A method for calculating proton-nucleus elastic cross sections

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Abstract:

Recently [20,21], we developed a method of extracting nucleon-nucleon (N-N) cross sections in the medium directly from experiment. The in-medium N-N cross sections form the basic ingredients of several heavy-ion scattering approaches including the coupled-channel approach developed at the NASA Langley Research Center. We investigated [22,23] the ratio of real to imaginary part of the two body scattering amplitude in the medium. These ratios are used in combination with the in-medium N-N cross sections to calculate proton-nucleus elastic cross sections. The agreement is excellent with the available experimental data. These cross sections are needed for the radiation risk assessment of space missions.

Introduction

The transportation of energetic ions in bulk matter is of direct interest in several areas including shielding against ions originating from either space radiations or terrestrial accelerators, cosmic ray propagation studies in galactic medium or radiobiological effects resulting from the work place or clinical exposures. For carcinogenesis, terrestrial radiation therapy, and radiobiological research, knowledge of beam composition and interactions is necessary to properly evaluate the effects on human and animal tissues. For the proper assessment of radiation exposures both reliable transport codes and accurate input parameters are needed. One such important input is elastic cross sections. The motivation of the work is to develop a method for calculating accurate elastic cross sections. These are needed in transport methods both deterministic and Monte Carlo.

Nucleon-nucleon (N-N) cross sections are the basic ingredients of many approaches [1 - 10] to heavy ion scattering problem. Most of the information about these N-N cross sections comes from the free two-body scattering. These cross sections are significantly modified in a nucleus, due to presence of other nucleons, which is affected [11] through the Pauli exclusion principle and modification of meson field coupling constants. The in-medium corrections are very important effects and nuclear-physics literature is quite rich on this subject [For example, see references [12-19]]. Most of these theoretical methods either use nonrelativistic many-body [12-14] or relativistic many-body [15-19] approaches. Unfortunately, there is no agreement between different approaches. As a result, there is no consensus on this issue. The essence of the problem is to answer a simple question as to how a nucleon behaves and interacts inside a nucleus. The theoretical approaches incorporate the renormalization effects to include the effects of the medium in the chosen approach. The goal, of course is, to be able to explain the experimental results. Our work [20-23] is technology based. As a result, we approach the problem from the other end. And ask ourselves the question as to what experiments tell us about the in-medium effects. Consequently, our results may also provide the meeting ground for various approaches on this issue and on the Coulomb effects discussed later in the text. Our theoretical approach is based on the coupled channel method [1-6] used at the NASA Langley Research Center. This method solves the Schrodinger equation with an eikonal approximation. The method needs modifications at low and medium energies. In an earlier work [20,21], we developed a unique method of extracting medium modified N-N cross sections from experiments and found that the renormalization of the free N-N cross sections is significant [20,21] at lower and medium energies. These modified in-medium N-N cross sections, in combination with the newly developed ratio of the real to imaginary part of the two-body scattering amplitudes in the medium, were used to calculate the total cross sections for proton-nucleus collisions [22,23]. We demonstrated that the blend of the renormalized N-N cross sections, the in-medium ratio of the real to imaginary part of the two-body amplitude and the coupled-channel method gave reliable approach to the total cross sections. The purpose of the current paper is three folds:

(i). To put in place a reliable method for calculating elastic cross sections for collisions of protons with ions. (ii) To use our previously developed N-N cross sections in the medium and modified two-body amplitudes to calculate elastic cross sections for proton-nucleus collisions;

(iii) To validate and compare the calculated results with the available experimental data. And provide theoretical results where data are not available (due to nonexistence of experimental facilities and/or difficulty in experimental data analysis).

Method

We briefly sketch here the essentials of the coupled-channel method for completeness (see reference 1 through 6 for details). In this approach the matrix for elastic scattering amplitude is given by,

$$f(\mathbf{q}) = -\frac{ik}{2\pi}\int d^2\mathbf{b}\,\exp(-i\mathbf{q}.\mathbf{b})\{\exp[i\chi(\mathbf{b})]-1\} \qquad 1$$

where *f* and χ represent matrices, k is the projectile momentum relative to the center of mass, **b** is the projectile impact parameter vector, **q** is the momentum transfer, and $\chi(\mathbf{b})$ is the eikonal phase matrix.

The total cross section is found from the elastic scattering amplitude by using the optical theorem as follows:

$$\sigma_{tot} = \frac{4\pi}{k}\, Im\, f(q=0) \qquad 2$$

Equations (1) and (2) give,

$$\sigma_{tot} = 4\pi\int_0^\infty db\, b\, \{1 - e^{-Im(\chi)}\cos(Re(\chi))\} \qquad 3$$

And the absorption cross sections (σ_{abs}) is given by [20,21]

$$\sigma_{abs} = 2\pi\int_0^\infty db\, b\, \{1 - e^{-2Im(\chi)}\} \qquad 4$$

Having calculated the total and absorption cross sections, for many nuclei, elastic cross section is the difference of these quantities,

$$\sigma_{el} = \sigma_{tot} - \sigma_{abs} \qquad 5$$

The eikonal phase matrix χ (see [1-6] for details) is given by,

$$\chi(\mathbf{b}) = \chi_{dir}(\mathbf{b}) - \chi_{ex}(\mathbf{b}) \qquad \mathbf{6}$$

The direct and exchange terms are calculated using the following expressions [1-6],

$$\chi_{dir}(\mathbf{b}) = \frac{A_P A_T}{2\pi k_{NN}}\int d^2\mathbf{q}\, \exp(i\mathbf{q}.\mathbf{b})\, F^{(1)}(-\mathbf{q})\, G^{(1)}(\mathbf{q})\, f_{NN}(\mathbf{q}) \qquad 7$$

