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Uncertainty Analysis of the Single-Vector Force Balance Calibration System

Peter A. Parker, Tianshu Liu
NASA Langley Research Center
Hampton, Virginia

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UNCERTAINTY ANALYIS OF THE SINGLE-VECTOR FORCE BALANCE CALIBRATION SYSTEM

Peter A. Parker[*], Tianshu Liu[†]
NASA Langley Research Center
Hampton, Virginia 23681

Abstract

This paper presents an uncertainty analysis of the Single-Vector Force Balance Calibration System (SVS). This study is focused on the uncertainty involved in setting the independent variables during the calibration experiment. By knowing the uncertainty in the calibration system, the fundamental limits of the calibration accuracy of a particular balance can be determined. A brief description of the SVS mechanical system is provided. A mathematical model is developed to describe the mechanical system elements. A sensitivity analysis of these parameters is carried out through numerical simulations to assess the sensitivity of the total uncertainty to the elemental error sources. These sensitivity coefficients provide valuable information regarding the relative significance of the elemental sources of error. An example calculation of the total uncertainty for a specific balance is provided. Results from this uncertainty analysis are specific to the Single-Vector System, but the approach is broad in nature and therefore applicable to other measurement and calibration systems.

Nomenclature

α	angle in the pitch direction
ϕ	angle in the roll direction
$\bar{x},\bar{y},\bar{z}$	linear distances to center of gravity
$\vec{e}$	position offset
λ	rotation matrix
A	axial force in the BCS
BCS	balance coordinate system
CG	center of gravity
F	force
GCS	gravitational coordinate system
g_x,g_y,g_z	projection of the gravity vector onto the balance axes
N	normal force in the BCS
P	pitching moment in the BCS
R	rolling moment in the BCS
S	side force in the BCS
x,y,z	linear distances
Y	yawing moment in the BCS

Subscripts

a	applied weight
bke	below the knife-edge system
f	load point on the fixture
FPS	force positioning system
IBS	inner bearing support
ig	initial geometrical offset
k	load point on the yoke at the location of the knife-edges
KE	knife-edge
KES	knife-edge attachment system
LT	load template
mc1	moment correction due to the yoke load point position
mc2	moment correction due to the yoke CG location
mc3	moment correction due to the 2-axis accelerometer CG location
p	weight pan
q2x	2-axis accelerometer system
s	fasteners to attach FPS to fixture
sub	sub assembly of the FPS (does not include fasteners)
w	precision weights
yk	yoke

Introduction

Direct measurement of aerodynamic loading is fundamental to wind tunnel testing. An instrument known as a force balance provides these measurements in six degrees of freedom. The balance is mounted internally in a scaled wind tunnel model and measures three orthogonal components of aerodynamic force (normal, axial, and side force) and three orthogonal

[*] Research Scientist
[†] Senior Research Scientist

components of aerodynamic torque (rolling, pitching, and yawing moments).

A force balance is a complex structural spring element instrumented with electrical strain sensors. Ideally, each strain sensor of the balance would respond only to its respective component of load, and it would have no response to other components of load. This is not entirely possible, even though balance designs are optimized to minimize these undesirable interaction effects. Ultimately, a calibration experiment is performed to obtain the necessary data to generate a mathematical model and determine the force measurement accuracy.

In general, force balance calibration consists of setting the independent variables and measuring the response of the dependent variables. The applied loads are the independent variables, and the electrical responses of the balance are the dependent variables. In a force balance calibration there are six independent variables and six dependent variables corresponding to each of the components. The uncertainty in setting the six independent variables of applied load must be determined in order to determine the attainable accuracy of the force balance under calibration.

Recently Langley Research Center (LaRC) has developed the Single-Vector Force Balance Calibration System (SVS) that integrates a unique single-vector load application mechanism with a "modern design of experiments" (MDOE) experimental approach.[1,2] This innovative system represents a significant advancement in force balance calibration technology and has enabled an order of magnitude reduction in calibration time and cost, while simultaneously increasing calibration quality. The system also features significantly fewer mechanical components than previous force balance calibration systems and therefore fewer sources of systematic error.

The SVS enables the complete calibration of a six-component force balance with a series of single force vectors. Calibrated dead-weight loads are applied in the gravitational direction generating six component combinations of load relative to the coordinate system of the balance. By utilizing this single force vector, load application inaccuracies caused by the conventional requirement to generate multiple force vectors are fundamentally reduced. The primary components include a non-metric positioning system, a multiple degree of freedom force positioning system, a three-axis orthogonal accelerometer system, and calibrated weights (see Figure 1).

