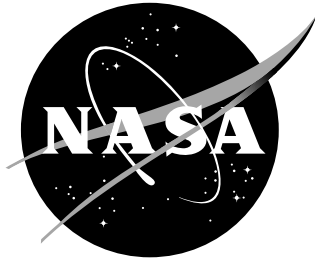# Robust Control of Uncertain Systems via Dissipative LQG-Type Controllers

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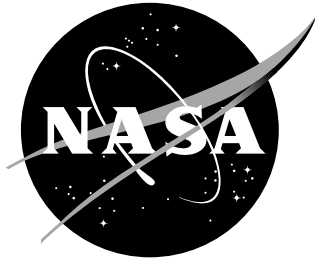- CONTRACTOR REPORT. Scientific and technical findings by NASA-sponsored contractors and grantees.

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March 2000

Robust Control of Uncertain Systems via Dissipative LQG-Type Controllers

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Abstract

Optimal controller design is addressed for a class of linear, time-invariant systems which are dissipative with respect to a quadratic power function. The system matrices are assumed to be affine functions of uncertain parameters confined to a convex polytopic region in the parameter space. For such systems, a method is developed for designing a controller which is dissipative with respect to a given power function, and is simultaneously optimal in the linear-quadratic-Gaussian (LQG) sense. The resulting controller provides robust stability as well as optimal performance. Three important special cases, namely, passive, norm-bounded, and sector-bounded controllers, which are also LQG-optimal, are presented. The results give new methods for robust controller design in the presence of parametric uncertainties.

1   Introduction and Problem Statement

Over the past three decades, a number of design methods have been developed for control of dynamic systems with uncertainties. The methods include H_∞, μ- synthesis, passivity, etc. The objective is to obtain closed-loop robust stability and optimal performance in spite of model uncertainties. A large class of dynamic systems can be characterized as "dissipative systems" which have the property that some of the energy put into the system gets dissipated. The term "energy" is defined in a very general sense, and thus the property of dissipativity

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encompasses a number of subclasses (for example, passive systems). A major advantage of dissipative systems is that they can be robustly stabilized by a controller that itself satisfies a certain dissipativity condition.

This paper presents an approach for the control of dissipative systems via linear quadratic Gaussian (LQG) controllers that are also restricted to be dissipative. The method employs the above-mentioned stabilization result to ensure robust closed-loop stability, and optimal LQG-type controller to achieve the required performance. The approach generalizes the LQG-optimal passive controller design of [1] to a broad class of dissipative systems. The method is subsequently specialized to three important subclasses of dissipative systems, namely, passive systems, norm-bounded systems, and sector-bounded systems.

1.1 Dissipative Systems

Consider a linear, time-invariant system Σ:

$$\dot{x} = Ax + Bu; \quad y = Cx \tag{1}$$

where x, u, y are n-, m-, and m- dimensional state, input, and output vectors respectively. We assume that A has all eigenvalues in the closed left-half plane.

Define the quadratic "power function"

$$p(y, u) = [y^T, u^T] \begin{bmatrix} Q & N \\ N^T & R \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} \tag{2}$$

The following definition is based on [2], [3].

Definition- The system Σ is said to be dissipative with respect to the power function $p(y, u)$ if there exists a positive definite quadratic "energy function" $E(x) = x^T P x$ with $P = P^T > 0$ such that the following dissipation inequality is satisfied:

$$\int_0^T p(y, u)dt \geq E[x(T)] - E[x(0)] \tag{3}$$

$\forall T \in [0, \infty)$ and $\forall u \in \mathcal{L}_{2e}^m$.

($\mathcal{L}_{2e}^m$ denotes the extended Lebesgue space of functions that are square-integrable over all finite intervals). The dissipativity condition above can also be expressed in the differential form as:

$$\frac{d}{dt}E(x) \leq p(y, u) \tag{4}$$

Three important special cases of dissipative systems are defined below.

Definition- A system which is dissipative with respect to the power function $p(y, u)$ in Eq. (2) is said to be

- *Passive* if $Q = 0$, $R = 0$, and $N = I$.