And

$$\chi_{ex}(\mathbf{b}) = \frac{A_P\, A_T}{2\pi\, k_{NN}} \int d^2\mathbf{q}\, \exp(i\mathbf{q}.\mathbf{b})\, F^{(1)}(-\mathbf{q})\, G^{(1)}(\mathbf{q}) \times \frac{1}{(2\pi)^2} \int d^2\mathbf{q}'\, \exp(i\mathbf{q}'.\mathbf{b})\, f_{NN}(\mathbf{q}+\mathbf{q}')\, C(\mathbf{q}') \qquad 8$$

where, $F^{(1)}$ and $G^{(1)}$ *are* projectile and target ground-state one-body form factors, respectively, k_{NN} is the relative wave number in the two-body center of mass system, and C is the correlation function [6]. The mass numbers of projectile and target nuclei are represented by A_P and A_T, respectively. The two-body amplitude, f_{NN}, is parameterized as,

$$f_{NN} = \frac{\sigma(\alpha+i)}{4\pi} k_{NN} \exp\left(-\frac{B\mathbf{q}^2}{2}\right) \qquad 9$$

where, σ is the two-body cross section, B is the slope parameter, and α is the ratio of the real part to imaginary part of the forward, two-body amplitude.

It is well known that the absorption cross section depends on the imaginary part of the eikonal phase matrix. This lead us to write [20,21] the two-body amplitude in the medium, $f_{NN,m}$, as

$$f_{NN,m} = f_m\ f_{NN} \qquad 10$$

where f_{NN} is the free NN amplitude and f_m is the system and energy dependent medium multiplier function [20,21]. It follows that the nucleon-nucleon cross sections in the medium ($\sigma_{NN,\,m}$) can be written as,

$$\sigma_{NN,m} = f_m \, \sigma_{NN} \qquad 11$$

where, σ_{NN} is the nucleon-nucleon cross section in free space, and the medium multiplier is given by,

$$f_m = 0.1 \exp(-E/12) + [1 - (\frac{\rho_{av}}{0.14})^{(\frac{1}{3})} \exp(-\frac{E}{D})] \qquad 12$$

where, E is the laboratory energy in units of A MeV. D is a parameter, in units of MeV, as defined below. The numbers 12 and 0.14 are in units of MeV and fm^{-3} respectively. For $A_T \leq 56$ (mass number for iron ion representing heavy elements considered in our transport phenomena),

$$D = 46.72 + 2.21 A_T - 2.25 x 10^{-2} A_T^2 \qquad 13$$

And, for $A_T \rangle 57$,

$$D = 100 \text{ MeV} \qquad 14$$

In Eq. (10), $\rho_{av,}$ refers to the average density of the colliding system,

$$\rho_{av} = \frac{1}{2} (\rho_{A_P} + \rho_{A_T}) \qquad 15$$

where the density of a nucleus A_i (i = P, T) is calculated in the hard sphere model, and is given by,

$$\rho_{A_i} = \frac{A_i}{\frac{4\pi}{3} r_i^3} \qquad 16$$

where the radius of the nucleus r_i is defined by,

$$r_i = 1.29\,(r_i)_{rms} \qquad 17$$

The root-mean-square radius, $(r_i)_{rms}$ is obtained directly from experiment [24] after "subtraction" of the nucleon charge form factor [2].

We also note from Eqs. (3), that total cross section depends on real component of eikonal phase matrix and hence (see Eqs. 5, 6 and 7) on the product of $\sigma\alpha$ in two-body amplitude. Since we have determined and tested thoroughly [20,21] the modification of the cross sections in the medium, we study here the modification of α - ratio of real to imaginary part of the two-body amplitude - in the medium in order to calculate the total cross sections. Some data for total cross sections are available for a few systems at high energies. Unfortunately, no data are available for total cross sections at low and medium energy range (there is some data for p+Pb in the 100 A MeV range). Therefore, values of the medium modified α have been tested for higher energies. At low and medium energies, our theoretical results, which incorporate the in-medium two-body amplitudes, can be validated, if and when experimental data becomes available.

A best estimate of medium modified α accounting the enhancement of the cross sections [33] and stability is given by,

$$\alpha_m = 3\exp(-(E - 13\,A^{1/3})^2/5000) + \frac{K}{1 + \exp\left(\frac{10 - E}{75}\right)} \qquad 18$$

where,

$$K = 0.35 + 0.65\exp(-\frac{2}{3}(N - Z)) \qquad 19$$

with N being the neutron number of the nucleus and Z its charge number.

We have also modified Eq. (3), to account for the Coulomb force in the proton-nucleus cross sections. Coulomb effects are very interesting and have been extensively worked on in the literature [See, for example references 25-30]. There is general agreement that there is a need to modify the free nucleon-nucleon coulomb interaction for the nucleus-nucleus collisions. There is model dependence on the modifications of the used theoretical approach. Here, again, we approach this from technology perspective and ask ourselves what the experiments tell us about this effect. Secondly, to keep the accepted form of the effect uniform we investigated [20-23] this using the available experimental data and found what medications are needed to explain the experimental results. In view of the importance of the problem and its wide use, in many different areas, in physics there will remain an active interest in this area. Our work may provide a useful bridge between various theoretical models and experiments. Our work indicated that this has significant effects at low energies and becomes less important as the energy increases and practically disappears for energies around 50 A MeV and higher. In the present work we use this formalism and recap the essentials features for completeness.

For nucleus-nucleus collisions the Coulomb energy is given by,

$$V_B = \frac{1.44\, Z_P \; Z_T}{R} \qquad 20$$

where, constant 1.44 is in units of MeV fm, Z_P and Z_T are charge numbers for the projectile and target respectively and R, the radial distance between their centers is given by,

$$R = r_P + r_T + 1.2(A_P^{(1/3)} + A_T^{(1/3)}) / E_{CM}^{(1/3)} \qquad 21$$

The number 1.2 in Eq.19 is in units of fm MeV $^{(1/3)}$. In our earlier work [20,21], these expressions were used also for the proton-nucleus collisions in order to have a unified picture of any colliding system. However, as shown in references [20,21], Eq. (21) over estimates the radial distance between proton-nucleus collisions and hence Eq. (20) under estimates the Coulomb energy between them. To compensate for this, we multiplied Eq. (20) by the following factor in these cases (see reference 20 and 21 for details), which gives the Coulomb multiplier to Eq. (3),

$$X_{Coul} = (1 + C_1 / E_{cm})(1 - C_2 V_B / E_{cm}) \qquad 22$$

For $A_T \leq 56$ (mass number for iron),

$$\begin{aligned} C_1 &= 6.81 - 0.17 A_T + 1.88x\,10^{-3} A_T^2 \\ C_2 &= 6.57 - 0.30 A_T + 3.6x\,10^{-3} A_T^2 \end{aligned} \qquad 23$$

The constant C_1 is in units of MeV. For $A_T \rangle$ 57,

$$\begin{aligned} C_1 &= 3.0 MeV \\ C_2 &= 0.8 \end{aligned} \qquad 24$$

For the nucleus-nucleus collisions, C_1 = 0 MeV and C_2 = 1. We have found that this form of Coulomb energy works well for the proton-nucleus absorption cross sections [20]. Eq. (5) is the main equation. The total cross section (Eq. 3) and absorption cross section (Eq.4) section are multiplied by Eq. (22) to get the total and absorption cross sections in the medium and then these are used in Eq. 5 to get the results shown in Figs. (1-6). For clarity we mention below the final expressions used for calculating the total and absorption cross sections in the medium,

$$\sigma_{tot} = 4\pi(1 + C_1 / E_{cm})(1 - C_2 V_B / E_{cm})\int_0^\infty db\, b\, \{1 - e^{-Im(\chi_m)}\cos(Re(\chi_m))\} \qquad 25$$

And,

$$\sigma_{abs} = 2\pi(1 + C_1 / E_{cm})(1 - C_2 V_B / E_{cm})\int_0^\infty db\, b\, \{1 - e^{-2Im(\chi_m)}\} \qquad 26$$

The constants C_1 and C_2 are given by Equations 21 and 22 and the in-medium eikonal phase matrix χ_m takes the form:

$$\chi_m(\mathbf{b}) = \chi_{m,dir}(\mathbf{b}) - \chi_{m,ex}(\mathbf{b}) \qquad 27$$

The direct and exchange terms in the medium are calculated using the following expressions,

$$\chi_{m,dir}(\mathbf{b}) = \frac{A_P A_T}{2\pi k_{NN}}\int d^2\mathbf{q}\exp(i\mathbf{q}.\mathbf{b})F^{(1)}(-\mathbf{q})G^{(1)}(\mathbf{q})f_{NN,m}(\mathbf{q}) \qquad 28$$

And

$$\chi_{m,ex}(\mathbf{b}) = \frac{A_P A_T}{2\pi k_{NN}}\int d^2\mathbf{q}\exp(i\mathbf{q}.\mathbf{b})F^{(1)}(-\mathbf{q})G^{(1)}(\mathbf{q}) \times \frac{1}{(2\pi)^2}\int d^2\mathbf{q}'\exp(i\mathbf{q}'.\mathbf{b})f_{NN,m}(\mathbf{q}+\mathbf{q}')C(\mathbf{q}') \qquad 29$$

The two-body amplitude in the medium, $f_{NN,\ m,}$ includes the in-medium nucleon-nucleon cross sections (σ_m),

$$f_{NN,m}(\mathbf{q}) = \frac{\sigma_m(\alpha_m + i)}{4\pi} k_{NN} \exp(-\frac{B\mathbf{q}^2}{2}) \qquad 30$$

Equations (25-30) are used to calculate the elastic cross sections shown in Figs. (1-6). The procedure is as follows: Using the in-medium NN cross sections calculate the two-body amplitude in the medium by Eq. (30). Then use Eqs. (28, 29) to calculate direct and exchange part of the eikonal phase matrix and Eq. (27) for the full eikonal matrix, which in turn is used in Eqs. (25) and (26) to calculate the total and absorption cross sections in the medium. And finally, Eq. (5) then gives the results shown in Figs. (1-6).

Results/Conclusions

Figures (1 - 6) show the results of our calculations for the elastic cross sections for proton on beryllium, carbon, aluminum, iron, lead and uranium targets respectively. The solid line includes the modifications discussed in the present work. The dotted line is without the corrections. The experimental data have been taken from the compilation of Ref. [31,32]. There is paucity of data at lower and intermediate energies, where the medium modifications play a significant role. For the energy ranges considered, where the data is unavailable, our results should provide good values of elastic cross sections, since many renormalization effects due to medium, which play important role in cross sections, have been built in the formalism. The reason for the enhancement in elastic cross sections in the intermediate energy range is mainly due to the fact that real part of the two-body scattering amplitude is more dominant compared to the imaginary of the two-body scattering amplitude in this energy range. Fig. 8 shows the in medium nucleon-nucleon cross sections as a function of energy for various systems, and Fig. 9 shows the in-medium ratio of real to imaginary part of the two-body amplitudes. It is good to note that these values are needed to explain the available experimental results and may also be useful for comparison with detailed theoretical calculations.

We find very good agreement with the experimental results for all the systems at higher energies where some data is available. We note that the in-medium cross sections derived earlier in combination with the modified ratio of real to imaginary part of the amplitude discussed here provide good results for the proton-nucleus elastic cross sections. It is gratifying to note that the present method gives a consistent approach for the total reaction and the total cross sections, hence, as discussed in the present work for the elastic cross sections for the entire energy range for all the systems studied here. The in medium two body modifications developed in the present approach can be used with ease in other nuclear processes.

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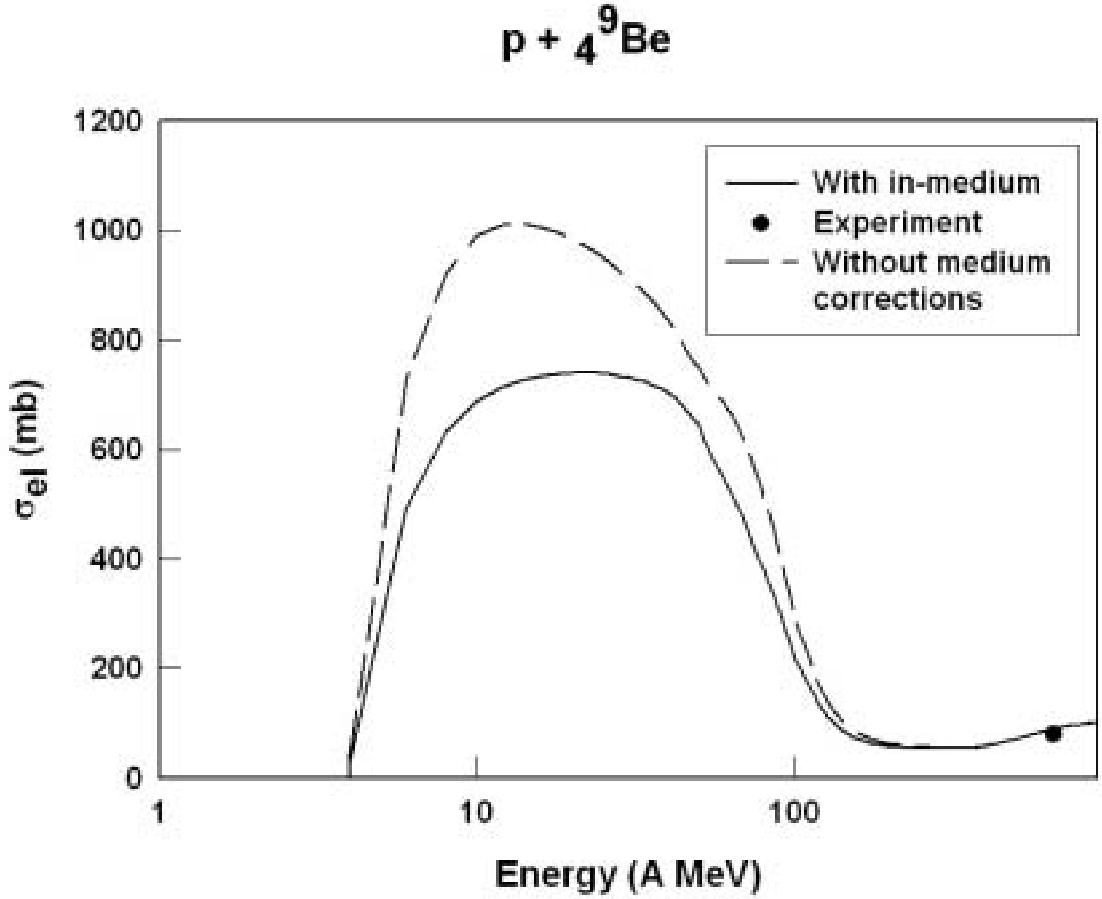

Fig 1.Elastic cross sections for proton-beryllium collision as a function of energy. The experimental points are taken from Ref. [31,32]. The solid line includes the modifications discussed in the present work. The dotted line is without the corrections

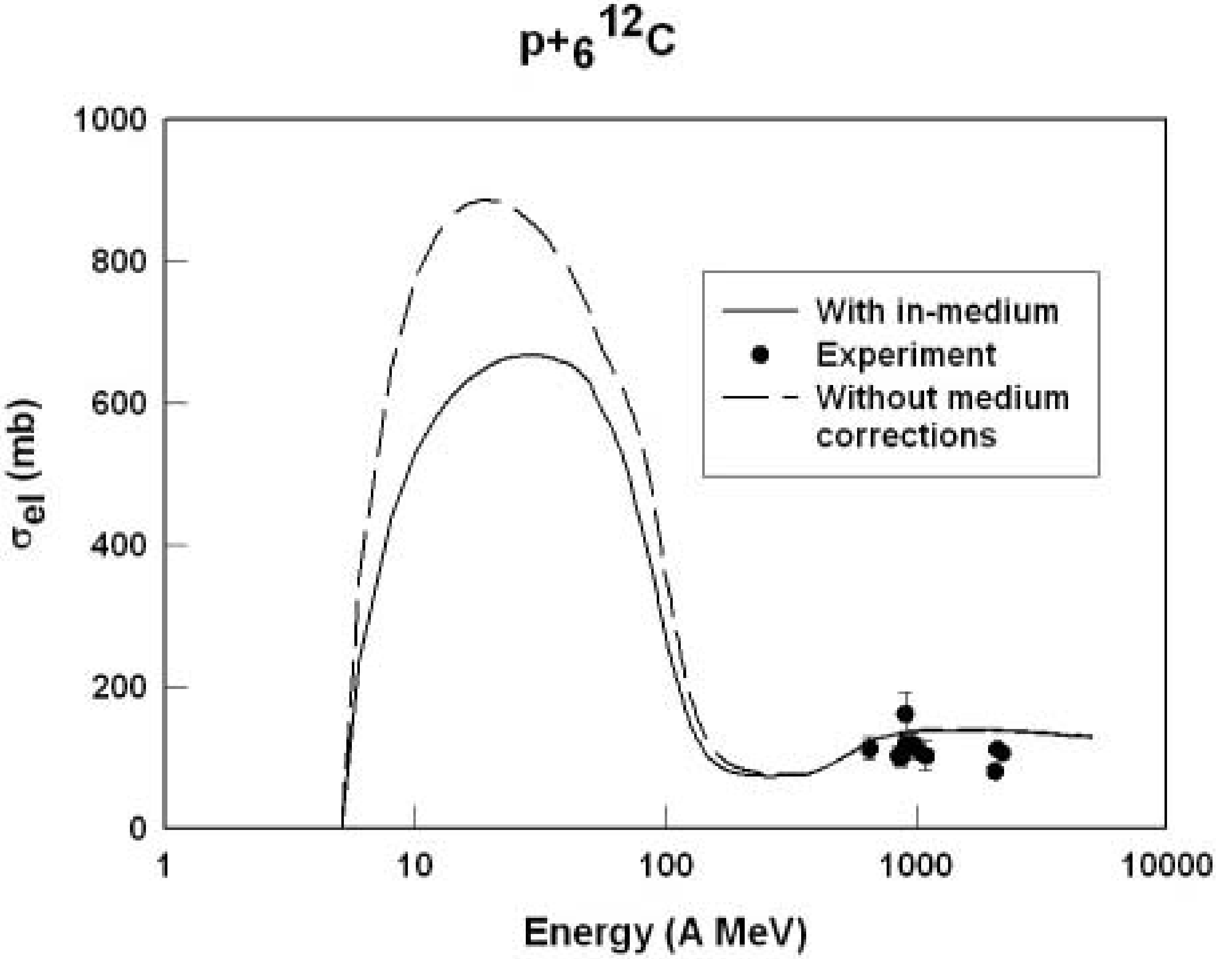

Fig 2 Elastic cross sections for proton-carbon collision as a function of energy. The experimental points are taken from Ref [31.32]. The solid line includes the modifications discussed in the present work. The dotted line is without the corrections

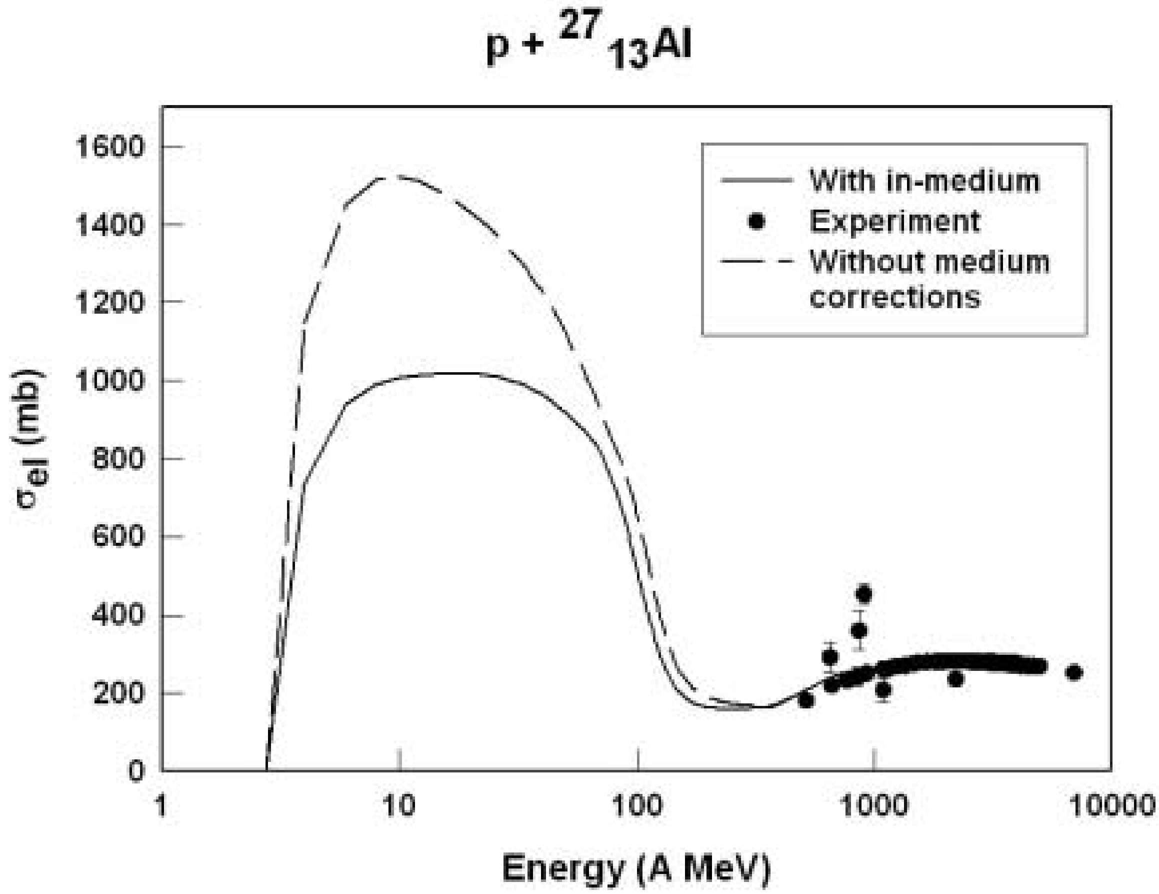

Fig 3 Elastic cross sections for proton-Aluminum collision as a function of energy. The experimental points are taken from Ref [31,32].]. The solid line includes the modifications discussed in the present work. The dotted line is without the corrections

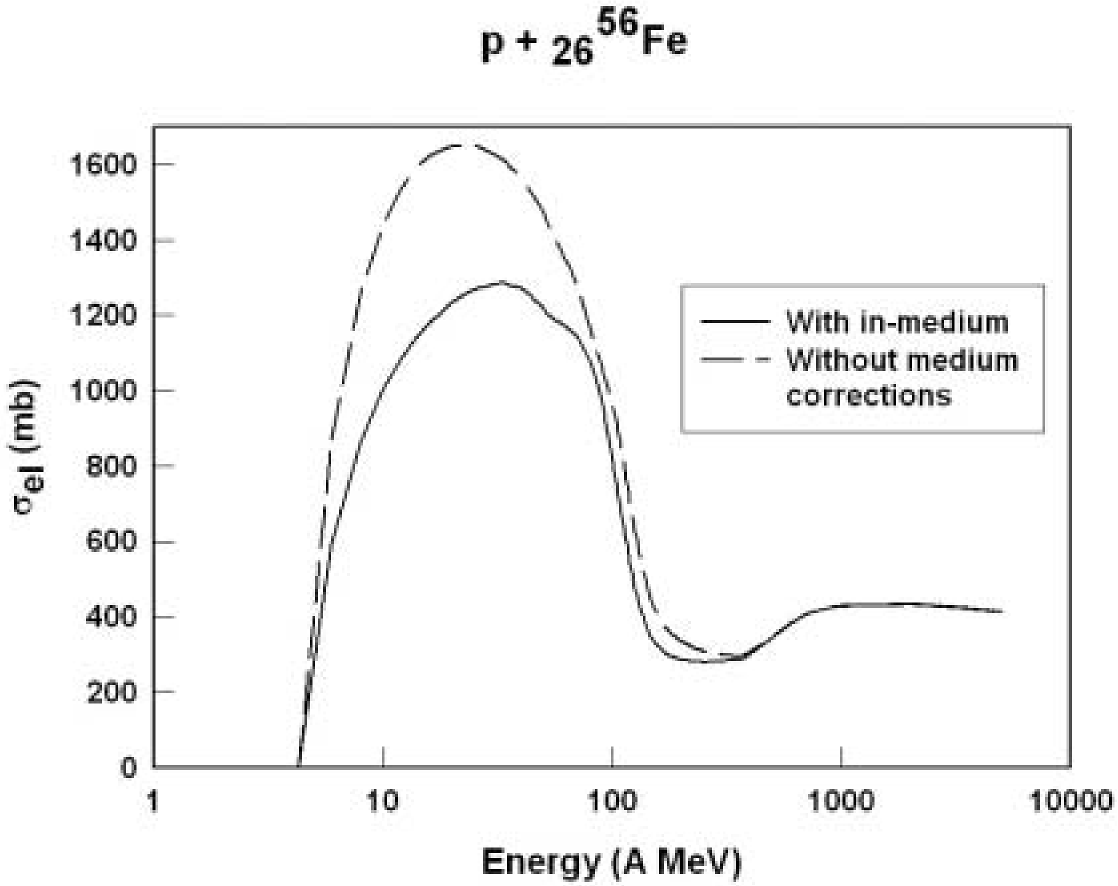

Fig 4 Elastic cross sections for proton-iron collision as a function of energy. The solid line includes the modifications discussed in the present work. The dotted line is without the corrections.