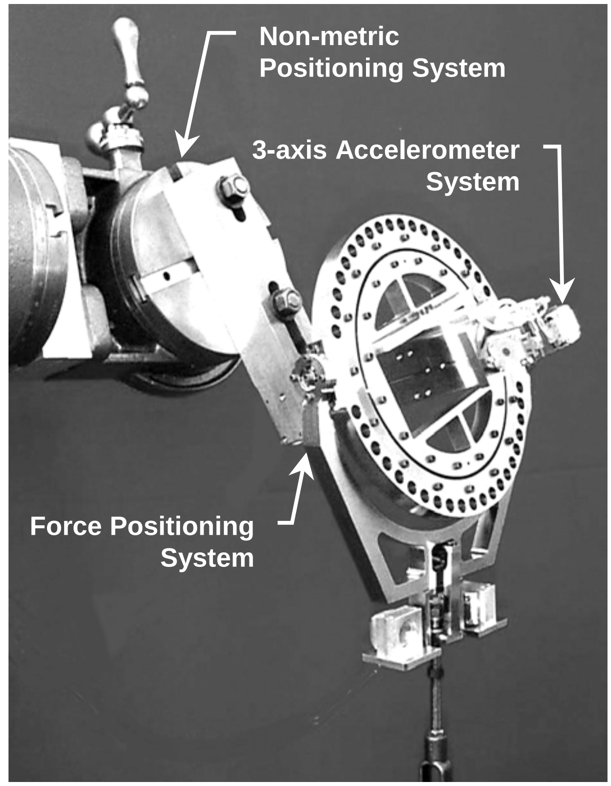

Figure 1. Single-Vector System

The MDOE approach provides an integrated view to the entire calibration process covering all three major aspects of an experiment; the design of the experiment, the execution of the experiment, and the statistical analyses of the data.

In order to quantify the uncertainty in setting the independent variables during the calibration experiment, a thorough uncertainty analysis of the SVS has been performed. A mathematical model is presented, followed by an uncertainty analysis.

Mathematical Model of the System

To perform the uncertainty analysis and the subsequent simulations, a mathematical model was required. This model provides the transfer function between the elemental sources of uncertainty and the resultant forces and moments applied to the balance under calibration. The basic principles involved in setting the forces and moments is presented first followed by a more detailed description that includes the mechanical implementation details.

The three balance force components are a function of the applied load and the orientation of the balance relative to the gravitational coordinate system. To generate a desired combination of the three forces, the balance is manipulated to a prescribed orientation using the non-metric positioning system, and the orientation of the balance is precisely measured on the metric end using the accelerometer system. This accelerometer system provides the components of the gravitational vector projected onto the three-axes of the balance coordinate system. Combining the measured gravitational components on the balance axes and the known applied weight enables the determination of the three force components.

The three balance moment components are a function of the force vectors and the position of the point of load application in three-dimensional space relative to the balance moment center (BMC). The BMC is a defined location in the balance coordinate system that serves as a reference point in which the moment components are described. The point of load application is set using the multiple degree-of-freedom force positioning system. This system utilizes a novel system of bearings and knife-edge rocker guides to maintain the load orientation, regardless of the angular orientation of the balance, which makes the point of load application independent of the angular orientation of the balance. Stated another way, when the balance is manipulated in three-dimensional space, the point of load application remains fixed.

A mechanical system provides the ability to set the independent variables during the calibration experiment as described above. While the SVS hardware system is fundamentally simpler than other force calibration systems, the nature of the six-component calibration experiment still requires multiple mechanical degrees of freedom. A drawing of the primary force application components of the SVS is provided in Figure 2. The system consists of a number of high-precision mechanical components that contribute to the total uncertainty of the forces and moments set during the calibration experiment. The ability to measure or calculate the geometric properties and weights of these components provides the elemental sources of error. The mathematical relationship between the component properties and the independent variables (forces and moments relative to the balance coordinate system) will now be presented.

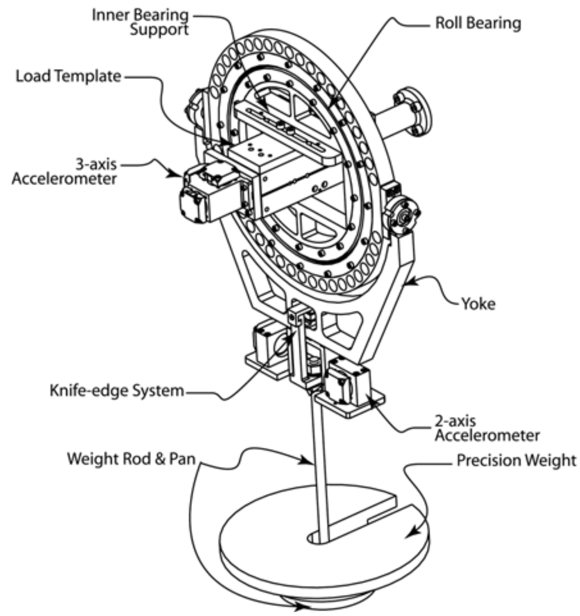

Figure 2. Single-Vector Diagram

<u>Forces Due to the Applied Weights</u>

The computation of the independent variables is divided into two parts. First, the forces are computed followed by the moments. The total applied force is a combination of the precision weights and the measured weight of the rod and pan system,

$$F_a = F_w + F_p \,. \tag{1}$$

The forces in the balance coordinate system (BCS) are

$$\begin{bmatrix} N_a \\ A_a \\ S_a \end{bmatrix} = F_a \begin{bmatrix} g_z \\ g_x \\ g_y \end{bmatrix}. \tag{2}$$

The applied force is multiplied by the projection of the gravitational vector onto the balance axes. In other words, the total force is decomposed into the vector components of the BCS.

<u>Forces Due to the Force Positioning System (FPS)</u>

The applied force due to the force positioning system (FPS) is a combination of the FPS subsystem weight and the fasteners that attach the FPS to the load template as follows,

$$F_{FPS} = F_{sub} + F_s \,. \tag{3}$$

The forces relative to the BCS are

$$\begin{bmatrix} N_{FPS} \\ A_{FPS} \\ S_{FPS} \end{bmatrix} = F_{FPS} \begin{bmatrix} g_z \\ g_x \\ g_y \end{bmatrix}. \tag{4}$$

<u>Moments Due to the Applied Weights</u>

The moments generated due to the applied weight are a function of the magnitude and position of the weight. To determine the position relative to the balance coordinate system, the measured locations on the load template (LT) are combined with the offset of the pin locations of the FPS from the nominal location as follows,

$$\begin{aligned}&\begin{bmatrix} x_f & y_f & z_f \end{bmatrix}^T = \\ &\qquad \begin{bmatrix} x_{LT} & y_{LT} & z_{LT} \end{bmatrix}^T + \begin{bmatrix} x_{FPS} & y_{FPS} & z_{FPS} \end{bmatrix}^T \end{aligned} . \quad (5)$$

The position vector and the computed forces are used to compute the moments relative to the BCS,

$$\begin{bmatrix} R_a \\ P_a \\ Y_a \end{bmatrix} = \begin{bmatrix} 0 & -N_a & -S_a \\ N_a & 0 & -A_a \\ S_a & A_a & 0 \end{bmatrix} \begin{bmatrix} x_f \\ y_f \\ z_f \end{bmatrix} . \quad (6)$$

<u>Moments due to the FPS</u>

Since the geometry of the FPS is not completely symmetric, there are moments generated as the orientation of the balance is changed. Therefore the center of gravity of the mechanical components is computed and used to determine location about which the forces due to the weight of the FPS act. The composite center of gravity (CG) of the FPS is computed from a combination of measured component weights and component CG from a high-fidelity solid model. The composite CG is computed according to,

$$\bar{z}_{FPS} = \frac{(F_{sub} - F_{IBS})(0) + (F_{IBS})(\bar{z}_{IBS}) + (F_s)(\bar{z}_s)}{(F_{sub} + F_s)} , \quad (7)$$

$$\bar{y}_{FPS} = \frac{(F_{sub} + F_s - F_{q2x})(0) + (F_{q2x})(\bar{y}_{q2x})}{(F_{sub} + F_s)} . \quad (8)$$

Once the location of the CG is known, the mounting location of the FPS on the load template is added to determine the position vector,

$$\begin{aligned}&\begin{bmatrix} \bar{x}_{FPS} & \bar{y}_{FPS} & \bar{z}_{FPS} \end{bmatrix}^T = \\ &\qquad \begin{bmatrix} x_f & y_f & z_f \end{bmatrix}^T + \begin{bmatrix} 0 & \bar{y}_{FPS} & \bar{z}_{FPS} \end{bmatrix}^T \end{aligned} . \quad (9)$$

The moments due to the FPS are

$$\begin{bmatrix} R_{FPS} \\ P_{FPS} \\ Y_{FPS} \end{bmatrix} = \begin{bmatrix} 0 & -N_{FPS} & -S_{FPS} \\ N_{FPS} & 0 & -A_{FPS} \\ S_{FPS} & A_{FPS} & 0 \end{bmatrix} \begin{bmatrix} \bar{x}_{FPS} \\ \bar{y}_{FPS} \\ \bar{z}_{FPS} \end{bmatrix} . \quad (10)$$

<u>Moments Corrections Due to the Eccentricity of the Load Point on the Yoke</u>

Due to frictional effects in the large roll bearing, the position of the knife-edge system on the yoke is not able to fully self-align under the load point position on the load template. The frictional effects only create a positioning error in the SVS, not a parallel load path as is commonly associated with frictional effects in other force measurement systems. The effects of the eccentricity of the load point on the yoke are computed in three separate steps as follows.

First the effect of the applied weight at the offset location is computed. This offset is initially computed relative to a gravitational coordinate system (GCS) according to rotational orientation of the yoke as measured by the two-axis accelerometer system,

$$\begin{aligned} e_{yGCS} &= y_{i_{yk}} - \sin(\phi_{yk}) * z_{KE} \\ e_{xGCS} &= x_{i_{yk}} - \sin(\alpha_{yk}) * z_{KE} \\ e_z &= 0 \\ \vec{e}_{ykGCS} &= \begin{bmatrix} e_{xGCS} & e_{yGCS} & e_z \end{bmatrix}^T \end{aligned} . \quad (11)$$

This offset must then be rotated into the balance coordinate system based on the measured attitude of the balance by the three-axis accelerometer system,

$$\vec{e}_{ykBCS} = \left([\lambda_{RM}][\lambda_{PM}]\vec{e}_{ykGCS} \right)^T . \quad (12)$$

Two rotation matrices are required to perform the rotation in the pitch direction and then in the roll direction,

$$\begin{aligned} \alpha_{BCS} &= -\sin^{-1}(-g_x) \\ \phi_{BCS} &= \arctan 2\left(\frac{g_z}{g_y}\right) + \frac{\pi}{2} \end{aligned} \quad (13)$$

$$\lambda_{RM} = \cos \begin{bmatrix} 0 & \frac{\pi}{2} & \frac{\pi}{2} \\ \frac{\pi}{2} & 0 & \left(\frac{\pi}{2} - \phi_{BCS}\right) \\ \frac{\pi}{2} & \left(\frac{\pi}{2} + \phi_{BCS}\right) & 0 \end{bmatrix} \quad (14)$$

$$\lambda_{PM} = \cos \begin{bmatrix} -\alpha_{BCS} & \frac{\pi}{2} & \left(\frac{\pi}{2} + \alpha_{BCS}\right) \\ \frac{\pi}{2} & 0 & \frac{\pi}{2} \\ \left(\frac{\pi}{2} - \alpha_{BCS}\right) & \frac{\pi}{2} & -\alpha_{BCS} \end{bmatrix} . \quad (15)$$

The total force that acts below the knife-edge where the offset occurs is based on a combination of the applied weight, the rod and pan assembly, and the knife-edge attachment system,

$$F_{bke} = F_w + F_p + F_{KES} . \quad (16)$$

Using this total force and the orientation of the BCS, the forces relative to the BCS is

$$\begin{bmatrix} N_{bke} \\ A_{bke} \\ S_{bke} \end{bmatrix} = F_{bke} \begin{bmatrix} g_z \\ g_x \\ g_y \end{bmatrix} . \quad (17)$$

The moments are then computed based on these forces and the eccentricity on the load point that has been determined relative to the BCS.

The moments due to the eccentricity of the load point are corrections to the previously computed moments, which assumed ideal alignment of the load point on the yoke under the load point on the template.

The first moment correction is

$$\begin{bmatrix} R_{mc1} \\ P_{mc1} \\ Y_{mc1} \end{bmatrix} = \begin{bmatrix} 0 & -N_{bke} & -S_{bke} \\ N_{bke} & 0 & -A_{bke} \\ S_{bke} & A_{bke} & 0 \end{bmatrix} \vec{e}_{yk_{BCS}} . \qquad (18)$$

Moments Correction Due to the Eccentricity of the Center of Gravity of the Yoke

A second moment correction is based on the effect of the center of gravity (CG) of the yoke being rotated relative to the BCS. In a similar manner, the position of the CG is determined by calculating its location relative to a GCS and then rotating it into the BCS,

$$\vec{e}_{ykCG_{GCS}} = \left[0 \quad -\sin(\phi_{yk}) * \bar{z}_{yk} \quad 0 \right]^T , \qquad (19)$$

$$\vec{e}_{ykCG_{BCS}} = \left([\lambda_{RM}][\lambda_{PM}] \vec{e}_{ykCG_{GCS}} \right)^T . \qquad (20)$$

The force applied at this rotated location is due to the weight of the yoke. The forces relative to the BCS is

$$\begin{bmatrix} N_{ykCG} \\ A_{ykCG} \\ S_{ykCG} \end{bmatrix} = F_{yk} \begin{bmatrix} g_z \\ g_x \\ g_y \end{bmatrix} . \qquad (21)$$

The second moment correction is equal to

$$\begin{bmatrix} R_{mc2} \\ P_{mc2} \\ Y_{mc2} \end{bmatrix} = \begin{bmatrix} 0 & -N_{ykCG} & -S_{ykCG} \\ N_{ykCG} & 0 & -A_{ykCG} \\ S_{ykCG} & A_{ykCG} & 0 \end{bmatrix} \vec{e}_{ykCG_{BCS}} . \qquad (22)$$

Moment corrections due to the Eccentricity of the Center of Gravity of the Two-Axis Accelerometer System

The third moment correction is based on the location of the center of gravity of the two-axis accelerometer system relative to the BCS. A similar procedure is followed to compute the location relative to the GCS,

$$\vec{e}_{q2xCG_{GCS}} = \left[0 \quad -\sin(\phi_{yk}) * \bar{z}_{q2x} \quad 0 \right]^T , \qquad (23)$$

rotate into the BCS,

$$\vec{e}_{q2xCG_{BCS}} = \left([\lambda_{RM}][\lambda_{PM}] \vec{e}_{q2x_{GCS}} \right)^T , \qquad (24)$$

compute the forces,

$$\begin{bmatrix} N_{q2x} \\ A_{q2x} \\ S_{q2x} \end{bmatrix} = F_{q2x} \begin{bmatrix} g_z \\ g_x \\ g_y \end{bmatrix} \qquad (25)$$

and calculate the moment correction,

$$\begin{bmatrix} R_{mc3} \\ P_{mc3} \\ Y_{mc3} \end{bmatrix} = \begin{bmatrix} 0 & -N_{q2x} & -S_{q2x} \\ N_{q2x} & 0 & -A_{q2x} \\ S_{q2x} & A_{q2x} & 0 \end{bmatrix} \vec{e}_{q2x_{BCS}} . \qquad (26)$$

Total Forces and Moments about the BCS

Combining the intermediate results of the forces acting relative to the BCS provides the total forces as

$$\begin{bmatrix} N \\ A \\ S \end{bmatrix} = \begin{bmatrix} N_a \\ A_a \\ S_a \end{bmatrix} + \begin{bmatrix} N_{FPS} \\ A_{FPS} \\ S_{FPS} \end{bmatrix} . \qquad (27)$$

To calculate the total moments relative to the BCS, the moments assuming no eccentricity on the load point are combined with the three moment corrections as

$$\begin{bmatrix} R \\ P \\ Y \end{bmatrix} = \begin{bmatrix} R_a \\ P_a \\ Y_a \end{bmatrix} + \begin{bmatrix} R_{FPS} \\ P_{FPS} \\ Y_{FPS} \end{bmatrix} + \begin{bmatrix} R_{mc1} \\ P_{mc1} \\ Y_{mc1} \end{bmatrix} + \begin{bmatrix} R_{mc2} \\ P_{mc2} \\ Y_{mc2} \end{bmatrix} + \begin{bmatrix} R_{mc3} \\ P_{mc3} \\ Y_{mc3} \end{bmatrix} . \qquad (28)$$

Note that the eccentricity of the load point does not require corrections to the total forces.

Uncertainty Analysis

Based on the mathematical models presented, the total uncertainty of each measured component $\{X_k\} = (N, A, P, R, Y, S)$ is given by the error propagation equation[3,4]

$$\frac{var(X_k)}{X_{FS\,k}^2} = \sum_{i,j=1}^{M} S_{ik} S_{jk} \rho_{ij} \frac{\left[var(\zeta_i) var(\zeta_j) \right]^{1/2}}{\zeta_i \zeta_j} \qquad (29)$$

where

$$\rho_{ij} = cov(\zeta_i \zeta_j) / [var(\zeta_i) var(\zeta_j)]^{1/2} \qquad (30)$$

is the correlation coefficient between the variables ζ_i and ζ_j, $var(\zeta_i) = < \Delta\zeta_i^2 >$ and $cov(\zeta_i \zeta_j) = < \Delta\zeta_i \Delta\zeta_j >$ are the variance and covariance, respectively, and the notation $< >$ denotes the statistical assemble average. Here the variables $\{\zeta_i, i = 1 \cdots M\}$ denote a set of the parameters in the mathematical models for all the system elements of the SVS. The normalized sensitivity coefficients S_{ik} are defined as

$$S_{ik} = \left(\zeta_i / X_{FS\,k} \right) \left(\partial X_k / \partial \zeta_i \right) \qquad (31)$$

where $X_{FS\,k}$ is the full-scale range of X_k.

A typical balance was chosen as an example to compute the total uncertainty. The SVS experiment design involves 54 unique combinations of the six independent variables (the forces and moments). Within the 54 runs there are combinations of six variables simultaneously, two variables, and single variable set points, also referred to as pure-loads. The uncertainty is dependent on the particular combination of variables applied. Stated another way, the uncertainty depends on the simultaneous levels of the variables for a particular required combination in the calibration experiment design. A detailed analysis was performed for each combination.

To explore the relationship between the elemental sources of error and the resultant forces and moments, a sensitivity study was performed. The objective of this study was to quantify the relative importance of the 27 elemental sources of error listed in Table I. The sensitivity coefficients were estimated numerically. In this case, the sensitivity coefficient matrix consists of 27 rows associated with the elemental sources of error and 6 columns associated with each component of load. Since a local derivative is calculated, the sensitivity coefficient matrix will be different depending on the particular combination of load.

As an example of this analysis, a six-component combination was chosen. The 27 parameters of the SVS are listed in Table I along with the typical elemental errors. In most calibration systems the combination of all six components is this most mechanically challenging and involves the maximum contribution of all sources of systematic error. The SVS experiment design involves the application of all six components simultaneously at approximately 41% of the full-scale load range. Figure 3 shows the estimated sensitivity coefficients for this six-component combination, where the elemental source index number refers to the parameters in Table I

The uncertainties of the force components (N, A, S) are dominated by the applied load and the attitude measurement of the balance coordinate system relative to the gravitational coordinate system. The forces are not influenced by the position of the force vector or the orientation of the yoke. This is expected since the forces are not dependent on the location of application. Therefore, the forces will always have a lower uncertainty as compared to the moment components. This is a general result and not dependent on the particular balance under calibration.

Furthermore, it is typically considered more difficult to accurately apply the force vectors when there exists a large ratio between the components, for example the ratio between normal force and axial force. With the SVS, the ratio does not influence the uncertainty of the application of axial force. A review of the elemental sources of error reveals that the uncertainty of all the forces will be the same regardless of force magnitude, since the elemental error associated with the precision of the weights is a constant percentage of the applied weight and the gravitational projection are also a constant percentage of the full scale applied load.

In terms of the moment uncertainties, they are influenced by the same sources as the forces plus the elemental distances. The elemental uncertainty of the distance is a constant in most cases. Therefore for a longer distance the elemental uncertainty becomes less significant in terms of a percentage. The results from the moment components are not general in nature, but are specific to the balance that is under calibration.

For example, a balance with a large normal force and a relatively small rolling moment will have a large uncertainty in the rolling moment component. This is due to the critical positioning of the normal force to avoid an unwanted rolling moment. The larger the ratio between rolling moment and the normal force, the less critical the positioning becomes.

The sensitivity coefficient matrix provides vital information about the elemental sources of error that have large contributions to the uncertainty of the setting of the independent variables. It also provides useful information about the elemental sources of the error that are not critical. Knowing which sources of error are dominant and which are not is crucial to the further development of the SVS. While the sensitivity coefficient matrix is a useful tool that provides a relative comparison of the influence of the elemental sources, it is also useful to quantify the total uncertainty.

The total uncertainty was calculated for all 54 experimental runs in the calibration experiment. The results are shown in Figure 4. On each of the six plots the total uncertainty for each component, denoted by the six symbols, is plotted versus the percentage applied of each component. For the plot in the top left corner of the figure, the total uncertainty of the components (N, A, P, R, Y, S) is plotted versus applied normal force expressed at a percentage of full-scale. This plot illustrates the maximum uncertainty of the rolling moment component occurring at the maximum normal force level. This due to the positioning error and is consistent with the previous discussion regarding this particular phenomenon.

The six component combinations occur on each plot with the levels of the applied load at approximately 41% of full-scale. The pure loads, or two components simultaneously in the case of a moment, occur at the 100% of full-scale level. The moments are loaded in combination with a force due to the inability to generate a pure moment with a single force vector.

Consistently across all plots the rolling moment has the highest total uncertainty of 0.025% at a six component combination, and 0.065% during a pure load. This is due to the short distance of elemental source number 5, which is one of the dominant elemental sources contributing to the rolling moment total uncertainty. This result is not general in nature, but is balance dependent as previously discussed.

The total uncertainty of the force components is approximately 0.01% of full-scale. This result is general and is not balance specific.

For this particular example balance, the maximum total uncertainties for the N, A, P, R, Y, and S components are 0.009%, 0.010%, 0.023%, 0.065%, 0.024%, 0.010% respectively.

Table I. Parameters and Elemental Error Sources of the SVS

Index	Variable	Description	Range Low	Range High	Mean Level	Standard Deviation	
1	g_z	projection of gravity vector on z-axis	-1	1		0.000020	g
2	g_x	projection of gravity vector on x-axis	-1	1		0.000020	g
3	g_y	projection of gravity vector on y-axis	-1	1		0.000020	g
4	x_{LT}	CMM measured *x* location on load template	-1.5625	1.5625		0.000200	inches
5	y_{LT}	CMM measured *y* location on load template	-0.625	0.625		0.000200	inches
6	z_{LT}	CMM measured *z* location on load template	constant		0.0000	0.000200	inches
7	F_w	measured force due to precision weights	0	160		0.01%	% of FS
8	F_p	measured force due to rod and pan assembly	constant		3.1566	0.000473	lbs.
9	F_s	measured force due to attachment screws	constant		0.0322	0.000005	lbs.
10	$\overline{z}_s$	calculated z-distance to the centroid of screws			-1.7500	0.001000	inches
11	F_{IBS}	measured force due to the inner bearing support	constant		6.7871	0.001018	lbs.
12	$\overline{z}_{IBS}$	calculated z-distance to CG of IBS	constant		-0.2867	0.001000	inches
13	F_{FPS}	measured force due to FPS	constant		30.1575	0.004524	lbs.
14	x_{FPS}	CMM measured *x* location of pins in FPS	constant		0.0025	0.000200	inches
15	y_{FPS}	CMM measured y location of pins in FPS	constant		-0.0006	0.000200	inches
16	z_{FPS}	CMM measured z location of pins in FPS	constant		0.0005	0.000200	inches
17	F_{yk}	measured force due to the yoke	constant		8.0639	0.001210	lbs.
18	$\overline{z}_{yk}$	calculated z-distance to CG of yoke	constant		4.7439	0.000500	inches
19	x_{yk}	CMM measured *x* location of hole in the yoke			0.0000	0.000200	inches
20	y_{yk}	CMM measured y location of hole in the yoke			0.0022	0.000200	inches
21	F_{q2x}	measured force due to the 2-axis accel. sys.	constant		1.4054	0.000211	lbs.
22	y_{q2x}	measured y-distance to the CG of 2-axis accel.	constant		0.0375	0.002000	inches
23	z_{q2x}	measured z-distance to the CG of 2-axis accel.	constant		10.5215	0.000200	inches
24	α_{yk}	measured angle of the yoke orientation in pitch	-1	1		0.001000	degrees
25	ϕ_{yk}	measured angle of the yoke orientation in roll	-1	1		0.001000	degrees
26	F_{KES}	measured force due to the knife-edge system	constant		1.2275	0.000184	lbs.
27	z_{KE}	CMM measured z loaction of knife-edge system	constant		8.0844	0.000200	inches