- *Norm-bounded* if $Q = -I$, $R = \gamma^2 I$, and $N = 0$ for some finite $\gamma > 0$. In this case, $\gamma \geq H_\infty$-norm of the system.

- *sector-bounded* inside the sector $[a, b]$, $a < 0 < b$ if $Q = -I$, $R = -abI$, and $N = \alpha I$ with $\alpha = (a + b)/2$.

It should be noted that norm-bounded and sector-bounded systems have to be stable (i.e., all eigenvalues of A must have negative real parts) while passive systems can be marginally stable.

1.2 Stability of Feedback Interconnection

The following result, referred to as the dissipativity lemma, is from [4].

Lemma- The system Σ is dissipative with respect to the power function $p(y, u)$ in Eq. (2) iff there exists a symmetric, positive-definite matrix P and matrices L and W such that the following equations are satisfied

$$A^T P + PA = C^T QC - L^T L \tag{5}$$

$$PB = C^T(QD + N) - L^T W \tag{6}$$

$$R + N^T D + D^T N + D^T QD = W^T W \tag{7}$$

Or equivalently, iff the following linear matrix inequality (LMI) is satisfied for some symmetric positive definite matrix P

$$\begin{bmatrix} A^T P + PA - C^T Q C & PB - C^T (QD + N) \\ B^T P - (QD + N)^T C & -(R + N^T D + D^T N + D^T QD) \end{bmatrix} \leq 0 \qquad (8)$$

A definition of strict dissipativity is given next, which is a generalization of the "weak strict positive real (WSPR)" definition given in [5].

Definition- If A is Hurwitz, (A, L) is observable, and $[A, B, L, W]$ is minimum-phase, the system Σ is said to be *strictly* dissipative with respect to power function $p(y, u)$.

We consider feedback-interconnected dissipative systems Σ_1 and Σ_2, which are dissipative with respect to power functions p_1 and p_2 defined as follows:

$$p_i(y, u) = [y^T, u^T] \begin{bmatrix} Q_i & N_i \\ N_i^T & R_i \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} \qquad (9)$$

The following result from [4] gives a sufficient condition for stability.

Theorem 1- Consider two systems Σ_1 and Σ_2 connected in the negative feedback configuration (Figure 1). Suppose Σ_1 is dissipative with respect to p_1 and Σ_2 is dissipative with respect to p_2. Then the interconnected system is Lyapunov stable if there exist scalars $\alpha_1 > 0$ and $\alpha_2 > 0$ such that

$$\alpha_1 p_1(y, u) + \alpha_2 p_2(-u, y) \leq 0 \quad \forall u, y \in \mathbf{R}^m \qquad (10)$$

Furthermore, if at least one of the systems is strictly dissipative, then the interconnected system is asymptotically stable.

The systems Σ_1 and Σ_2 can be considered to be the plant and the controller, respectively. The significance of above result is that the stability is robust to model uncertainties; as long as the plant is dissipative with respect to power function p_1, any controller which is dissipative with respect to power function p_2 will stabilize it, provided that the condition of Theorem 1 is satisfied. For example, if the plant is passive, it is stabilized by any strictly passive controller [5]; if the plant is norm-bounded by γ, any controller which is strictly norm-bounded by $1/\gamma$ will stabilize it; and if the plant belongs to sector $[a, b]$, any controller which belongs (strictly)

to sector $[-\frac{1}{b}, -\frac{1}{a}]$ will stabilize it. In the case of passive systems, it is possible to further weaken the requirement of strict passivity to marginally strict passivity [6].