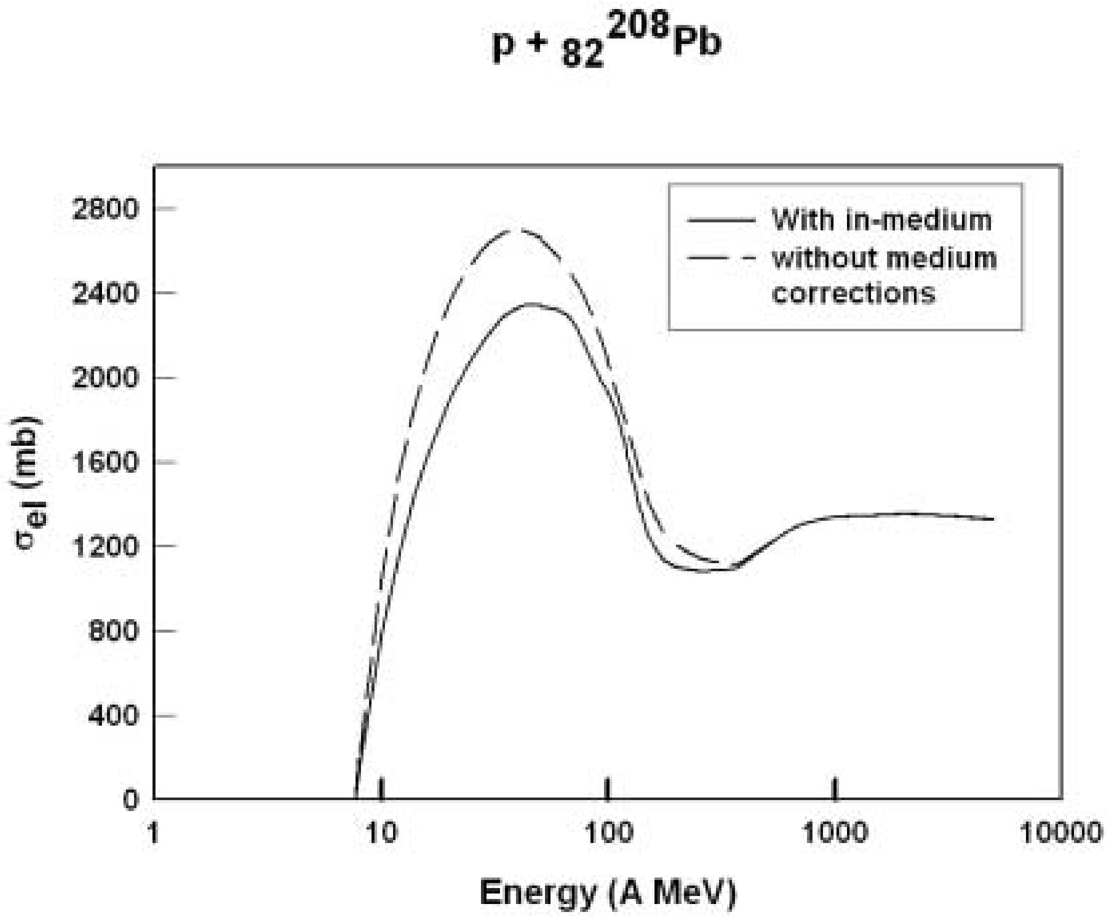

Fig 5 Elastic cross sections for proton-lead collision as a function of energy. The solid line includes the modifications discussed in the present work. The dotted line is without the corrections.

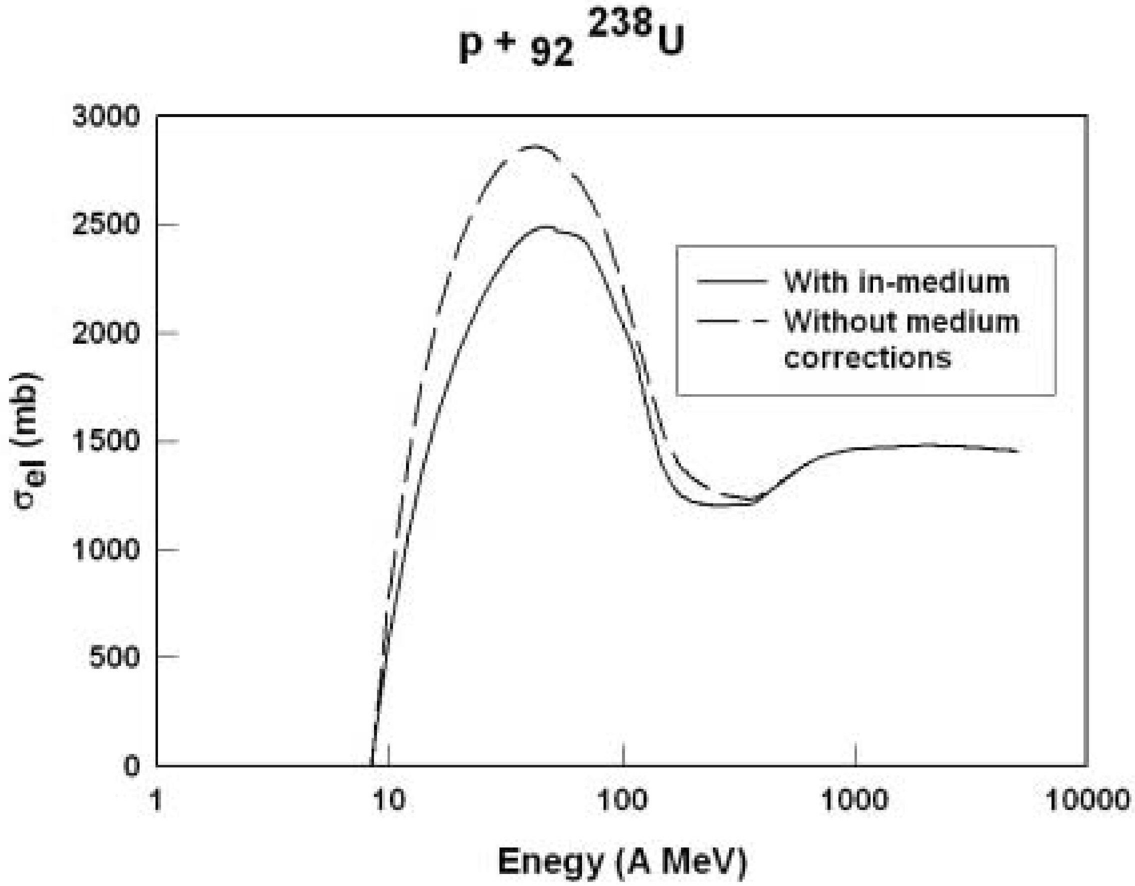

Fig 6 Elastic cross sections for proton-uranium collision as a function of energy. The solid line includes the modifications discussed in the present work. The dotted line is without the corrections.

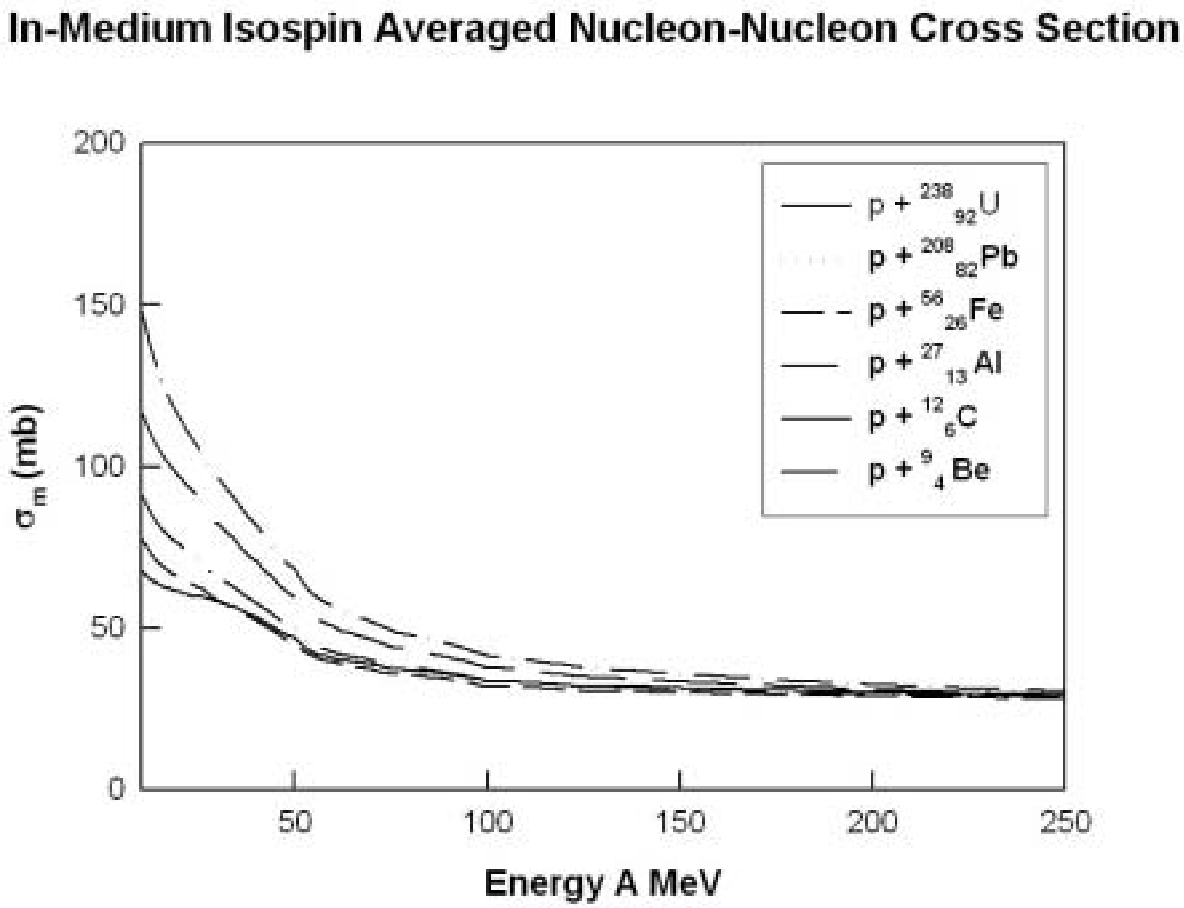

Fig 7: In-medium isospin average nucleon-nucleon cross section as a function of energy for various cases.

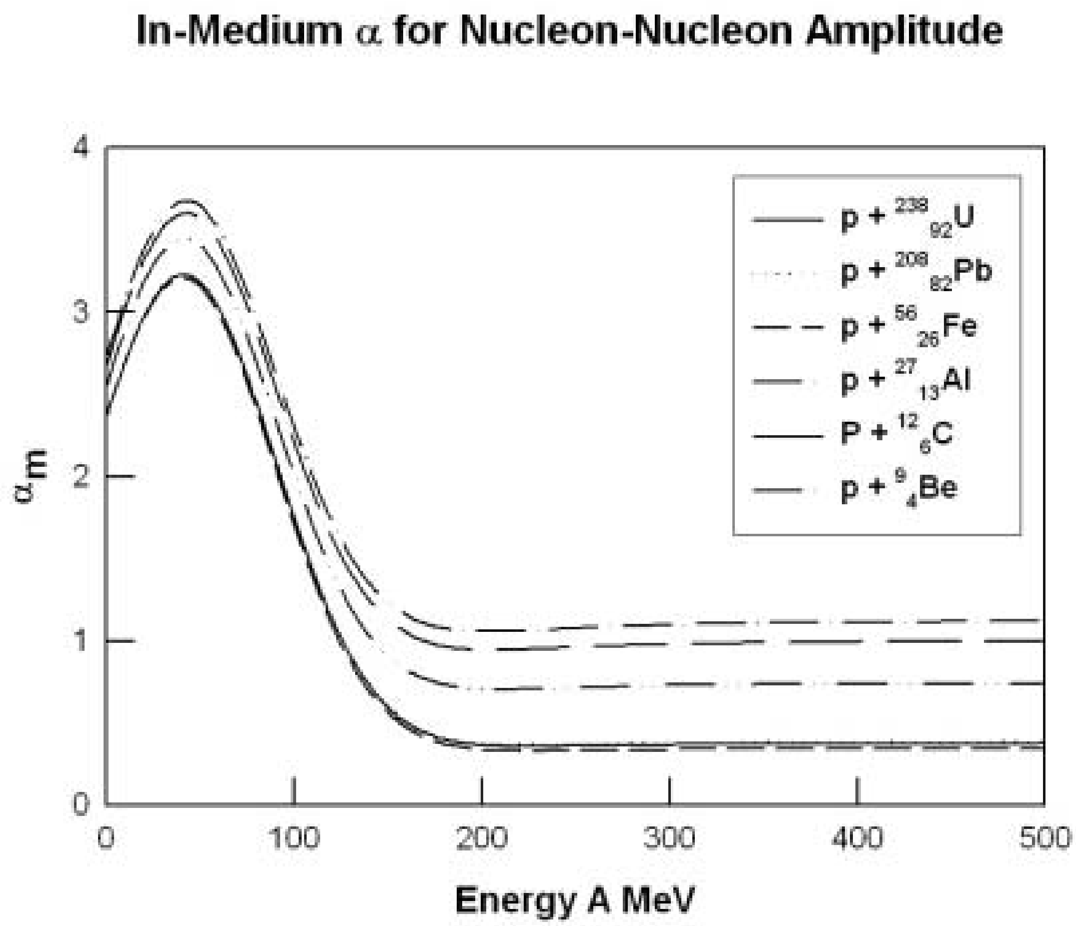

Fig 8: In-medium ratio of real to imaginary part of the two body amplitude as a function of energy for various cases.