COMPUTATION OF SOUND GENERATED BY FLOW OVER A CIRCULAR CYLINDER: AN ACOUSTIC ANALOGY APPROACH*

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SUMMARY

The sound generated by viscous flow past a circular cylinder is predicted via the Lighthill acoustic analogy approach. The two dimensional flow field is predicted using two unsteady Reynolds-averaged Navier-Stokes solvers. Flow field computations are made for laminar flow at three Reynolds numbers ($Re = 1000$, $Re = 10,000$, and $Re = 90,000$) and two different turbulent models at $Re = 90,000$. The unsteady surface pressures are utilized by an acoustics code that implements Farassat's formulation 1A to predict the acoustic field. The acoustic code is a 3-D code—2-D results are found by using a long cylinder length. The 2-D predictions overpredict the acoustic amplitude; however, if correlation lengths in the range of 3 to 10 cylinder diameters are used, the predicted acoustic amplitude agrees well with experiment.

INTRODUCTION

The sound generated by a viscous flow over a cylinder has been widely studied but is still difficult to compute at moderate and high Reynolds numbers. This flow is characterized by the von Karman vortex street—a train of vorticies alternately shed from the upper and lower surface of the cylinder. This vortex shedding produces an unsteady force acting on the cylinder which generates the familiar aeolean tones. This problem is representative of several bluff body flows found in engineering applications (e.g., automobile antenna noise, aircraft landing gear noise, etc.). For the workshop category 4 problem, a freestream velocity of Mach number $M = 0.2$ was specified with a Reynolds number based on cylinder diameter of $Re = 90,000$. This Reynolds number is just below the drag crisis, hence, the flow is very sensitive to freestream turbulence, surface roughness, and other factors in the experiment. Numerical calculations of the flow at this Reynolds number are also very sensitive—2-D laminar calculations are nearly chaotic and the transition of the boundary layer from laminar to turbulent flow occurs in the same region that vortex shedding takes place. These aspects of the workshop problem significantly increases the difficulty of prediction and interpretation of results.

In this work, the unsteady, viscous flow over a two-dimensional circular cylinder is computed by two different flow solvers, CFL3D and CITY3D. Two-dimensional (2-D) flow-field calculations were performed at this stage of the investigation to reduce the computational resources required. The noise prediction utilizes the Lighthill acoustic analogy as implemented in a modified version of the helicopter rotor noise prediction program WOPWOP. The 2-D flow field data is utilized in WOPWOP by assuming that the loading does not vary in the spanwise direction.

In the remainder of this paper we will first briefly describe both the aerodynamic and acoustic predictions for both laminar flow and turbulent flows. The Lighthill acoustic analogy [1] utilized in this work effectively

*Presented at the Second Computational Aeroacoustics Workshop on Benchmark Problems, Tallahassee, FL, November 1996.

separates the flow field and acoustic computations, hence, the presentation is divided in this manner. This paper focuses on the acoustic predictions. More emphasis placed on the computational fluid dynamics (CFD) calculations in a companion paper written by the authors [2].

FLOW-FIELD PREDICTIONS

CFD Methodology

Two unsteady Reynolds-averaged Navier-Stokes (RANS) solvers (CFL3D and CITY3D) were utilized in this work. Note that the term *Reynolds averaged* is used here not in its conventional sense (which implies averaging over an infinite time interval) but, rather, to denote averaging over a time interval which is longer than that associated with the slowest turbulent motions but is much smaller than the vortex shedding period. Thus it is possible to cover a complete vortex shedding cycle with a reasonable number of time steps (typically 2000 or less) without the need to resolve the details of the turbulent motions as would be necessary, for example, with either Direct or Large-Eddy Simulations.