We consider a class of systems $\mathcal{S}(\theta)$ given by

$$\dot{x} = A(\theta)x + B(\theta)u + v; \quad y = C(\theta)x + w \tag{11}$$

where $\theta \in \mathbf{R}^k$ denotes the vector of uncertain parameters and x, u, y are n-, m-, and m- dimensional state, input, and output vectors respectively. v and w denote zero-mean Gaussian white noise processes with covariance intensities Q_f and R_f respectively. $A(\theta)$, $B(\theta)$, $C(\theta)$ are appropriately dimensioned matrices that are assumed to be affine functions of the parameter vector θ. The parameter vector θ assumed to lie in a convex polytopic region $\mathcal{P}$ in the parameter space, bounded by vertices θ^j, $j = 1, 2, \cdots l$. For example, if each component of θ lies in an interval $[\underline{\theta}_i, \overline{\theta}_i]$, $\mathcal{P}$ would be a hyper-rectangular box in $\mathbf{R}^k$, and the number of vertices $l = 2^k$. We assume that the system is stable (i.e., $A(\theta)$ has all eigenvalues in the open left-half plane), and the realization is minimal in the entire region $\mathcal{P}$. The system is assumed to be strictly proper because the objective is to design an LQG-type controller.

Suppose the *nominal* system is represented by the following minimal realization

$$\dot{x} = Ax + Bu + v; \quad y = Cx + w \tag{12}$$

The problem is to obtain a controller which minimizes the performance function

$$J = \lim_{T \to \infty} \frac{1}{T} \mathcal{E} \int_0^T (x^T Q_r x + u^T R_r u) dt \tag{13}$$

and maintains closed-loop stability in the presence of parametric uncertainties, where $Q_r \geq 0$ and $R_r > 0$ are appropriately dimensioned symmetric matrices.

1.3 Dissipativity in the Presence of Uncertain Parameters

The following theorem gives a sufficient condition for the system to be dissipative in the presence of parametric uncertainties, and is a generalization of the result in [7] for passive systems.

Theorem 2-The system $[A(\theta), B(\theta), C(\theta)]$ is dissipative with respect to the quadratic power function $p(y, u)$ defined in Eq. (2) $\forall \theta \in \mathcal{P}$ if there exists a matrix $P = P^T > 0$ such that

$$Z(\theta^j, P) := \begin{bmatrix} A(\theta^j)^T P + PA(\theta^j) - C(\theta^j)^T QC(\theta^j) & PB(\theta^j) - C(\theta^j)^T N \\ B(\theta^j)^T P - N^T C(\theta^j) & -R \end{bmatrix} \leq 0$$

$$for \ j = 1, 2, \cdots l \qquad (14)$$

Proof- The proof is similar to that for the quadratic stability case [8]. Because $\mathcal{P}$ is a convex region in the parameter space, the function $\mathcal{F} = x^T Z(\theta, P)x$ is convex in $\theta \ \forall x \in \boldsymbol{R}^{n+m}$. Therefore $\mathcal{F}$ takes on its maximum value at one or more of the vertices of $\mathcal{P}$, and thus $Z(\theta, P) \leq 0 \ \forall \theta \in \mathcal{P}$ if (14) is satisfied. ∎

The significance of this property ("robust" dissipativity) is that any controller satisfying condition (10) can robustly stabilize the system. For example, for passive systems, one can determine the largest region $\mathcal{P}$ in which the system remains passive [7]. For norm-bounded systems, one can determine the largest $\mathcal{P}$ in which the system norm remains below a certain value (γ), or for given $\mathcal{P}$, the smallest norm of the system. For sector-bounded systems, one can determine the largest $\mathcal{P}$ for a given sector $[a, b]$, or the smallest sector for a given $\mathcal{P}$. For these three cases, any controller that is (respectively) passive, or norm-bounded (by γ^{-1}), or inside the sector $[-b^{-1}, -a^{-1}]$, will robustly stabilize the system. The next section addresses design of such robust controllers that are also optimal.

2 Optimal Controller

Given that an uncertain plant is dissipative with respect to a quadratic power function p_1, the approach considered in this paper is to design an optimal controller which is restricted to be dissipative with respect to a quadratic power function p_2 chosen in such a way that the conditions of Theorem 1 are satisfied.

