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Effects of Stiffening and Mechanical Load on Thermal Buckling of Stiffened Cylindrical Shells

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EFFECTS OF STIFFENING AND MECHANICAL LOAD ON THERMAL BUCKLING OF STIFFENED CYLINDRICAL SHELLS

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Abstract

A study of thermal buckling of stiffened cylindrical shells with the proportions of a preliminary supersonic transport fuselage design (1970) is presented. The buckling analysis is performed using an axisymmetric shell-of-revolution code, BOSOR4. The effects of combined mechanical (axial loading) and thermal loading (heated skins) are investigated. Results indicate that the location of longitudinal eccentric stiffening has a very large effect on the thermal buckling strength of longitudinally stiffened shells, and on longitudinally stiffened shells with rings.

I. Introduction

New aerospace systems studies indicate a renewal of interest in supersonic transports (SST's) and single-stage-to-orbit (SSTO) launch vehicles. Because of the moderate to high heating rates, both vehicles will experience thermal stresses from temperature differences between the skin and stiffening members. If the primary structures are large stiffened shells, it is possible for them to buckle under moderate temperature increases.

The purpose of the present paper is to numerically investigate the buckling behavior of a generic cylindrical shell model under a combination of mechanical and thermal loading. To restrict the number of computations, the shell is limited to a length where general instability is infeasible and the buckling interaction is mainly between panel instability (from mechanical loads) and local instability (from circumferential thermal stresses at the ends due to end restraints).

In the body of the paper, the technical approach is described in more detail. Results are presented in the form of tables and buckling interaction curves with axial load and temperature change as parameters. In performing the study, it is found that the eccentricity of the longitudinal stiffening of the shell is important (i.e., whether the stiffening is located on the inner or outer surface of the shell). Hence, surface plots are presented for the cases that vary the following parameters: axial load, temperature change and eccentricity. Finally, the effect of boundary conditions is investigated by comparing models with and without end effects.

II. Approach

A non-linear, finite difference, shell-of-revolution code, BOSOR4[1] is used as the analysis tool. BOSOR4 is based upon the Vlasov-Sanders shell theory and is used to conduct both nonlinear pre-buckling and buckling analyses[2] for all cases presented herein. Modal iteration is used in BOSOR4 to determine the eigenvalues. BOSOR4 is an efficient code for developing buckling interaction curves, but for the models analyzed in this paper, BOSOR4's solution algorithm (modal iteration) is sensitive to the starting eigenvalue and eigenvector as the eccentricity varies. Multiple solutions occur along some points of the interaction curves and BOSOR4 needs to be directed along the curve (the previous solution is used as a starting point for the next analysis).

The model's material properties, Table 1, and structural configuration are adopted from an earlier (1970) SST design, Fig. 1.[3] The fuselage structure is simplified to an axisymmetric shell of revolution without imperfections, Fig. 2.[4] The shell is modeled with at least six nodes per inch of the shell length to ensure the capture of local instability modes. Ring frames are modeled as discrete beams and the stringer's stiffness is smeared into the skin properties. Thus, ring and stringer crippling or rollover is not investigated. Simple support boundary conditions are used to represent the bulkheads assumed at each end. At both ends, the shell is allowed to grow axially and is constrained from expanding in the radial direction. The ends must be allowed to move axially, so that axial forces can be introduced into the shell. Rotations at the ends are permitted because the skin is considered extremely flexible compared to the bulkheads. Rigid body motion in the longitudinal direction is eliminated by restricting axial motion at the center plane of the shell. In Fig. 3, the coordinate system of the shell model is presented. A complete list of the boundary conditions used in the analysis is given in Tables 2, 3 and

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4. Table 2 contains the boundary conditions for studies of longitudinally stiffened shells without ring frames where symmetry is employed at one end of the shell. Table 3 displays the boundary conditions for full-length ring- and stringer-stiffened shells. Table 4 contains boundary conditions used for a study of the interior behavior of the shell where symmetry is employed at both ends.

Table 1. Material properties and geometry of members of an internally stiffened shell.

Member	E ksi.	ν	α per °F	A in^2	I in^4	GJ $lbs\text{-}in^2$	e in.
Skin	14.5×10^3	.30	5.0×10^{-5}	-	-	-	-
Ring	16.4×10^3	-	-	.246	.1377	3477	-1.992
Stringer	16.4×10^3	-	-	.0495	.004083	.000	-.342

E - Modulus of Elasticity
ν - Poisson's ratio
α - Coefficient of thermal expansion
A - Area
I - Moment of inertia
GJ - Torsional Rigidity
e - Eccentricity measured from mid-plane of the shell wall to the centroid of the stiffening member (positive if located on the outer surface, Fig. 3)

Table 2. Boundary conditions for interior shell models employing symmetry.