The first code, CFL3D [3], is a 3-D thin-layer compressible Navier-Stokes code which employs the finite volume formulation in generalized coordinates. It employs upwind-biased spatial differencing for the convective and pressure terms, and central differencing for the viscous terms. It is globally second order accurate in space, and employs Roe's flux difference splitting. The code is advanced implicitly in time using 3-factor approximate factorization. Temporal subiterations with multigrid are employed to reduce the linearization and factorization errors. For the current study, CFL3D was run in a 2-D time-accurate mode which is up to second-order accurate in time. Viscous derivative terms are turned on in both coordinate directions, but the cross-coupling terms are neglected as part of the thin-layer assumption.

CFL3D has a wide variety of turbulence models available, including zero-equation, one-equation, and two-equation (linear as well as nonlinear). For the current study, the code was run either laminar-only (i.e., no Reynolds averaging), or else employed the shear stress transport (SST) two-equation k-ω turbulence model of Menter [4]. This model is a blend of the k-ω and k-ϵ turbulence models, with an additional correction to the eddy viscosity to account for the transport of the principal turbulent shear stress. It has been demonstrated to yield good results for a wide variety of steady separated turbulent aerodynamic flows [5], but its capabilities for unsteady flows remain relatively untested.

CITY3D is a finite-volume code for the solution of the incompressible, 3-D Navier-Stokes equations in generalized coordinates. A pressure-correction technique is used to satisfy mass and momentum conservation simultaneously. Temporal and spatial discretization are first- and third-order accurate, respectively. The turbulence model used in this study is the k-ϵ model modified as described in [6] to account for the effects of superimposing organized mean-flow periodicity on the random turbulent motions. The modification takes the form of an additional source to the ϵ equation which represents the direct energy input into the turbulence spectrum at the Strouhal frequency. Further details are reported in [2] which also gives details of the high Reynolds-number treatment adopted in specifying the near-wall boundary conditions.

CFD Results

Both the shedding frequency and mean drag coefficient for flow past a circular cylinder are known to exhibit only small Reynolds number dependence in the range $1000 < Re < 100,000$. A little above $Re = 100,000$ the drag crisis occurs and the mean drag coefficient $\bar{C}_d$ decreases significantly (from $\bar{C}_d \approx 1.2$ to $\bar{C}_d \approx 0.3$ – see [7] for representative figures). The exact Reynolds number where the drag crisis occurs can decrease significantly with any increase in free-stream turbulence intensity or surface roughness. Because the workshop problem specified $Re = 90,000$, we decided it would be prudent to make a series of computations for both laminar and turbulent flow. Laminar computations were made for $Re = 1000$, $Re = 10,000$, and $Re = 90,000$ with a flow Mach number $M = 0.2$, cylinder diameter $D = 0.019$ m, and freestream speed of sound 340 m/s. Turbulent calculations at $Re = 90,000$ were made for both the SST turbulence model in CFL3D and for the modified k-ϵ model in CITY3D. A portion of the lift and drag