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2.1 A Special Realization

Suppose the system (12) is dissipative with respect to the quadratic power function p_1 with weights (Q_1, N_1, R_1). Then $\exists$ a matrix $P_1 = P_1^T > 0$ such that (8) is satisfied. Suppose $\Gamma \in \mathbf{R}^{n \times n}$ is a square root of P_1, i.e., $P_1 = \Gamma^T \Gamma$. Using the coordinate transformation $\xi = \Gamma x$, the system (omitting the noise terms) becomes

$$\dot{\xi} = \hat{A}\xi + \hat{B}u \tag{15}$$

$$y = \hat{C}\xi \tag{16}$$

where $\hat{A} = \Gamma A \Gamma^{-1}$, $\hat{B} = \Gamma B$, $\hat{C} = C \Gamma^{-1}$. Premultiplying (8) (with $D = 0$) by $diag[\Gamma^{-T}, I_m]$ and postmultiplying by its transpose, we have

$$\begin{bmatrix} \hat{A}^T + \hat{A} - \hat{C}^T Q_1 \hat{C} & \hat{B} - \hat{C}^T N_1 \\ \hat{B}^T - N_1^T \hat{C} & -R_1 \end{bmatrix} := - \begin{bmatrix} Q_A & Q_{12} \\ Q_{12}^T & R_1 \end{bmatrix} \le 0 \tag{17}$$

Therefore, without loss of generality, it will be assumed that the system is in this form, i.e., (8) is satisfied with $P = I$, and the "hat" notation will be dropped in the subsequent material.

2.2 LQG-Optimal Controller

For the nominal system, it is well-known that the controller which minimizes J consists of a linear-quadratic regulator (LQR) and a Kalman-Bucy filter (KBF), and is given by:

$$\dot{x}_c = A_c x_c + B_c y \tag{18}$$

$$y_c = C_c x_c \quad u = -y_c \tag{19}$$

where

$$A_c = A - BR_r^{-1}B^T P_r - P_f C^T R_f^{-1} C \quad B_c = P_f C^T R_f^{-1} \quad C_c = R_r^{-1} B^T P_r \tag{20}$$

$$P_r A + A^T P_r - P_r B R_r^{-1} B^T P_r + Q_r = 0 \tag{21}$$

$$P_f A^T + A P_f - P_f C^T R_f^{-1} C P_f + Q_f = 0 \tag{22}$$

The problem is to find an LQG controller that is dissipative with respect to the quadratic power function p_2 with given weights $(Q_2,\ N_2,\ R_2)$. It is assumed that $Q_2 \leq 0$, which is usually the case for most dissipative systems.

Two cases, $R_2 > 0$ and $R_2 = 0$, will be considered separately. The following theorem gives an LQG controller that is dissipative with respect to p_2 for the case $R_2 > 0$.

Theorem 3- Suppose $R_2 > 0$, and the LQG performance function weights are such that $R_r > 0$, $Q_r > 0$, and

$$R_r \geq N_2 R_2^{-1} N_2^T - Q_2 \tag{23}$$

$$Q_r > P_r[C^T R_f^{-1} R_2^{-1} R_f^{-1} C - C^T R_f^{-1} R_2^{-1} N_2^T R_r^{-1} B^T - B R_r^{-1} N_2 R_2^{-1} R_f^{-1} C]P_r$$
$$-C^T R_f^{-1} C P_r - P_r C^T R_f^{-1} C \tag{24}$$

$$Q_f = Q_A + C^T (R_f^{-1} - Q_1)C \geq 0 \tag{25}$$

Then the resulting LQG controller is strictly dissipative with respect to the power function

$$p_2(y, u) = [y^T, u^T] \begin{bmatrix} Q_2 & N_2 \\ N_2^T & R_2 \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} \tag{26}$$

Proof- The controller (A_c, B_c, C_c) is dissipative with respect to p_2 iff $\exists$ a matrix $P_c = P_c^T > 0$ such that

$$Z_c := \begin{bmatrix} A_c^T P_c + P_c A_c - C_c^T Q_2 C_c & P_c B_c - C_c^T N_2 \\ B_c^T P_c - N_2^T C_c & -R_2 \end{bmatrix} \leq 0 \tag{27}$$

