

Receptivity and Bypass Dynamics

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Abstract. Problems concerning laminar-turbulent transition are addressed by solving a series of initial value problems. Solutions to the temporal, initial-value problem with an inhomogeneous forcing term imposed upon the flow are sought. It is shown that: (1) A transient disturbance lying located outside of the boundary layer can lead to the growth of an unstable Tollmein-Schlicting wave; (2) A resonance with the continuous spectrum may provide a mechanism for bypass transition; and (3) The continuum modes of a disturbance feed directly into the Tollmein-Schlicting wave downstream through non-parallel effects.

1 Introduction and Results

In previous work [2],[5], the authors have shown a strong correlation to the solution of a temporal, three-dimensional, initial-value problem and the direct numerical simulation of the spatial problem. The methodology consisted of solving the linear disturbance equations subject to a series of initial values. These solutions are relatively easy, fast, and inexpensive to calculate. The corresponding spatially evolving flow was then determined by direct numerical simulation using the full Navier-Stokes equations and the two solutions were compared. During the period of transient growth for both channel flow and the laminar boundary layer, the two approaches agree quite well. Thus, it is reasonable to use the inexpensive and fast solutions of the temporal, initial-value problem as a means to conduct numerical experiments that can lead to greater understanding of the mechanisms at work. It is only natural that we extend our studies to investigate the effects that freestream disturbances have on the laminar boundary layer.

Receptivity has traditionally been divided into the broad categories of forced receptivity or natural receptivity; however, based on the physical and mathematical descriptions, we use three categories: forced receptivity, natural receptivity, and naturally forced receptivity. Forced receptivity is characterized by the experiments of Nishioka and Morkovin [6] where disturbances of limited spatial extent are introduced in the freestream *downstream* of the leading edge. The case of natural receptivity is characterized by the experiments of Boiko, *i.e.*, [1] where a disturbance field *upstream* of the leading edge of a smooth plate is generated. Naturally forced receptivity is characterized by the experiments of Dietz [3] where freestream disturbances are scattered

by localized surface irregularities. If disturbances are kept at a level in which linear theory applies, then forced receptivity problem is governed by a set of inhomogeneous linear partial differential equations in time and space, natural receptivity by the homogeneous problem, and naturally forced receptivity is either governed by the inhomogeneous problem (if viewed as a perturbational problem) or by the homogeneous problem (if viewed as a changing mean flow). Downstream of the imposed disturbance, all three problems are the same mathematically as the natural receptivity problem.

The procedure used here, integrating the linear disturbance equations of temporal stability theory as an initial value problem, is straightforward and simple. In every numerical calculation, the complete solution, namely, the continuum of eigenfunctions to the Orr-Sommerfeld problem and all discrete modes, is determined. Only afterwards is this solution interpreted in terms of the individual modes of the Orr-Sommerfeld equation.

For the flat-plate boundary layer, the fluid is taken as one of constant density with the basic flow approximated as parallel with $U = U(y), V = W = 0$. The instantaneous flow is decomposed into a basic state, (U, V, W, P) , plus a time-dependent disturbance to this basic state, (u, v, w, p) . The length scale chosen for non-dimensionalization is the displacement thickness, δ^* , of the Blasius boundary layer solution. and $U(2.856) = .99$ gives the outer edge of the boundary layer.

By using the Fourier transformations defined with respect to x and by decomposing the forcing as $\mathbf{F} = \text{curl free } \mathbf{F} + \text{divergence free } \mathbf{F} = (A_c, B_c, C_c) + (A_d, B_d, C_d)$, the time-dependent Orr-Sommerfeld equation governing two-dimensional disturbances, with $\Delta = D^2 - \alpha^2$, becomes

$$\left[\frac{\partial}{\partial t} - i\alpha U \right] \Delta \check{v} + i\alpha U'' \check{v} = R^{-1} \Delta \Delta \check{v} + \Delta \check{B}_d. \quad (1)$$

This equation is solved numerically by the method of lines. All solutions of the forced problem are subject to an initial condition that produces the quantifiable, unstable Tollmien-Schlichting wave seen in Figure (1A) labeled by $B_0 = 0$. The forcing is chosen to be identically zero at $t = 0$ and to increase smoothly as a function of time. Receptivity is measured by the gain in the amplitude to the unstable Tollmien-Schlichting wave due to forcing terms.