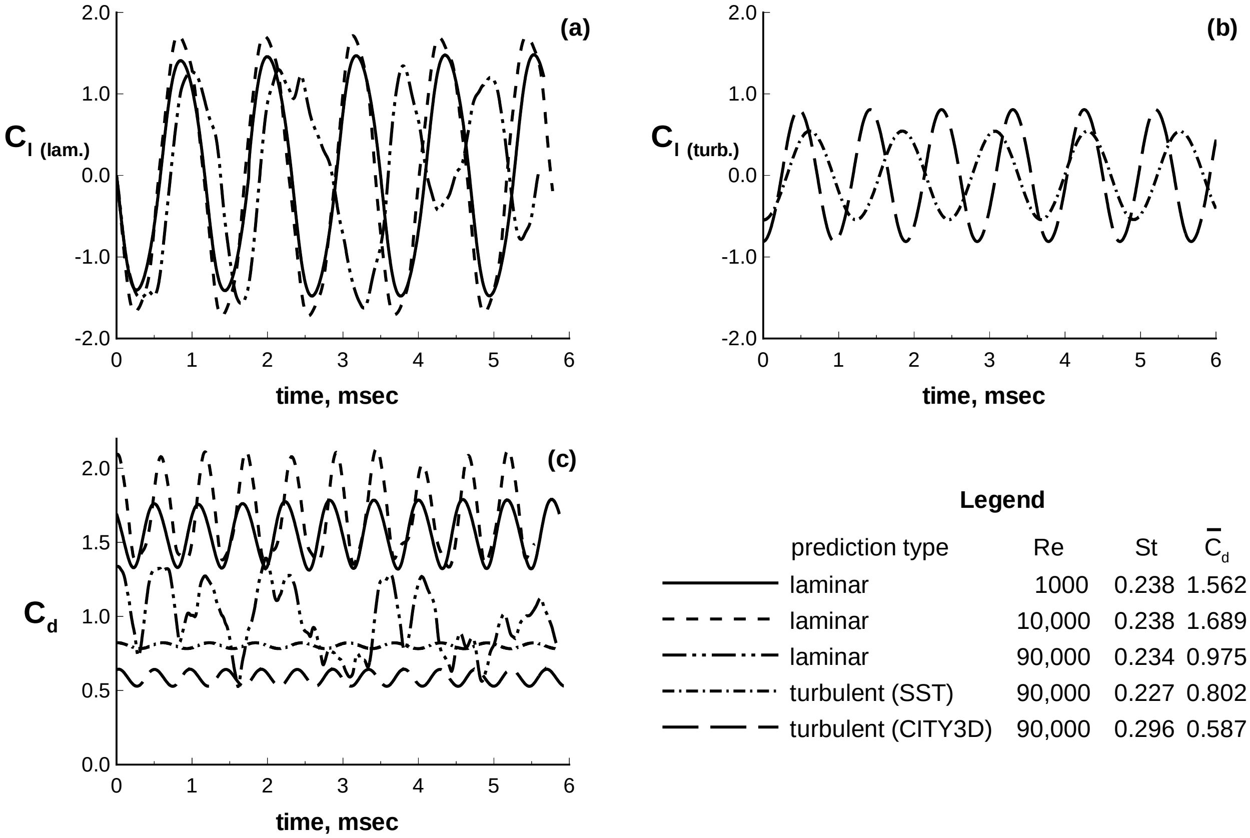

Figure 1. Comparison of predicted C_l and C_d time histories for $M = 0.2$ flow past a 2-D circular cylinder. (a) laminar C_l predictions; (b) turbulent C_l predictions; (c) C_d predictions.

coefficient time history is shown in figure 1. The predicted Strouhal number St and mean drag coefficient $\bar{C}_d$ values are given in the legend of figure 1.

Figure 1 shows that the laminar C_l time histories have approximately the same amplitude, but the laminar $Re = 90,000$ computation is somewhat irregular. The turbulent computations have both lower C_l fluctuation amplitude and lower mean and fluctuating drag levels. These lower levels are in general agreement with experiments which have a higher level of turbulence. For example, Revell et al. [8] measured $\bar{C}_d = 1.312$ for a smooth cylinder and $\bar{C}_d = 0.943$ for a rough cylinder, both at $M = 0.2$ and $Re = 89,000$. Notice that the CITY3D codes calculates a Strouhal number somewhat higher than CFL3D and more in the range of a higher Reynolds number data. This is probably related to the fact that CITY3D used a 'wall function' and hence has a turbulent boundary layer profile throughout (as would be the case for flow at a higher Reynolds number). More discussion of these results is given in reference 2.

ACOUSTIC PREDICTIONS

Acoustic Prediction Methodology

The unsteady flow-field calculation from CFL3D or CITY3D is used as input into an acoustic prediction code WOPWOP [9] to predict the near- and far-field noise. WOPWOP is a rotor noise prediction code based upon Farassat's retarded-time formulation 1A [10], which is a solution to the Ffowcs Williams – Hawkings (FW-H) equation [11] with the quadrupole source neglected. Formulation 1A may be written as