Since $R_2 > 0$, the above is equivalent to the following condition [9]:

$$A_c^T P_c + P_c A_c - C_c^T Q_2 C_c + (P_c B_c - C_c^T N_2)R_2^{-1}(B_c^T P_c - N_2^T C_c) \leq 0 \tag{28}$$

After substituting for $A_c,\ B_c,\ C_c$, (28) becomes

$$(A - B R_r^{-1} B^T P_r - P_f C^T R_f^{-1} C)^T P_c + P_c(A - B R_r^{-1} B^T P_r - P_f C^T R_f^{-1} C)$$
$$-P_r B R_r^{-1} Q_2 R_r^{-1} B^T P_r$$
$$-(P_c P_f C^T R_f^{-1} - P_r B R_r^{-1} N_2)R_2^{-1}(P_c P_f C^T R_f^{-1} - P_r B R_r^{-1} N_2)^T$$
$$\leq 0 \tag{29}$$

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Noting from (17) and (25) that the filter algebraic Riccati equation (ARE) (22) is satisfied with $P_f = I$, using the control ARE (21), and letting $P_c = P_r$, (29) can be written as

$$-Q_r - P_r B[R_r^{-1} + R_r^{-1} Q_2 R_r^{-1} - R_r^{-1} N_2 R_2^{-1} N_2^T R_r^{-1}]B^T P_r$$
$$+P_r[C^T R_f^{-1} R_2^{-1} R_f^{-1} C - C^T R_f^{-1} R_2^{-1} N_2^T R_r^{-1} B^T - B R_r^{-1} N_2 R_2^{-1} R_f^{-1} C]Pr$$
$$-C^T R_f^{-1} C P_r - P_r C^T R_f^{-1} C \leq 0 \qquad (30)$$

From (23),

$$R_r + Q_2 - N_2 R_2^{-1} N_2^T \geq 0 \qquad (31)$$

Pre- and post-multiplying the above by R_r^{-1},

$$R_r^{-1} + R_r^{-1} Q_2 R_r^{-1} - R_r^{-1} N_2 R_2^{-1} N_2^T R_r^{-1} \geq 0 \qquad (32)$$

From (24) and (32), (30) [and therefore (28)] is satisfied (with strict inequality "< ") and the LQG controller (A_c, B_c, C_c) is dissipative with respect to $p_2(y, u)$.

Inequality (27) holds in the strict sense; hence in (27),

$$Z_c = - \begin{bmatrix} L_c^T \\ W_c^T \end{bmatrix} [L_c \ W_c] < 0 \qquad (33)$$

where $L_c \in \boldsymbol{R}^{p \times n}$ and $W_c \in \boldsymbol{R}^{p \times m}$ $(p \geq n + m)$ are of rank n and m respectively. Because (27) holds in the strict sense and $Q_2 \leq 0$, A_c is Hurwitz. (A_c, L_c) is observable because L_c is of rank n. The transfer function of (A_c, B_c, L_c, W_c) is non-square and has no transmission zeros. Thus the controller is strictly dissipative with respect to the power function p_2. ∎

Remark 1- For most common dissipative systems, usually $Q_1 \leq 0$; therefore, (25) is satisfied for any $R_f > 0$.

Remark 2- It should be noted that it is always possible to choose the performance function weights to satisfy the conditions of Theorem 3. The procedure for designing an LQG controller that is dissipative with respect to a given power function p_2 consists of the following steps:

- Obtain a solution P_1 to the dissipativity LMI for the nominal system and transform to the special realization of (15), (16)

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- Choose LQR control weighting matrix $R_r > 0$ satisfying (23), and the state weighting matrix Q_r based on the performance requirements

- Solve the control ARE (21) to get P_r

- Choose R_f such that (24) is satisfied; choose Q_f as in (25)

- The controller is given by (20) with $P_f = I$

Remark 3- It should be noted that the KBF weighting matrices Q_f and R_f are used as design parameters and have no statistical significance.