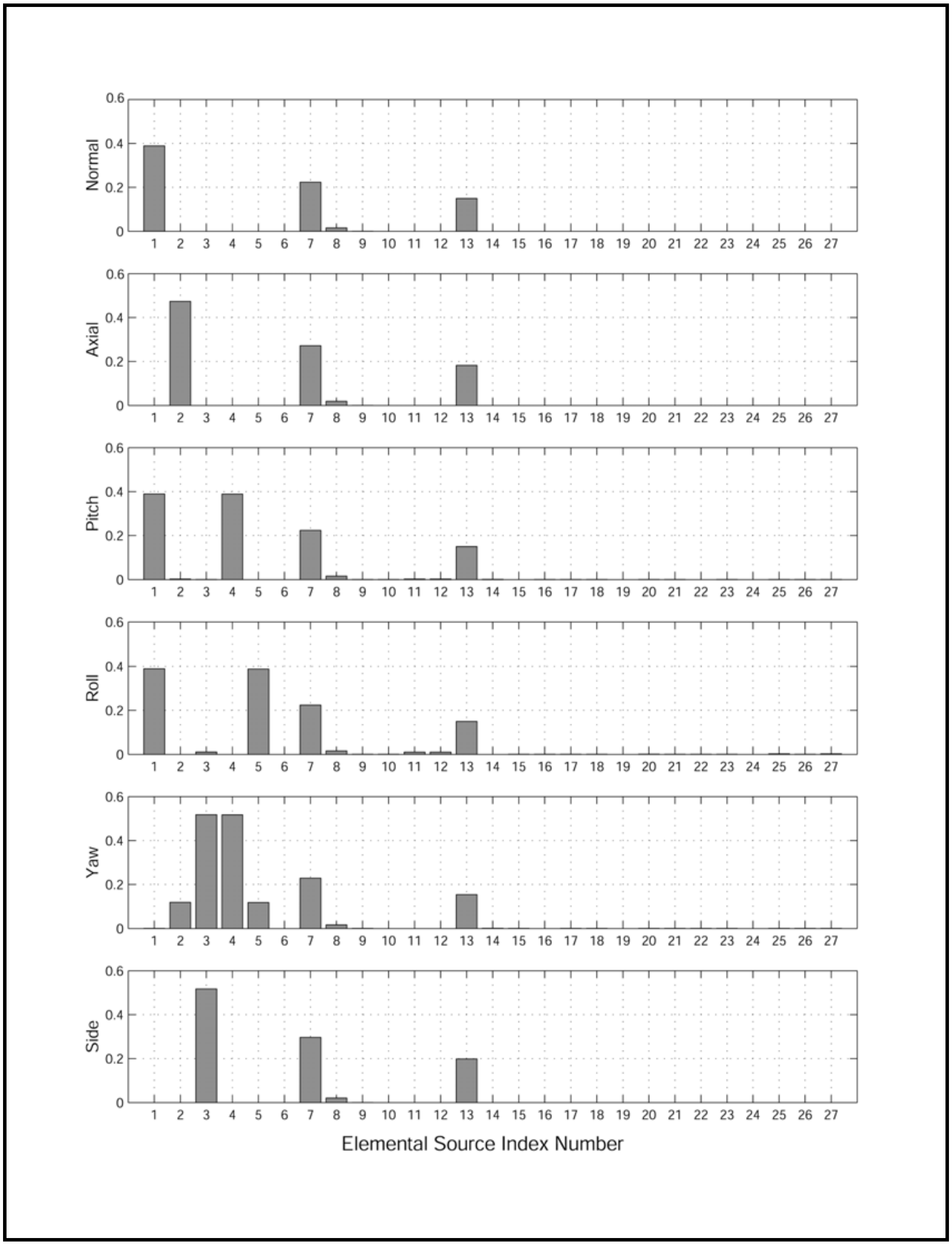

Figure 3. Sensitivity Coefficients for a Six Component Combination.