A general purpose forcing function, localized about $y = y_0$, is

$$\check{B}(y) = B_0 r(t) e^{i\omega_f t} \frac{(y/y_0)^2 e^{-(y-y_0)^2/\sigma_y^2}}{\left(\int_0^\infty [(y/y_0)^2 e^{-(y-y_0)^2/\sigma_y^2}]^2 dy \right)^{1/2}}. \quad (2)$$

The time histories of the perturbation energy using the parameters values $B_0 = 1000$, $R = 1000$, $\alpha = \tilde{\gamma} = .25$, $\phi = 0$, $\sigma_y = .5$, $y_0 = 6$ and $r(t) = 1 - e^{-t^2/\sigma_t^2}$ with $\sigma_t = 100$, are shown in Figure (1). The forcing frequencies are $\omega_f = 0.05$ and $\omega_f = 0.25$ (for comparison, the Tollmien-Schlichting frequency

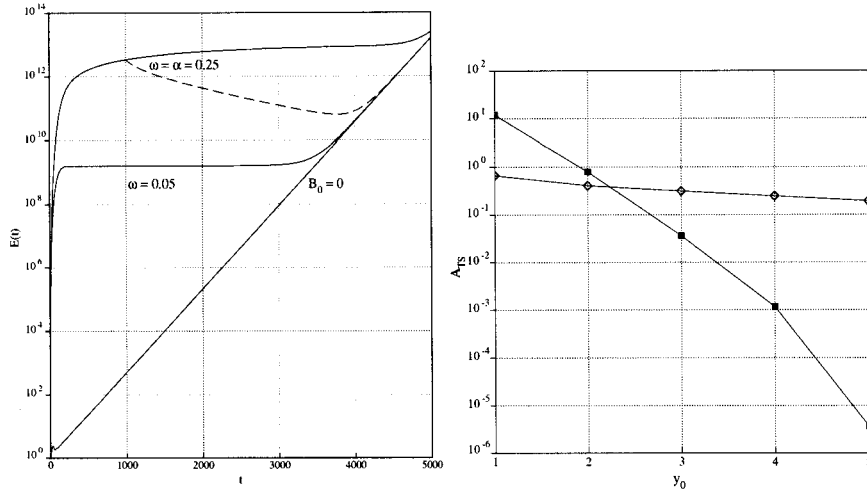


Fig. 1. **A.** Response to slowly ramped single frequency forcing. Forcing stopped at $t = 1000$ (dashed line). **B.** Receptivity factor for forced modes (diamonds) and naturally-forced modes (squares)

is .0874). The energy approaches a constant for $\omega_f \neq \alpha$. This behaviour mimics the particular solution when the governing equations are forced at a single real frequency. The energy remains constant until the energy of the forced solution and the energy of the unstable Tollmien-Schlichting wave introduced by the initial conditions are of equal value. The curve with $\omega_f = \alpha$ does not level off to a finite value on this time scale and shows much greater initial growth as compared with $\omega_f \neq \alpha$. This demonstrates that there is a resonance with the continuum. The dashed line in Figure (1A) shows the results of smoothly removing the forcing at $t = 1000$. A significant Tollmien-Schlichting wave is not generated by forcing external to the boundary layer at this resonant frequency even though the energy of the disturbance shows tremendous growth prior to removing the forcing. The effects of transient forcing are explored by calculating the response to the forcing with

$$r(t) = \left(\frac{t}{t_0}\right)^2 e^{-(t-t_0)^2/\sigma_t^2}. \quad (3)$$

Figure (2B) shows the time histories of the perturbation energy for eighteen cases: $t_0 = 200$, $B_0 = 10^6$, $\alpha = \tilde{\gamma} = .25$, $\phi = 0$, $\sigma_y = .5$, $\sigma_t^2 = 10, 25, 50$, $\omega_f = 0.05, 0.25$ and $y_0 = 1.0, 3.0, 5.0$. Of important note is the slow algebraic decay after cessation of the forcing when $y_0 = 3$ and 5. This is the hallmark of the response as predicted in Grosch and Salwen [7] for the continuum modes. When considering the generation of the Tollmien-Schlichting waves, the dominant parameter is, clearly, the vertical position of the localized forcing function. This might even be unexpected since the Tollmien-Schlichting wave and its adjoint are proportional to $\exp(-\tilde{\gamma}y_0)$ at the point of forcing. This factor cannot account for the more than five orders of magnitude de-

crease seen between groups of solutions as the parameter y_0 varies. In the previous calculations, the function \check{B} has been specified, thus, representing the case of naturally forced receptivity. In Figure (1B), the same function was used to specify $\Delta^2\check{B}$ instead of \check{B} thus representing the case of forced receptivity. The differences in A_{TS} as a function of y_0 is shown. As the disturbance location moves toward the freestream, the immediate generation of Tollmien-Schlichting waves have strengths proportional to $e^{-\gamma y_0}$ in the case of forced receptivity. The strength of immediate generation of Tollmien-Schlichting waves for the naturally forced receptivity drops off at a much faster rate as the forcing location moves toward the freestream.

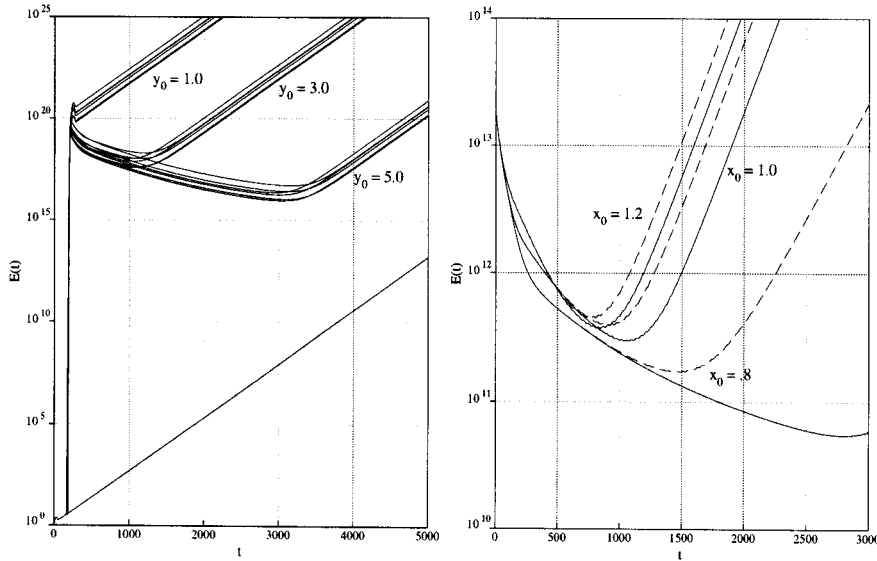


Fig. 2. A. Response to transient forcing. B. Response to initial conditions with Tollmien-Schlichting cancellation (dashed lines) and without cancellation (solid lines).

The development of the theory used here for non-parallel boundary layers is straightforward and relies on the assumption that the parallel theory is at least locally applicable for each value of the Reynolds number, *i.e.*, the same assumption used to derive the Orr-Sommerfeld equation and as a basis for analyzing the effects of non-parallelism on a single Tollmien-Schlichting wave. If the forcing term is written as

$$\Delta\check{B}_d = f(y, t)e^{i\omega_f t}, \quad (4)$$

where the frequency ω_f is considered as the primary frequency of the forcing and the time dependence is such that $f(y, 0) = 0$ and $f(y, t) = 0$ for $t >$

T , then the solution of (1), subject to zero initial conditions and Reynolds number $R = R_1$, is

$$\check{v}(y, t) = \sum_{i=1}^N a_i e^{i\omega_i(t-T)} \phi_i(y) + \int_0^\infty A(k) e^{(i\alpha\gamma - \epsilon\alpha^2 - \epsilon k^2)(t-T)} \phi_k(y) dk \quad (5)$$

for $t > T$. The finite set $\phi_i(y)$, $n = 1, 2 \dots N$ are the discrete eigenfunctions to the Orr-Sommerfeld equation and are normalized to have unit energy. The set $\phi(y, k)$ is the continuum of eigenfunctions to the Orr-Sommerfeld equation, also normalized with respect to energy.