When $R_2 = 0$, perhaps the only case of interest is when $Q_2 = 0$ and $N_2 = I$, which corresponds to passivity. In this case, the LQG controller is passive iff

$$
\begin{bmatrix} A_c^T P_c + P_c A_c & P_c B_c - C_c^T \\ B_c^T P_c - C_c & 0 \end{bmatrix} \le 0 \tag{34}
$$

As shown in [1], the above condition is satisfied with $P_c = P_r$ if

$$
\begin{align}
Q_r &> BR_r^{-1}B^T \tag{35} \\
R_f &= R_r \tag{36} \\
Q_f &= -(A + A^T) + BR_r^{-1}B^T \tag{37}
\end{align}
$$

It should be noted that the above LQG controller is weakly strict positive real (WSPR), i.e., it is stable, minimum-phase, and positive real, even when the open loop system is only marginally stable.

When $R_2 > 0$, two important special cases are norm-bounded and sector-bounded LQG controllers.

Corollary 3.1: Norm-bounded controller- Suppose the LQG performance functions weights are such that $Q_r > 0$ and

$$
R_r \ge I \tag{38}
$$

$$
Q_r > \gamma^{-2} P_r C^T R_f^{-2} C P_r - C^T R_f^{-1} C P_r - P_r C^T R_f^{-1} C \tag{39}
$$

$$
Q_f = -(A + A^T) + C^T R_f^{-1} C \tag{40}
$$

Then the controller is norm bounded by γ, i.e., $\|G_c(s)\|_\infty \le \gamma$, where $G_c(s) = C_c(sI - A_c)^{-1}B_c$.

Note that (38), (39), and (40) correspond to (23), (24), and (25) of Theorem 3 (with $Q_2 = -I$, $R_2 = \gamma^2 I$, $N_2 = 0$).

Corollary 3.2: Sector-bounded controller- Suppose the LQG performance functions weights are such that $Q_r > 0$ and

$$R_r \ge \left[1 - \frac{(a+b)^2}{4ab}\right]I \tag{41}$$

$$Q_r > -\left(\frac{1}{ab}\right)P_r[C^T R_f^{-2}C - (C^T R_f^{-1}R_r^{-1}B^T + BR_r^{-1}R_f^{-1}C)(a+b)/2]P_r$$
$$-C^T R_f^{-1}CP_r - P_r C^T R_f^{-1}C \tag{42}$$

$$Q_f = -(A + A^T) + C^T R_f^{-1}C \tag{43}$$

Then the controller belongs to the sector $[a, b]$ $(a < 0 < b)$ in the strict sense.

Remark 4- Design of LQG controllers that belong to the $[0, -\frac{1}{a})$ sector for systems belonging to the $[a, \infty)$ sector was considered in [10], [11].

3 Concluding Remarks

Design of linear quadratic Gaussian (LQG) controllers was considered for a class of uncertain systems which are dissipative with respect to a quadratic power function. The system matrices were assumed to be affine functions of parameters belonging to a convex polytopic region. A method was given for designing an LQG controller that is dissipative with respect to a given quadratic power function. Three important special cases, passive, norm-bounded, and sector-bounded controllers, were presented. By appropriately choosing the weighting functions, the controller can be designed to provide optimal performance as well as robust stability in the presence of parametric uncertainties.

References

[1] Lozano, R., and Joshi, S. M.: On the Design of Dissipative LQG-Type Controllers. *Recent Advances in Robust Control* (P. Dorato and R. Yedavalli, Eds.), IEEE Press, 1990.

[2] Willems, J. C.: Dissipative Dynamical Systems- Parts I and II: General Theory. *Arch. Rational Mechanics and Analysis*, Vol. 45, 1972, pp. 321-391.

[3] Hill, D., Ortega, R., and van der Schaft, A.: *Nonlinear Controller Design Using Passivity and Small-Gain Techniques*. Notes from Tutorial Workshop at the IEEE Conference on Decision and Control, Orlando, FL, December 1994.