$$p'(\mathbf{x}, t) = p'_T(\mathbf{x}, t) + p'_L(\mathbf{x}, t) \tag{1}$$

where

$$4\pi p'_T(\mathbf{x},t) = \int\limits_{f=0} [\frac{\rho_o(\dot{v}_n + v_{\dot{n}})}{r(1-M_r)^2}]_{ret} dS + \int\limits_{f=0} [\frac{\rho_o v_n (r\dot{M}_r + c(M_r - M^2))}{r^2(1-M_r)^3}]_{ret} dS$$

$$4\pi p'_L(\mathbf{x},t) = \frac{1}{c}\int\limits_{f=0} [\frac{\dot{\ell}_r}{r(1-M_r)^2}]_{ret} dS + \int\limits_{f=0} [\frac{\ell_r - \ell_M}{r^2(1-M_r)^2}]_{ret} dS + \frac{1}{c}\int\limits_{f=0} [\frac{\ell_r (r\dot{M}_r + c(M_r - M^2))}{r^2(1-M_r)^3}]_{ret} dS$$

Here p' is the acoustic pressure, v_n is the normal velocity of the surface, ℓ_i are the components of the local force intensity that act on the fluid, M is the velocity of the body divided by the freestream sound speed c, and r is the distance from the observer position $\mathbf{x}$ to the source position $\mathbf{y}$. The subscripts r and n indicate a dot product of the main quantity with unit vectors in the radiation and surface normal directions, respectively. The dot over variables indicates source-time differentiation. The square brackets with the subscript ret indicates that the integrands are evaluated at the retarded (emission) time.

Notice that the integration in equation (1) is carried out on the surface $f = 0$ which describes the body—in our case a circular cylinder. Unlike the CFD calculations, the integration performed for the acoustic calculation is over a three-dimensional cylinder that is translating in a stationary fluid. For the predictions in this paper, we assume that the surface pressures are constant along the span at any source time. To model a 2-D cylinder in the 3-D integration, we use a long cylinder length and do not integrate over the ends of the cylinder. Experiments and computational work (e.g., [12–15]) have shown that vortex shedding is not two-dimensional and the shedding is correlated only over some length (typically $< 10D$). We have modeled the effect of vortex shedding correlation length by truncating the cylinder used in the acoustics prediction.

Acoustic Results

To test the coupling of the CFD and acoustic codes, we chose to predict the noise generated by flow past the circular cylinder for an observer position at a location 90 deg from the freestream direction and 128 cylinder diameters away from the cylinder. This corresponds to a microphone location in the experiment conducted by Revell et al. [8]. The predicted acoustic spectra for each of the CFD inputs are compared with experimental data in figure 2. One period of surface pressure data (repeated as necessary) was used to predict the noise. (Approximately 62 cycles of input data were used in the noise calculation of the laminar $Re = 90,000$ case because the loading time history was irregular.) A 0.5 m ($26.3D$) cylinder length was used in the prediction, matching the physical length of the cylinder used in the experiment. In figure 2 we see that both the Strouhal number and the amplitude are overpredicted. The CFL3D turbulent (SST model) prediction yields a slightly lower amplitude and Strouhal number, but the CITY3D turbulent prediction again has a high Strouhal value at the fundamental frequency and overpredicts the amplitude. The first harmonic of the vortex shedding frequency can be clearly seen in the predictions, but the experimental data is lower in amplitude and frequency.

One explanation for the discrepancy in the noise predictions is that the vortex shedding has been modeled as completely coherent in figure 2. In experiments, however, the vortex shedding has been found to be coherent only over a relatively short length, usually less that $10D$. To investigate the effect of vortex shedding correlation length on predicted noise levels, we varied the length of the cylinder L over the range $3D < L < 250D$ and plotted the overall sound pressure level predicted at the 90 deg, $128D$ microphone location. Figure 3 show that the length of the cylinder has a strong effect on the peak noise level. For example, a cylinder length of $10D$ (which is a long correlation length) yields a peak amplitude at the 90 deg

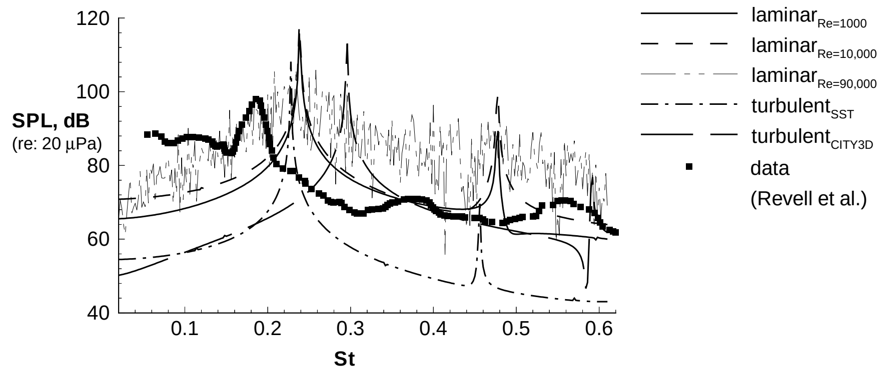

Figure 2. Comparison of predicted and measured sound pressure level for a microphone located $128D$ away from the cylinder at a 90 deg angle to the freestream flow.

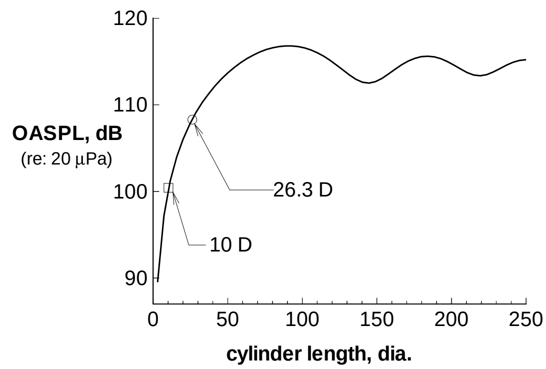

Figure 3. Overall sound pressure level (OASPL) plotted versus cylinder length in the 3-D acoustic computation. The 2-D CFD input data for the turbulent case with the SST model was used for this plot. The CFD data was assumed to be constant along the span for any given source time.

observer location that is within 2 dB of the experiment. Clearly then a true 2-D noise prediction should be *expected* to overpredict the measured noise, possibly by as much as 25 dB!

Requested Workshop Predictions

Now that the noise prediction procedure has been compared with experimental data we have enough confidence to present the results requested for the workshop. For these predictions, a cylinder length of $10D$ is used and the microphone locations are set to $35D$ away from the cylinder. Rather than just show the spectra at a few angles we have chosen to plot the entire directivity pattern around the cylinder for the laminar $Re = 1000$ and turbulent SST cases, which are representative. The overall sound pressure level, fundamental frequency, first harmonic are shown in figure 4. In the figure, the cylinder is at the origin and the flow moves from left to right. The 90 deg location is at the top of the figure and the axes units are in dB (re: 20μPa). Figure 4(b) and (c) show the expected dipole directivity pattern. The dipole shape in figure 4(c) is not symmetric right and left because of the left-to-right direction of the flow.

The dipole directivity pattern in figure 4 can be understood in more detail if we assume that the cylinder cross section is acoustically compact, that is that the acoustic wavelength is large compared to the diameter of the cylinder. This is actually a very good approximation in this flow condition. By assuming the cylinder has a compact cross section, we can predict the noise by using the section lift and drag directly rather than integrating the pressure over the cylinder surface. Figure 5 shows the directivity of the lift and drag