Pre-Buckling

Left End	Right End (Plane of Symmetry)
u = free	u = fixed
v = fixed	v = free
w = fixed	w = free
$w,_x$ = free	$w,_x$ = fixed

Buckling

Left End	Right End (Plane of Symmetry)
u = free	u = fixed
v = fixed	v = free
w = fixed	w = free
$w,_x$ = free	$w,_x$ = fixed

The shell is loaded with axial line loads and a uniform temperature change, Fig. 3. The axial line loads are applied at the two ends in opposing directions on the centroid of the shell and stringer cross-section. The uniformly distributed axial line load represents an approximation to the maximum load experienced in bending by the fuselage. A uniform temperature change is applied to the skin only. The temperature change is not applied to the rings, stiffeners or bulkheads. A zero temperature change is applied to the skin where the bulkheads and rings come into contact with the skin; as a result, a temperature gradient is formed in the axial direction. The described temperature loading occurs as the vehicle climbs to altitude. As the SST ascends, the outside temperature rises and the internal stiffeners and bulkheads remain cool, not only because of the thermal mass of the members, but also because of climate conditions inside the cabin. All of the loads on the shell are applied to produce the worst case conditions, yielding conservative results.

Table 3. Boundary conditions for full-length ring- and stringer-stiffened shells.

Pre-Buckling

Left End	Center	Right End
u = free	u = fixed	u = free
v = fixed	v = free	u = free
w = fixed	w = free	w = fixed
$w,_x$ = free	$w,_x$ = free	$w,_x$ = free

Buckling

Left End	Center	Right End
u = free	u = fixed	u = free
v = fixed	v = free	v = fixed
w = fixed	w = free	w = fixed
$w,_x$ = free	$w,_x$ = free	$w,_x$ = free

The impact of length of the shell and the number of rings on thermal buckling results is the first study conducted using BOSOR4. In Fig. 4, results are presented showing the difference in the axial or thermal buckling load as the number of rings increase. When considering four or more rings, the results do not vary for either temperature loading or axial loading. There is a small change in wave number, but not a significant difference in temperature or axial buckling load.

Since the addition of rings would not result in a more accurate solution, a six-ring model is used to represent a complete shell. The restriction of the shell to a length of six rings eliminates general instability as a mode of failure in the present studies. Thus, the buckling interaction investigated herein is between panel instability from axial compression and local buckling from circumferential stress due to end constraints and temperature change.

Table 4. Boundary conditions for single-ring models employing symmetry at both ends.

Left End	Right End
Pre-Buckling	
u = free	u = fixed
v = fixed	v = free
w = free	w = free
$w,_x$ = fixed	$w,_x$ = fixed
Buckling	
u = free	u = fixed
v = fixed	v = free
w = free	w = free
$w,_x$ = fixed	$w,_x$ = fixed

III. Results and Discussion

It was suspected that two distinct buckling modes would have a direct impact on the shape of the interaction curve. Hoff[5] showed that when a shell buckles because of thermal loads, local instability at the boundaries was usually the mode of failure. Local instability can occur when the boundaries were rigid compared to the shell and caused high circumferential stress. Panel instability was usually associated with axial loading or end shortening, Anderson.[6]

In the present study of shell buckling behavior, it is found that eccentricity of the longitudinal stiffening has a large effect. Eccentricity is measured from the centroid of the stiffening member to the reference surface of the shell, Fig. 3. To highlight this effect, results for shells without rings are presented first.

Longitudinally Stiffened Shells

Results for the buckling interaction curve for longitudinally stiffened shells are presented in Figs. 5 and 6. The buckling interaction curve and mode shapes are for the geometry given in Fig. 2, but without ring frames. The longitudinal stiffeners are located either on the internal or external surface of the shell. The results in Figs. 5 and 6 suggest that the interaction between buckling modes (panel vs. local) are quite separate with no transitions between modes.

The shell buckles because of panel instability in the portion of the buckling interaction curve with circular symbols in Figs. 5 and 6. Panel instability is due to axial load. The mode shape for this type of failure is indicated on the figures. The location of failure in the shell is depicted by the maximum peaks on the graphs. The shell buckles because of local instability in the portion of the curve with square symbols in Figs. 5 and 6. Local instability is due to high circumferential stresses caused by the imposed temperature change and end constraints. The associated mode shape is indicated on the figures.

The internally stiffened shell under only temperature loading, Fig. 5, can tolerate a temperature change of almost three times the temperature change withstood by the externally stiffened shell, Fig. 6 ($T_{int.}$ = 1003°F vs. $T_{ext.}$ = 340°F). This extreme difference is a very interesting phenomena for thermally loaded shell structures. The externally stiffened shell can withstand almost twice the axial load carried by the internally stiffened shell ($Nx_{int.}$ = 561 lbs/in. vs. $Nx_{ext.}$ = 922 lbs/in.) for axial loading only. The latter result is well known for longitudinally stiffened shells when comparing the relative strength of external stiffening to internal stiffening.

