>- 0.64</td>
<td>- 0.26</td>
<td>+ 0.08</td>
</tr>
</tbody>
</table>

Notes:  
(a) For the Australian models, the policy experiment involves raising the short-term real interest rate, while for the US models, the short-term nominal rate is raised. Given the sluggish adjustment of inflation, this difference should not matter much.  
(b) The results for the current model are those for the underlying CPI model, bias-adjusted assuming $\gamma = 0.5$.  
(c) See Murphy (1995) and The Treasury (Australia) (1996) for a description of the models. Assuming the models are linear for small changes, the results are derived by adding up the impact on output of a series of nominal shocks which together imply a one percentage point rise in the short-term real interest rate for 12 quarters. The results using the TRYM model should in no way be regarded as being Treasury analyses of the effect of a given policy change or as having the sanction of the Treasury, the Treasurer or the Commonwealth Government.  
(d) The MPS model is maintained by the staff of the Federal Reserve Board, the DRI model is commercially available, the FAIR model is maintained by Ray Fair of Yale University, the FRBSF model is maintained by the Federal Reserve Bank of San Francisco, while the VAR model is a standard vector autoregressive model (see Rudebusch (1995) for further details).

jump appreciation of the real exchange rate, and therefore a significant contractionary impact on output via its effect on net exports.
3.6 Implications for Policy

On the basis of our empirical results, we conclude that output growth falls by about one-third of a per cent in the first and second years after a one percentage point rise in the short-term real interest rate and by about one-sixth of a per cent in the third year, implying that the majority of the effect on growth occurs more than a year after a change in interest rates. These relatively long lags, combined with the problems inherent in forecasting economic growth more than a year into the future, point to the difficulty of trying to use monetary policy to iron-out fluctuations in the business cycle.

Two points are worth making in response to this observation. First, although our discussion has focussed on the response to a once-off permanent interest rate change, monetary-policy-induced changes in the real interest rate are, in fact, temporary. Consider a temporary policy change: a one percentage point rise in the short-term real interest rate that is reversed after a year. The biggest impact of this is in the first year, with output growth falling by about one-third of a per cent. Combining the effect of the interest rate rise and subsequent fall, output growth is then unchanged in the second year, with the policy effect being unwound by one-sixth of a per cent in each of the following two years. The implication is clear enough. The maximal effect on growth of temporary changes in short-term real interest rates is concentrated in the near-term, where useful information is available on expected developments in inflation and output.

Secondly, a key focus of monetary policy is on medium-term price stability. While output is an important leading indicator of inflation, successful pursuit of a medium-term price target does not depend on being able to ‘fine-tune’ the business cycle. Prolonged swings in output can and should be avoided, but the policy lags are long enough, and uncertain enough, that it is futile to try to use monetary policy to fine-tune the business cycle.
4. Conclusions

We draw three broad conclusions about the transmission lags from the short-term real interest rate to output in the Australian economy.

First, there is strong econometric evidence that the level of the short-term real interest rate has a sizeable, and statistically significant, impact on output in the Australian economy. Ordinary least squares estimation suggests that a one percentage point rise in the short-term real interest rate lowers output growth by one-fifth to one-quarter per cent in the first year, one-third per cent in the second year and one-sixth per cent in the third year, although these estimates are subject to considerable uncertainty.

Second, we examine the implications of monetary policy being based not only on past information, but also on current and expected future developments in the economy. We show that, as a consequence, the strength of monetary policy’s impact on output is underestimated by ordinary least squares estimation, while the length of the monetary policy lags are overestimated. Under plausible assumptions, the underestimation is concentrated in the first year, while the estimated impact on output growth in the second and third years is largely unaffected. This leads us to conclude that the effect of a one percentage point rise in the short-term real interest rate on output growth is probably similar in the first and second years – at about one-third per cent in each year – declining to about one-sixth per cent in the third year. These results imply an average lag of about five or six quarters in monetary policy’s effect on output growth. All these estimates are, however, subject to considerable uncertainty.

Third, we look for evidence of changes in the lags of monetary policy over the 1990s. Our results, based on both ordinary least squares and instrumental variable estimation, do not suggest any shortening in these lags. If anything, they suggest that the lags of monetary policy may have become slightly longer over time, although our estimates are not sufficiently precise to be very confident of this conclusion.
Appendix A: Monte Carlo Procedure

This appendix outlines the Monte Carlo procedure used to generate confidence intervals for the OLS, IV and recursive regressions.

A.1 Ordinary Least Squares Regressions

The non-farm output equation, rewritten here for convenience, is

$$\Delta y_t = \alpha + \sum_{j=0}^{6} \beta_j r_j + [\gamma_2 \Delta y_{t-2} + \gamma_4 \Delta y_{t-4}] + \delta y_{t-1} + \chi w_{t-1} + \sum_{j=0}^{1} \phi_j \Delta w_{t-j} + \epsilon_t, \quad (A1)$$

which may be simplified to

$$\Delta y_t = \delta y_{t-1} + N_t \lambda + \epsilon_t, \quad (A2)$$

where $N_t$ is the vector of explanatory variables excluding $y_{t-1}$.

A sustained one per cent rise in the real interest rate leads to an effect on the level of output after $j$ quarters ($m_j$) of:

$$m_0 = 0$$

$$m_j = (1 + \delta) m_{j-1} + \sum_{i=0}^{k} \beta_i \text{ where } k = \min(j - 1, 6) \quad (A3)$$

$$m_{\infty} = -\sum_{i=0}^{6} \beta_i / \delta.$$ 

Estimating Equation (A2) by OLS over the 63 quarters 1980:Q3 to 1996:Q1 leads to parameter estimates $\hat{\delta}$ and $\hat{\lambda}$, and an estimate of the standard deviation of the errors, $\sigma_\epsilon = 0.56$, for both the underlying and headline models. The Monte Carlo distribution is then generated by running 1 000 trials with each trial, $i$, proceeding as follows:

1. draw a sequence of observations $\{\epsilon_i\}_{i=1}^{63}$ from a normal distribution with mean 0 and variance $\sigma_\epsilon^2$;
2. generate sequences of synthetic data \( \{ \Delta y_t^i \}_{t=1}^{63}, \{ y_t^i \}_{t=1}^{63} \) using
\[
\Delta y_t^i = \hat{\delta} y_{t-1}^i + N_t \hat{\lambda} + \varepsilon_t^i \quad \text{and} \quad y_t^i = y_{t-1}^i + \Delta y_t^i,
\]
where \( \hat{\delta} \) and \( \hat{\lambda} \) are from the OLS estimation using the original data;

3. use the synthetic data to estimate the equation \( \Delta y_t^i = \delta^i y_{t-1}^i + N_t \lambda^i \), by OLS and hence generate parameter estimates \( \hat{\delta}^i \) and \( \hat{\lambda}^i \); and

4. with these parameter estimates, use Equation (A3) to calculate, for this \( i^{th} \) iteration, the effect of a one per cent rise in the real interest rate on the level of output \( (m_j^i, j = 1, \ldots, 12, \infty) \) and the year-ended growth rate of output \( (m_j^i - m_{j-4}^i) \) after \( j \) quarters.

The figures in the text show the 5th, 50th and 95th percentile values for the effect on the level of output, \( m_j^i \), and on the year-ended growth rates, \( m_j^i - m_{j-4}^i \).

A.2 Instrumental Variable Regressions

The policy reaction function, rewritten for convenience, is

\[
\tilde{r}_t = a_1 d_{t-1}^0 + b_1 \pi_{t-1}^o + c_1 gap_{t-1}^o + e_1 c d_{t-1}^o + a_2 d_{t-1}^n + b_2 \pi_{t-1}^n + c_2 gap_{t-1}^n + u_t. \tag{A4}
\]

Estimating the underlying CPI version of Equation (A4) by OLS over the 63 quarters 1980:Q3 to 1996:Q1 leads to fitted values \( \hat{r}_t \), and an estimate of the standard deviation of the errors, \( \sigma_u = 1.32 \). Diagnostic tests on the sample errors reveal strong signs of first-order autocorrelation, with an estimated autocorrelation coefficient, \( \hat{\rho} = 0.31 \).

Estimating Equation (A2) by IV, using \( \hat{r}_t \) as an instrument for \( r_t \) over the period 1980:Q3 to 1996:Q1 leads to parameter estimates \( \tilde{\delta} \) and \( \tilde{\lambda} \), and an estimate of the variance-covariance matrix of the errors from Equations (A2) and (A4), \( \tilde{V} \). The
Monte Carlo distribution is then generated by running 1 000 trials with each trial, \( i \), proceeding as follows:

1. draw two sequences of observations \( \{ e_t^i \}_{t=1}^{63} \) and \( \{ u_t^i \}_{t=1}^{63} \) from a bivariate normal distribution with mean 0 and covariance matrix \( \tilde{V} \), such that \( u_t^i = \hat{\rho} u_{t-1}^i + \eta_t^i \), where \( \eta_t^i \) are independent and identically distributed;

2. generate sequences of synthetic data \( \{ \Delta y_t^i \}_{t=1}^{63}, \{ y_t^i \}_{t=1}^{63} \) using
\[
\Delta y_t^i = \tilde{\delta} y_{t-1}^i + N_t \tilde{\lambda} + \epsilon_t^i \quad \text{and} \quad y_t^i = y_{t-1}^i + \Delta y_t^i,
\]
where \( \tilde{\delta} \) and \( \tilde{\lambda} \) are from the IV estimation using the original data;

3. generate a sequence of synthetic data \( \{ r_t^i \}_{t=1}^{63} \) according to \( r_t^i = \hat{r}_t + u_t^i \). Re-estimate Equation (A4) by OLS using \( r_t^i \) instead of \( r_t \) and obtain a new set of fitted values, \( \hat{r}_t^i \);

4. estimate the equation \( \Delta y_t^i = \delta^i y_{t-1}^i + N_t \lambda^i \) by IV, using \( r_t^i \) as an instrument for \( r_t \), and hence generate parameter estimates \( \delta^i \) and \( \lambda^i \); and

5. with the parameter estimates \( \delta^i \) and \( \lambda^i \), use Equation (A3) to calculate, for this \( i^{th} \) iteration, the effect of a one per cent rise in the real interest rate on the year-ended growth rate of output, \( m_j^i - m_{j-4}^i \), after \( j \) quarters.

The figures in the text show the 5th, 50th and 95th percentile values for the year-ended growth rates.

### A.3 Recursive Regressions

For the recursive regressions, a new Monte Carlo distribution is estimated from 1 000 trials after each new quarter of data is added.
Appendix B: Estimating the Bias from Ordinary Least Squares

It is convenient to rewrite the model for non-farm GDP growth, Equation (1) in the text, as

$$\Delta y_t = \sum_{j=0}^{6} \beta_j r_{t-j} + \delta Z_{t-1} + W_t \phi + \varepsilon_t,$$

(B1)

where $Z_{t-1} = y_{t-1} - \chi^* w_{t-1}$ ($\chi^*$ is the cointegrating vector between $y$ and $w$) and $W_t$ is the matrix of exogenous variables, $W_t = [1 \quad \Delta w_t \quad \Delta w_{t-1} \quad \Delta f_{t-2} \quad \Delta f_{t-4}]$. Equation (B1) may be further simplified to

$$\Delta y_t = X_t \alpha + W_t \phi + \varepsilon_t,$$

(B2)

where $X_t = [r_t \quad r_{t-1} \quad r_{t-2} \quad r_{t-3} \quad r_{t-4} \quad r_{t-5} \quad r_{t-6} \quad Z_{t-1}]$ is the matrix of regressors presumed to be correlated with the disturbance term, $\varepsilon_t$.

OLS on Equation (B2) yields the following estimate for $\alpha$,

$$\hat{\alpha}_{OLS} = \alpha + (X' M_W X)^{-1} X' M_W \varepsilon,$$

(B3)

where $M_W = I - W(W'W)^{-1} W'$.

Now,

$$\text{plim} \frac{X' M_W \varepsilon}{T} = \text{plim} \frac{X' \varepsilon}{T} - \text{plim} X' W(W'W)^{-1} W' \varepsilon$$

$$= \text{plim} \frac{X' \varepsilon}{T} - \text{plim} X' W(W'W)^{-1} \cdot \text{plim} \frac{W' \varepsilon}{T}$$

$$= \text{plim} \frac{X' \varepsilon}{T}$$
as $\text{plim} \frac{W'\varepsilon}{T} = 0$ since $W$ is exogenous. In the limit, as the sample size increases, the true value of the vector $\alpha$ is then

$$\alpha = \hat{\alpha}_{OLS} - \text{plim} \left( \frac{X'MWX}{T} \right)^{-1} \cdot \text{plim} \frac{X'\varepsilon}{T}. \quad (B4)$$

We presume that the short-term real interest rate, $r_t$, can be expressed as

$$r_t = f(\text{exogenous variables}) + u_t, \quad (B5)$$

where ‘exogenous’ implies uncorrelated with the error term in Equation (B2) and $u_t$ is determined by the policy-maker on the basis of information about current and future output not available to the econometrician estimating the output equation (B2).

As explained in the text, the correlation coefficient between real interest rates and the error term in Equation (B2) is assumed to be a geometrically declining function of the lag of the real interest rate, with no correlation after the sixth lag. The covariance between $Z_{t-1}$ and $\varepsilon_t$ (which is identical to the covariance between $y_{t-1}$ and $\varepsilon_t$) is denoted $\sigma_\varepsilon \sigma_u \theta$ and is derived below. In symbols we have

$$\text{plim} \frac{X'\varepsilon}{T} = \sigma_\varepsilon \sigma_u \left[ \gamma \quad \gamma^2 \quad \gamma^3 \quad \ldots \quad \gamma^7 \quad \theta \right]. \quad (B6)$$

where $\sigma_\varepsilon$ and $\sigma_u$ are estimates of the standard deviations of the errors in Equations (B1) and (B5). For the underlying model, $\sigma_\varepsilon = 0.56$, while for $\sigma_u$, we use the value derived from estimating Equation (3) in the text (which is a simple version of Equation (B5)). This gives the estimate $\sigma_u = 1.32$.

Now define the variables $C_i$, $i = 0, \ldots, 6$ by

$$C_i = \sigma_\varepsilon \sigma_u \sum_{j=0}^{6-i} \beta^j \gamma^{i+j+1}. \quad (B7)$$
Denote the covariance between $\varepsilon_t$ and $y_{t-i}$ as $PL_i$, and between $\varepsilon_t$ and $\Delta y_{t-i}$ as $PC_i$. The model, Equation (B2), implies the recursive structure,

\begin{align*}
PL_7 &= 0 \\
PC_6 &= C_6 \\
PL_i &= PL_{i+1} + PC_i \\
PC_i &= C_i + \delta PL_{i+1}.
\end{align*}

We require $\theta = PL_i / \sigma_{\varepsilon} \sigma_u$, which is a function of the true vector $\alpha$. For given $\gamma$ in the range zero to 0.75, we proceed as follows. First, we use the sample value of $\begin{pmatrix} X' M W X \end{pmatrix}^{-1}$ as our estimate of $\text{plim} \begin{pmatrix} X' M W X \end{pmatrix}^{-1}$. (This requires an estimate of the cointegrating vector, $\chi^*$, between $y$ and $w$; we use the OLS estimate for this.) Next, we use the OLS estimate, $\hat{\alpha}_{OLS}$ to generate an estimate $\hat{\theta}$ via Equation (B8).

We now have an estimate for $\text{plim} \frac{X' \varepsilon}{T}$ via Equation (B6). This enables us to generate an estimate, $\hat{\alpha}$, of the ‘true’ vector $\alpha$ via Equation (B4). We now iterate: $\hat{\alpha}$ implies a new estimate for $\theta$, $\hat{\theta}$, which, in turn, implies a new estimate for $\alpha$, $\hat{\alpha}$. This process is continued until it converges, yielding $\bar{\alpha}$. The estimated response to a permanent 1 per cent increase in the real interest rate on year-ended growth shown in Figure 4 and on the average lag of monetary policy shown in Figure 5 are generated using $\bar{\alpha}$. 
References