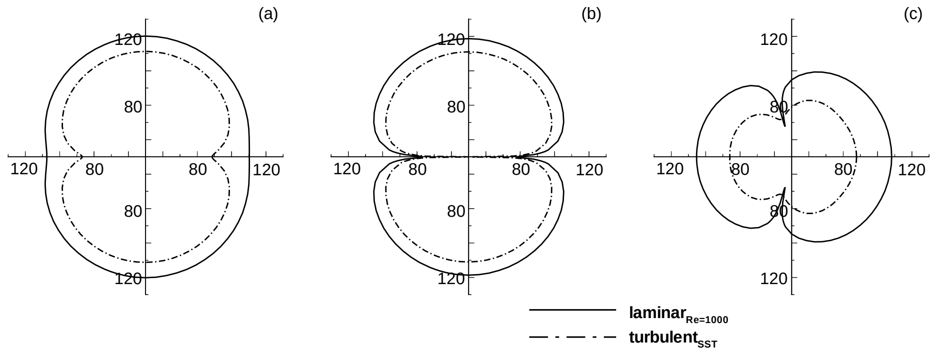

Figure 4. Predicted directivity patterns for $M = 0.2$ flow traveling left to right. Axes units are decibels (dB, re:20μPa). (a) overall sound pressure level; (b) fundamental frequency; (c) first harmonic.

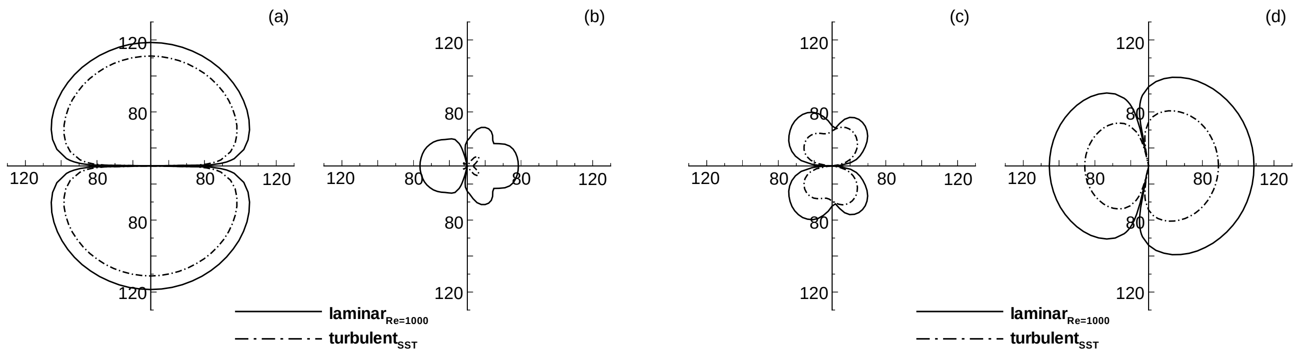

Figure 5. Comparison of the C_l and C_d noise components directivity pattern for $M = 0.2$ flow traveling left to right. Axes units are decibels (dB, re: 20μPa). (a) C_l fundamental frequency; (b) C_d fundamental frequency; (c) C_l first harmonic; (d) C_d first harmonic.

separately for both the fundamental and first harmonic. By separating the lift and drag, we can see clearly in the figure that the noise produced at the fundamental frequency is entirely from the lift dipole (except at the nulls of the dipole), while the drag completely dominates the first harmonic frequency. This is what should be expected because the period of the drag oscillation is half the lift oscillation period.

CONCLUDING REMARKS

The choice of Reynolds number $Re = 90,000$ makes the calculation of noise generated by flow past a circular cylinder particularly difficult. This difficulty is due to the transitional nature of the flow at this Reynolds number. Laminar flow calculations at such a high Reynolds number are irregular and nearly chaotic. The turbulent calculations are sensitive to both grid and turbulence model (See reference 2).

Although we have performed only 2-D flow calculations in this paper, the amplitude of the noise prediction seems to agree fairly well with experimental data if a reasonable correlation length of the cylinder is used. To understand all of the details of the flow the problem must ultimately be solved as a 3-D problem to properly account for partial coherence of vortex shedding. The acoustic model does not require any changes for 3-D computations, but the CFD calculations will be very demanding. The CFL3D calculations for two dimensions already require approximately 4.5 CPU hrs on a Cray Y/MP (CITY3D – 80 hrs on workstation) to reach a periodic solution. This will be much longer for an adequately resolved 3-D computation. In contrast the acoustic calculation for a single observer position required about 70 CPU sec on a workstation.

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