Equation (5) represents the exact solution for a parallel flow subjected to forcing. However, the boundary layer thickens as time progresses, and for a more realistic solution the effects of this must somehow be included. This thickening of the boundary layer is of course a continual process, but insight can be gained by re-expanding the solution at $t = T$ in terms of the eigenfunctions for a larger Reynolds number. The eigenfunctions and adjoint eigenfunctions for $R = R_2$ are generated by replacing $U(y)$, $U'(y)$, and $U''(y)$ with

$$U\left(\frac{y}{\sqrt{x_0}}\right), \quad \frac{1}{\sqrt{x_0}} U'\left(\frac{y}{\sqrt{x_0}}\right), \quad \text{and} \quad \frac{1}{x_0} U''\left(\frac{y}{\sqrt{x_0}}\right), \quad (6)$$

respectively. The parameter x_0 is related to the new Reynolds number by $\sqrt{x_0} = R_2/R_1$. If $\psi_i(y)$ and $\psi_k(y)$ are the eigenfunctions at the new downstream position, then at $t = T$, the solution can be expanded as

$$\check{v}(y, T) = \sum_{i=1}^N b_i \psi_i(y) + \int_0^\infty B(k) \psi_k(y) dk \quad (7)$$

with

$$b_i = - \int_0^\infty \Psi_i^*(y) (D^2 - \tilde{\gamma}^2) \left(\sum_{j=1}^N a_j \phi_j(y) + \int_0^\infty A(k) \phi_k(y) dk \right) dy, \quad (8)$$

where $\Psi_i^*(y)$ is the adjoint eigenfunction of the downstream discrete mode. By taking the extreme example of $a_i = 0$, *i.e.*, the forcing producing no Tollmien-Schlichting waves at $R = R_1$, the coefficient $b_i \neq 0$ since $\Psi_i^*(y)$ is not orthogonal to $\phi_k(y)$ and $\phi_j(y)$ when $j \neq i$. This is a receptivity mechanism due to non-parallelism alone since no further disturbance within or without the boundary layer is required to initiate the gain in amplitude of the Tollmien-Schlichting wave.

These non-parallel effects are explored numerically through a series of initial value problems. The solution for initial values of the form (2) with $y_0 = 3$ and $x_0 = 0.8$ is calculated twice: once as is and once with an additional Tollmien-Schlichting wave added that is amplitude and phase matched to nearly cancel the instability. The results are seen in Figure (2B) where the

divergence of the solutions clearly show that the value of a_{TS} for the second case is near zero. The solution using the same two initial values are calculated with $x_0 = 1$ and $x_0 = 1.2$. Surprisingly, it is seen that the majority of the Tollmien-Schlichting wave at these higher values of x_0 does not come from the Tollmien-Schlichting wave at $x_0 = 0.8$ but rather from the part of the solution that produces no Tollmien-Schlichting wave at $x_0 = 0.8$. The results here are rather ominous. If there is any additional disturbance in the outer edges of the boundary layer or near freestream, these disturbances feed directly into the Tollmien-Schlichting wave and will produce a growth rate greater than (sometimes very much greater than) the predicted value, *even when that predicted value accounts for all of the non-parallel effects associated with a single Tollmien-Schlichting wave.*

2 Conclusions

It has been shown that the techniques previously developed by the authors to investigate various aspects of the temporal stability problem can also be applied to investigate the problem of receptivity. A resonance with the continuum is discovered and must be considered when investigating bypass mechanisms. The form and vertical location of the forcing function is shown to have great significance when determining the strength of the generated Tollmien-Schlichting wave. Perhaps most importantly, when transferring the solution from one downstream location to another the continuum at the upstream position feeds into the Tollmien-Schlichting at the downstream location. More details can be found in [4].

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