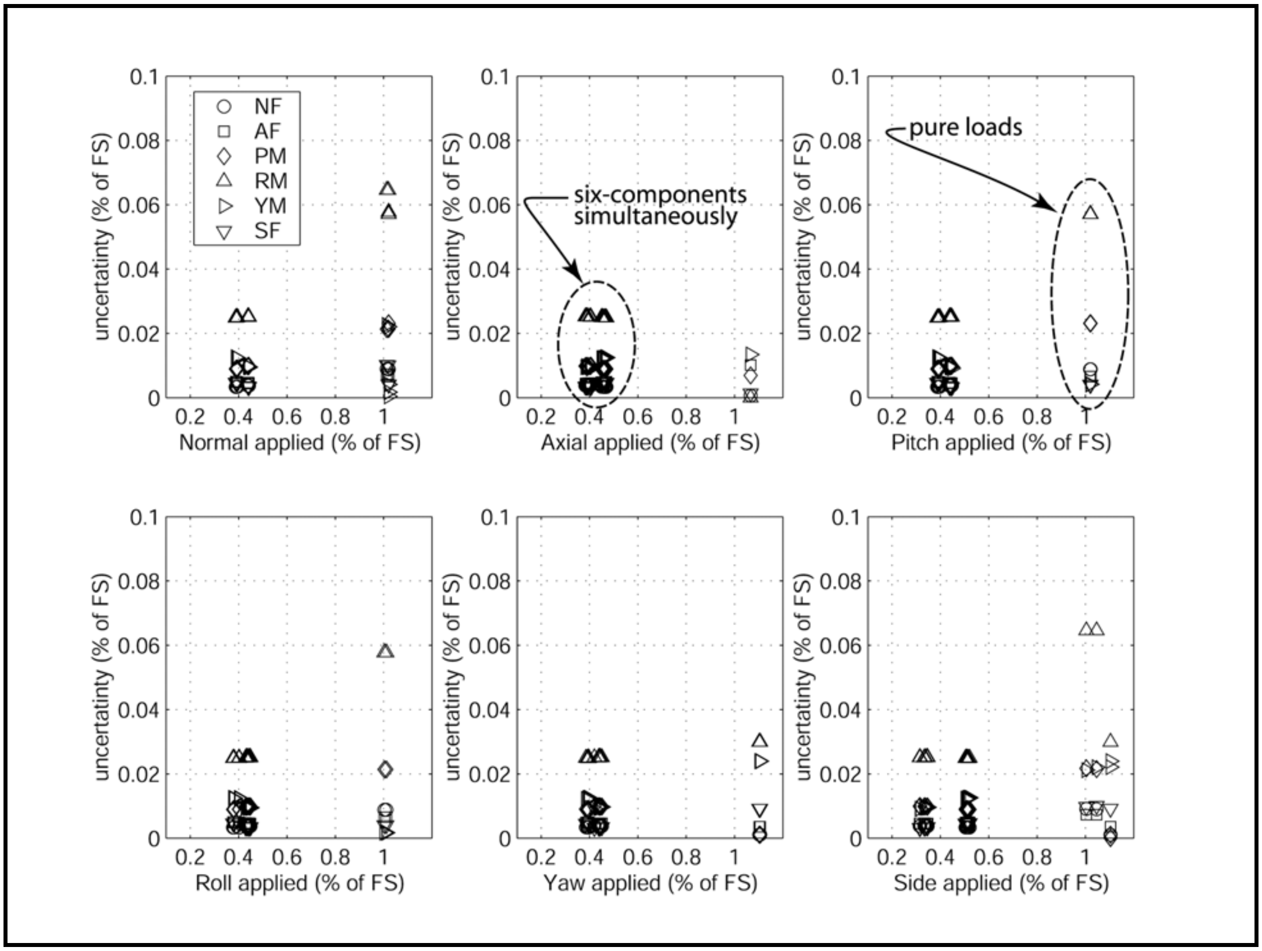

Figure 4. Total Uncertainty of Experimental Design Points

Concluding Remarks

An uncertainty analysis of the Single-Vector System has been presented. This analysis has provided insight into the uncertainty in setting the independent variables during a calibration experiment. The computation of the total uncertainty is necessary to define the attainable accuracy of a specific balance under calibration.

A complete mathematical model that defines the relationship between the elemental sources of error and the desired forces and moment has been presented. The sensitivity of the forces and moments to each elemental source of error has illustrated the dominant sources. An example of the calculation of the total uncertainty has been provided for a specific balance

A rigorous uncertainty analysis of the force calibration system is necessary to be able to quantify the limits for a specific balance calibration. This paper documents the uncertainty analysis of the SVS and presents a general method that is applicable to other force balance calibration systems.

References

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