[4] Gupta, S.: Robust Stabilization of Uncertain Systems Based on Energy Dissipation Concepts. NASA CR 4713, March 1996.

[5] Lozano-Leal, R., and Joshi, S. M.: Strictly Positive Real Functions Revisited. *IEEE Trans. Auto. Contr.*, vol. AC-35, No. 11, November 1990, (pp. 1243-1245).

[6] Joshi, S. M., and Gupta, S.: On a Class of Marginally Stable Positive Real Systems. *IEEE Trans. Auto. Contr.*, vol. AC-41, No. 1, January 1996 (pp. 152-155).

[7] Joshi, S. M., and Kelkar, A. G.: Robust Passification via Optimal Sensor Blending and Control Allocation. Proc. 1999 American Control Conference, San Diego, CA, June 2-4, 1999.

[8] Horisberger, H. P., and Belanger, P. R.: Regulators for Linear Time Invariant Plants with Uncertain Parameters. *IEEE Trans. Auto. Contr.*, vol. AC-12, No. 5, October 1976 (pp. 705-708).

[9] Boyd, S., El-Ghaoui, L., Feron, E., and Balakrishnan, E.: *Linear Matrix Inequalities in System and Control Theory*. Philadelphia: SIAM, 1994.

[10] Drai, R.: Design of LQG-Dissipative Controllers Using Linear Matrix Inequalities. Fifth IEEE Mediterranean Conference, Cyprus, July 1997.

[11] Drai, R.: H_2 Dissipative Controllers for Non-Positive Systems. Sixth IEEE Conference on Control Applications, Hartford, CT, October 1997.
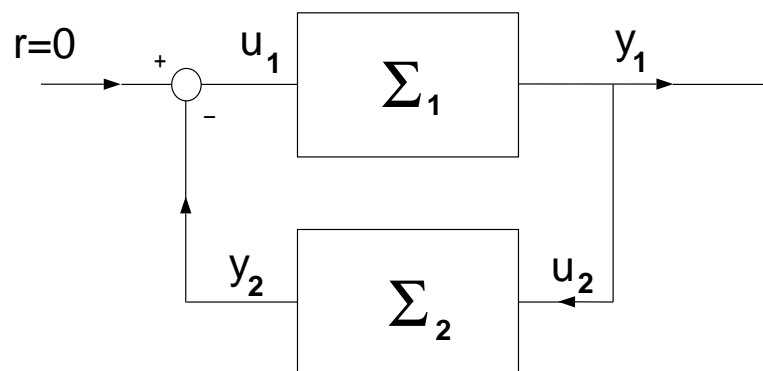
Figure 1: Interconnected dissipative systems

REPORT DOCUMENTATION PAGE

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	March 2000	Technical Memorandum

4. TITLE AND SUBTITLE
Robust Control of Uncertain Systems via Dissipative LQG-Type Controllers

5. FUNDING NUMBERS
706-11-21-02

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
NASA Langley Research Center
Hampton, VA 23681-2199

8. PERFORMING ORGANIZATION REPORT NUMBER
L-17961

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Washington, DC 20546-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA/TM-2000-209866

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified-Unlimited
Subject Category 63   Distribution: Nonstandard
Availability: NASA CASI (301) 621-0390

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)
Optimal controller design is addressed for a class of linear, time-invariant systems which are dissipative with respect to a quadratic power function. The system matrices are assumed to be affine functions of uncertain parameters confined to a convex polytopic region in the parameter space. For such systems, a method is developed for designing a controller which is dissipative with respect to a given power function, and is simultaneously optimal in the linear-quadratic-Gaussian (LQG) sense. The resulting controller provides robust stability as well as optimal performance. Three important special cases, namely, passive, norm-bounded, and sector-bounded controllers, which are also LQG-optimal, are presented. The results give new methods for robust controller design in the presence of parametric uncertainties.

14. SUBJECT TERMS
Robust Control, Dissipative Systems, Optimal Control

15. NUMBER OF PAGES
18

16. PRICE CODE
A03

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL