Robust Control of Non-Passive Systems via Passification

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Abstract

This paper presents methods which enable the use of passivity-based control design techniques to control *non-passive* systems. For inherently non-passive finite-dimensional linear time-invaraint systems, ***passification*** methods are presented to render such systems passive by suitable compensation. The passified system can then be controlled by a class of passive linear controllers. The idea is to exploit the robust stability properties of passivity-based control laws for uncertain systems. The proposed passification methods are demonstrated by application to the ACC benchmark problem and to pitch-axis control of an F-18 High Alpha Research Vehicle (HARV) model.

1 Introduction

A number of stability results exist in the literature for the control of naturally passive systems. Some examples of such systems include large flexible space structures or multilink flexible robots with collocated actuators and sensors [Kel.96]. For such systems, model-based controllers are often found to be extremely sensitive to parametric uncertainties[Jos.89]. Passivity-based controllers, however, have proven to be highly effective in robustly controlling such plants. Being model-independent, such controllers are robust to modeling errors and parametric uncertainties. For non-passive systems, however, passivity-based control techniques cannot be used directly. One way of making non-passive systems amenable to passivity-based control is to render such systems passive, i.e., to ***passify*** them using a suitable compensation. The compensated plant can then be robustly controlled by any marginally strictly positive-real (MSPR) controller [Jos.96]. This technique essentially converts the problem of ***robust controller design*** into the problem of ***robust passification*** which in some cases may be easier to accomplish. The numerical examples given in this paper can help demonstrate this concept more clearly.

A brief overview of robust stability results for passive systems is first presented, followed by four passification methods which include series, feedback, combination of series and feedback, and feedforward compensation. The proposed methodology is then demonstrated by application to the ACC benchmark problem and to the longitudinal control of an F-18 High Alpha Research Vehicle (HARV) model.

2 Stability of Passive Systems

The notion of passivity is one of the oldest in the network theory literature. For linear time-invariant (LTI) systems passivity is equivalent to the positive-realness of system's transfer function. It is well known that a positive-real (PR) system is robustly stabilized by a strictly PR (SPR) controller. However, various definitions of SPR systems exist in the literature, for example, see [Jos.96]. The weakest SPR systems known to date are referred to as marginally strictly positive-real (MSPR) systems [Jos.96]. An $m \times m$ rational matrix $G(s)$ is said to be ***marginally strictly positive real*** if it is positive real, and

$$G(j\omega) + G^*(j\omega) > 0 \ for \ \omega \in (-\infty, \infty). \qquad (1)$$

Numerous stability results exist [Hil.94] on the feedback interconnection of two passive systems. For the LTI case, it was proved in [Jos.96] that the negative feedback interconnection of $G(s)$ and $H(s)$ is asymptotically stable if (i) $G(s)$ is MSPR, (ii) $H(s)$ is PR, and (iii) None of the $j\omega$ -axis poles of $G(s)$ is a transmission zero of $H(s)$.

The appealing feature of passivity-based control laws is the stability robustness to model uncertainties including unmodeled dynamics and parametric uncertainties. If a non-passive system is ***robustly passified*** (i.e., if it remains passive in the presence of model uncertainties), then it can be robustly stabilized by any MSPR controller.

3 Passification Methods

Application of passivity-based methods to non-passive systems has been addressed in the literature [Bar.87], [Kau.94], [Sun.94]. Much of the literature, however, has focused on "almost strictly PR" (ASPR) systems, which are systems that can be passified by sufficiently high constant-gain output feedback. Such systems represent a rather restrictive class since they have to be stable and minimum-phase, and cannot have a relative degree of more than one.

In this section, we present four passification methods which can be used to passify LTI non-passive single-input, single-output (SISO) systems. For many non-passive systems (e.g., systems with relative degree > 1) passification by a proper compensator may not be possible; however, it may be possible to obtain a compensation that can keep the phase of the system within $\pm 90^o$ in a finite frequency range of interest. Such systems will be referred to as Band-Limited Positive Real (BLPR) systems.

3.1 Series Compensation

Consider the block diagram shown in Fig. 1 wherein the plant $P(s)$ is non-passive. The idea of series passification is to design a compensator $C_s(s)$ such that the compensated

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plant $P_c(s) = P(s)C_s(s)$ is positive-real. (For multi-input, multi-output (MIMO) plants, both right and left multipliers can be used). For the SISO case, the compensated system is passive if the phase of $P_c(s)$ remains within $\pm 90^0$ for all frequencies. For stable minimum-phase systems which have a relative degree of zero or one, the violation of phase condition can take place only over a finite or semi-finite range of frequencies. For such systems, a "proper" series compensation can be obtained which can render these systems positive-real.

An intuitive yet important observation that can be made is that a positive-real transfer function cannot have two consecutive occurrences of poles or zeros, i.e., it should have an alternating pole-zero pattern. For example, consider a SISO plant with distinct poles and zeros that are only on the imaginary axis. For such a plant to be PR its poles and zeros must alternate. Thus, a series compensator can be designed to insert poles/zeros at appropriate locations. In the case of real poles and zeros, a similar observation can be made. In fact, the maximum phase contribution by a pole-zero pair ($\frac{s+z}{s+p}$)can be shown to be equal to ϕ_m, where:

$$\phi_m = tan^{-1}(\frac{p-z}{2\sqrt{pz}}).$$

Then, positive-realness of a plant having only real poles/zeros can be ensured by restricting sum of all such maximum phase contributions to $\pm 90^\circ$. Similarly, for plants with complex-conjugate poles and zeros, a similar phase computation, although cumbersome, can be done using the closed-form expressions. For general plants which have real, imaginary, and complex poles and zeros, it was found in a number of examples that alternating pole-zero pattern of both real and imaginary parts resulted in positive realness.

With this insight, one of the techniques to obtain a series passification is to select a compensator whose pole-zero pattern along with plant's poles and zeros forms an alternating pole-zero pattern. An alternate method is to inspect the Bode diagram of P(s) and to compute the phase required to make $P_c(s)$ PR. Limitations of series compensation are that it cannot passify unstable or non-minimum phase plants, and plants having repeated poles or zeros on the imaginary axis. For systems with relative degree greater than one, the

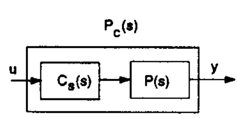

Figure 1: Series Compensation

series passification would have to be improper. For physical realizability, however, the compensation must be made proper by placing high frequency poles that are sufficiently outside the closed-loop bandwidth, which results in a BLPR system. Such high frequency dynamics can then be modeled as multiplicative uncertainty and the controller design can be obtained to ensure robustness to these unmodeled dynamics.

Another technique for series passification is the formulation of the problem in the LMI (linear matrix inequality) setting based on the Kalman-Yakubovich lemma. This approach can also be used for multi-input multi-output (MIMO) systems to yield left- and right-series compensators. This approach is not discussed in this paper.

3.2 Feedback Compensation

As stated previously, certain non-passive systems such as unstable systems or systems having repeated poles/zeros on the imaginary axis, cannot be passified by series compensation alone. For such systems, passification can sometimes be achieved by feedback compensation (Fig. 2). For minimum-phase systems, the condition for passification by feedback compensation can be easily derived as:

$$Re(C_f(j\omega)) \geq -\frac{Re(P(j\omega))}{|P(j\omega)|^2}.$$

Another example of sytems where feedback compensation can be used for passification is ASPR systems [Bar.87],[Kau.94], which can be passified by a constant-gain output feedback. The problem of feedback passification can also be formulated in the LMI setting as shown in [Sun.94]. Such a compensation is also robust [Gu.90] to uncertainties which satisfy certain boundedness condition. Although this result guarantees that a constant feedback gain can passify such systems, the feedback gain required in some cases could be very large.

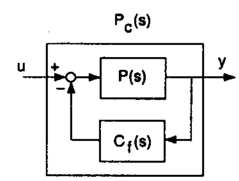

Figure 2: Feedback Compensation

3.3 Hybrid Compensation

In certain cases, solely series or solely feedback passification may not be possible or desirable. For example, the feedback gain required to passify certain ASPR systems may be very large or, in some cases, the series passification alone may be very sensitive to plant variations. In such cases, a hybrid compensator which is a combination of series and feedback passification, may be more desirable. Figure 4 shows the configuration for hybrid compensation. In preliminary numerical trials, it has been found that hybrid compensation significantly increases the robustness of series passification. An alternate configuration for hybrid passification would include $C_s(s)$ inside the feedback loop.

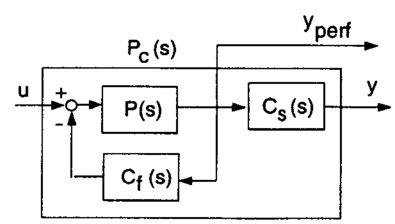

Figure 3: Hybrid Compensation

3.4 Feedforward Compensation

For certain systems, such as non-minimum phase systems or systems with high relative degree, the first three passification methods cannot be used. To passify such systems, a possible solution is to use feedforward compensation $D(s)$ [Bar.87], [Kau.94] as shown in Fig. 4. If D is a constant matrix, it has the effect of reducing the relative degree of the modified system to zero. In general, $D(s)$ does not have to be a constant matrix but can be a proper transfer function. The condition for passification would be to obtain transfer function $D(s)$ such that $Re(P(j\omega)) + Re(D(j\omega)) \geq 0$ $\forall\ \omega \in\ (0, \infty)$.

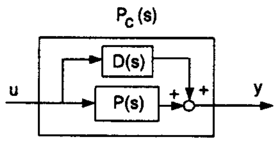

Figure 4: Feedforward Compensation

4 Numerical Examples

Two numerical examples are given to demonstrate the use of some of the passification methods for obtaining robust controller design. It is also shown that in certain cases one type of passification method alone is not adequate for passification and the combination of different methods may be warranted.

4.1 ACC Benchmark Problem

The "ACC Benchmark Problem" [Wie.92] consists of two masses (m_1 and m_2) attached by a single spring with stiffness k, moving on a friction-free horizontal surface (Figure 5). This fourth-order system has one rigid-body mode and one undamped elastic mode. The control input is the force u applied to m_1, and the output y is the position of m_2. In addition, two disturbance forces w_1 and w_2 act on the masses. The transfer function from u to y is given by:

$$P(s) = \frac{k}{m_1 s^2\{m_2 s^2 + [1 + \frac{m_2}{m_1}]k\}} \qquad (2)$$

The nominal values of m_1, m_2, and k are unity. The problem is to design a controller which will robustly stabilize the system for values of k between 0.5 and 2.0. It is also desired that the impulse response from w_1 and w_2 to the position output y should have a settling time of about 15 seconds.

The relative degree of $P(s)$ is four and the system is clearly not passive. It has a double pole at the origin and a pair of purely imaginary poles at $s = \pm j1.414$. In order to design a passivity-based controller, the system has to be "passified". To keep the phase between $\pm 90°$, the poles and zeros on the imaginary axis must be interlaced. Therefore, the double-pole of the origin must be moved. This cannot be accomplished by series compensation. However, simple constant-gain negative feedback ($C_f(s) = \gamma$) can easily accomplish this. A feedback gain $\gamma = 0.135$ was chosen, which moved the poles from the origin to $\pm j0.265$. The poles at $\pm j1.414$ moved slightly closer to the origin. To passify the resulting system, zeros must be introduced between the imaginary axis poles. A series compensator was chosen to have zeros at $s = 0$ and $\pm j0.55$. The compensator must, however, be proper; therefore, it was chosen as:

$$C_s(s) = \frac{s(s^2 + 0.55^2)}{(T_c s + 1)^3} \qquad (3)$$

The value of T_c was chosen as 0.01 by trial and error to obtain passification in the frequency range of interest. The passification must be designed to be robust, i.e., the compensated system must remain passive for all values of the spring stiffness k in the given range. Variation of k would cause the imaginary axis poles to move. As long as the poles remain interlaced with the zeros, the positive realness will be retained. The resulting pole-zero map is shown in Figure 6, which also shows the variation of the poles as k varies between 0.5 and 2.0. Only the poles at $\pm j1.414$ undergo significant variation. The poles do not cross the zeros, and hence the passification is robust.

This system, passified using hybrid feedback/series compensation, is only *approximately* positive real (BLPR). Figure 7 shows the Bode plot of $C_s P(1+\gamma P)^{-1}$ which indicates that the phase remains within $\pm 90°$ up to nearly 5 rad/sec frequency, i.e., the sytem has been rendered "passive" only in a finite frequency range. A check of the Bode plots also confirms that the system's phase remains within $\pm 90°$ for up to 5 rad/sec frequency as k varies within its range.

Because the (ideal) passified system is positive real, it can be stabilized by any MSPR controller, the simplest being a constant gain. After a few trial response computations, the constant gain γ_{PR} was chosen to be 2.75. The responses due to unit impulse disturbance inputs for the nominal case (k=1)and for perturbed values of k are shown in Figures 8 and 9. The responses are satisfactory, with a settling time of approxomately 15 seconds for the nominal case. There is very little degradation of the resonse due to variation of k; that is, the controller provides robust stability as well as performance. A more systematic way to ensure robustness of the design is to represent the compensated plant as $(1+\Delta)P_c$ where $P_c = P(1+\gamma P)^{-1}[ks(s^2+0.55^2)]$ and multiplicative uncertainty $\Delta = \frac{1}{(T_c s+1)^3} - 1$. Then the sufficient condition for stability of feedback loop of Figure 10 in the presense of multiplicative uncertainty is given by

$$\overline{\sigma}(\Delta(j\omega)) < \frac{1}{\overline{\sigma}[\gamma_{PR} P_c(1+\gamma_{PR} P_c)^{-1}(j\omega)]} \quad \forall \text{ real } \omega.$$

In an attempt to improve the performance further, an optimal LQG controller, which is restricted to be weakly SPR (WSPR) [Loz.90], was next designed for the hybrid-passified system. The LQG weights were varied to obtain good performance. However, the best performance obtained by WSPR LQG controller was no better than that obtained by the constant-gain controller. Therefore, for this problem, a simple controller employing third-order compensation (for realizable passification), can provide robust stability as well as performance.

4.2 Flight Control Design Using Passification

A mathematical model of an F-18 HARV configuration [Ost.94] is used as a second example to demonstrate the effectiveness of passivity-based control design methodology described above for non-passive systems. The HARV configuration is a modified version of an F-18 airplane model which includes multi-axis thrust vectoring capability for pitch and yaw control power. The longitudinal models for pitch-axis control of HARV for four different flight conditions at the altitude of 15,000 ft are considered as focus configurations for controller design. The four configurations had the following combinations of speed and vertical acceleration, respectively: (1) 0.7 Mach and 1g, (2) 0.6 Mach and 1g, (3) 0.49 Mach and 1g, and (4) 0.3 Mach and 0.37g. The controller design was obtained based on a nominal 4th-order plant model for the second flight condition, i.e., altitude of 15,000 ft, speed 0.6 Mach, and acceleration of 1g. The controller was designed to be robust for varying flight conditions with Mach number in the range 0.3 to 0.7 and vertical acceleration in the range 0.37g to 1g. The controller design process is summarized below.

First, the passification of the nominal plant model was achieved by using a proper, third-order series compensator with poles at -10, -.05, and $-.0035$ and zeros at -1, -0.5, and -0.08. The passification was chosen so as to be robust to mach number variation between 0.30 to 0.70 and g-variation between .37 to 1, i.e., a single series compensator was obtained which could passify plant models at all four flight conditions. The flight conditions represent a large variation in the parameters. For example, the short-period frequency (eigenvalue) variation for these flight conditions was between 0.79 and 2.71 rad/sec. Figure 11 shows phase plots of the passified plants. Having robustly passified the plant, a short-period approximation of the plant was used as the design model for the plant. An LQG-optimal, fifth-order WSPR controller [Loz.90] was then designed for the design flight condition to obtain satisfactory response. The final controller was a combination of the passifying series compensator and the LQG-optimal WSPR compensator. The controller design was found to give satisfactory response even for the other three flight conditions. Figure 12 shows step responses for the four flight conditions using this fixed eigth-order controller.

Robust Gain-Scheduling via Convex Combination of PR Controllers-Once a plant is robustly passified, the passifying compensation can be fixed, but the WSPR feedback controller can be tuned to each model (for each flight condition). The following fact can then be used to obtain robust gain-scheduling and consistent performance.

Fact- A convex combination of PR (WSPR) systems is PR (WSPR).

To obtain optimal performance at all four flight conditions, LQG-optimal WSPR controllers $[C_i(s), i = 1, 2, 3, 4]$ can be designed, and a controller of the form: $C(s) = \sum_{i=1}^{4} \alpha_i C_i(s)$ where the coefficients $\alpha_i \geq 0$; $(\sum_{i=1}^{4} \alpha_i = 1)$ are chosen corresponding to the actual flight condition which may be in-between the four given flight conditions. This approach was followed with two controllers corresponding to flight conditions (2) and (4) (since (1) is quite close to (2) and (3) is close to (4)). The resulting controller gave guaranteed stability and satisfactory performance for all four flight conditions and also for intermediate flight conditions. The order of such a controller is the sum of the orders of $C_i(s)$'s.

5 Concluding Remarks

Methods for extending passivity-based controller design techniques to non-passive systems were investgated. In particular, series, feedback, hybrid, and feedforward passification were discussed. It was shown that a robust gain-scheduling controller can be obtained by using a convex combination of weakly strictly positive real controllers. The methods were applied to two example problems, and were shown to provide satisfactory robust control. Future work should address extension of the methods to multi-input multi-output systems, and the development of more systematic methods for robust passification.

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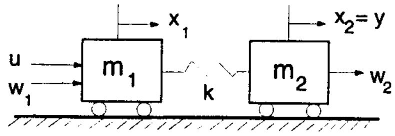

Figure 5: ACC benchmark problem

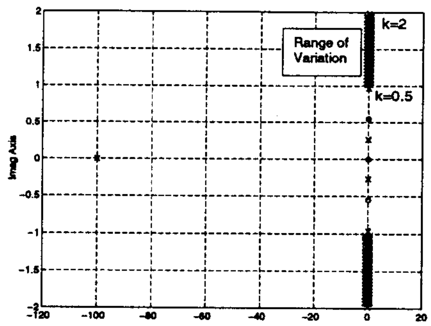

Figure 6: Pole-zero map of passified system

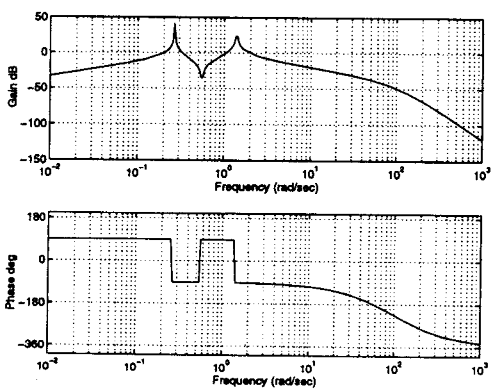

Figure 7: Bode plot of passified system

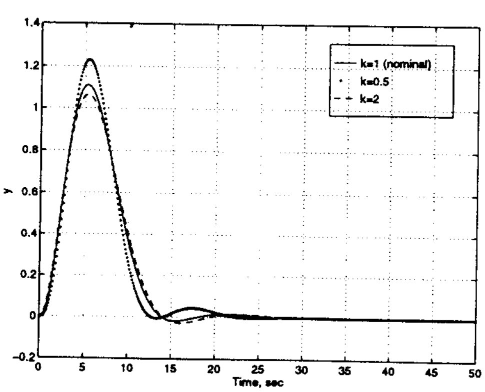

Figure 8: Response for $\dot{w}_1 = \delta(t)$

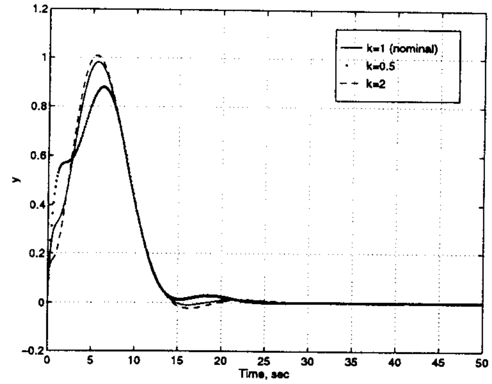

Figure 9: Response for $w_2 = \delta(t)$

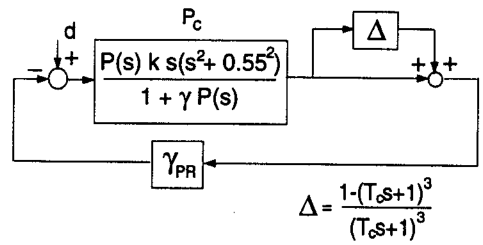

Figure 10: Multiplicative uncertainty formulation

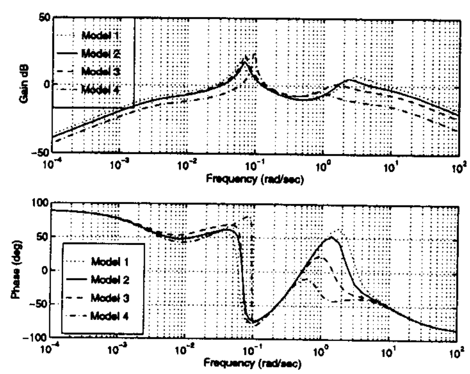

Figure 11: Bode plots of passified systems

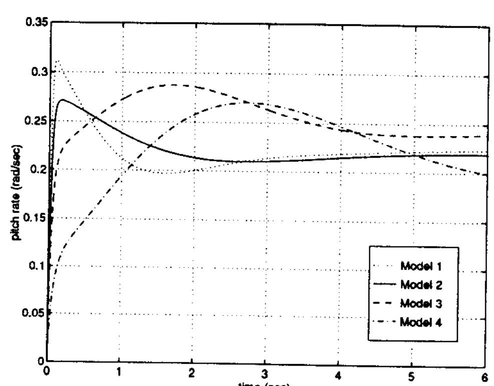

Figure 12: Step response of systems