Ring- and Stringer-Stiffened Shells

The buckling interaction curve for a ring- and stringer-stiffened shell with internal longitudinal stringers at an eccentricity of -0.342 inches is presented in Fig. 7. The buckling interaction curve has a distorted but similar shape compared to that produced by Chang and Card.[4] In the buckling interaction curve presented in Fig. 7, two distinct buckling modes are present and both influence the shape of the curve. The shell fails from axial load, i.e., panel instability, in the portion of the curve with circular symbols. The associated mode shape is indicated in Fig. 7. Buckling under axial loading is caused by high stresses in the axial direction of the shell. In the portion of the curve with square symbols, failure is caused by temperature change or local instability (see Fig. 7). Buckling under temperature load is induced by high circumferential stresses developed in the skin at the cool bulkheads (edge effect). The center portion of the curve (portion with triangular symbols) exhibits where the two mode shapes interact with each other forming a hybrid or combined mode shape. Failure of the shell is caused by combined panel and local instability, Fig. 7.

During the generation of the buckling interaction curve in Fig. 7 with axial load as the eigenvalue parameter, it was found that multiple solutions could exist beyond the portion of the curve dominated by axial load (portion of curve with circular symbols). Temperature was used as the eigenvalue parameter to overcome this difficulty.

In Table 5, it can be seen that the wave numbers decrease as the axial load changes from tension to compression (compression is positive). This trend shows that the mode shapes mutate along the curve, Fig. 7, in a continual fashion. The mode shape quickly changes from panel instability to the hybrid mode shape at the distortion in the curve as the curve is traversed from above. However, there is a smooth transition from local instability to the hybrid mode shape as the buckling interaction curve is traversed from below.

As the centroid of the longitudinal stiffeners is moved through the thickness of the shell, the buckling interaction curve distorts even more. The results in Fig. 8 are for the shell with the centroid of the stringers at the shell's reference surface or mid-plane (eccentricity = 0.000 inches). The longitudinal stiffeners in Fig. 9 are external to the shell, at an eccentricity of 0.342 inches. The buckling modes remain the same in Figs. 7, 8 and 9, but their shape varies with eccentricity. When the eccentricity of the stringers is varied, the shape of the buckling interaction curve develops a large distortion. This distortion occurs where hybrid buckling modes are located. The distortion is most pronounced when the longitudinal stiffeners are fully external to the shell. All of the results characterized in the two previous paragraphs for the internally stiffened shell (eccentricity = -0.342 inches) remain the same, but the shape of the buckling interaction curve changes significantly, Fig. 8 and Fig. 9.

In Fig. 10, the buckling interaction curves for the three discussed eccentricities are superimposed on one graph. In this figure the dramatic effect of changing the eccentricities can be seen. As the centroid of the stringers is moved through the mid-plane of the shell wall (internal to external), the maximum axial load decreases then increases, but the maximum temperature load continually decreases. The axial load increases by a factor of 1.4 ($Nx_{int.}$ = 4122 lbs/in. vs. $Nx_{ext.}$ = 5568 lbs/in.) for longitudinal stiffening with no temperature change. The maximum temperature change decreases by a factor of over 2.0 between internal and external stiffening ($T_{int.}$ = 626°F vs. $T_{ext.}$ = 300°F when there is no axial load).

The three-dimensional plot, Fig. 11, shows a surface envelope of buckling results that displays how the distortion of the buckling interaction curve changes as the eccentricity is varied The reduction in buckling temperature and increase in axial load can also be seen in Fig. 11. The solution surface shows that as the centroid of the stringers is moved closer to the reference surface, not only does the ability of the shell to resist temperature load decrease, so does its ability to withstand axial load.

Table 5. Buckling Results from BOSOR4 for an internally stiffened shell with ring frames.

Critical Pressure Psig.	Critical Temp. ° F.	Critical Nx Lbs/In.	Circum-ferential Wave Number	Eccen-tricity* In.
0.	623.5	-600.0	67	-0.3420
0.	623.9	-400.0	66	-0.3420
0.	624.5	-200.0	65	-0.3420
0.	626.5	000.0	63	-0.3420
0.	624.2	200.0	62	-0.3420
0.	619.2	400.0	60	-0.3420
0.	610.3	600.0	59	-0.3420
0.	598.3	800.0	58	-0.3420
0.	583.9	1000.0	57	-0.3420
0.	566.6	1200.0	56	-0.3420
0.	549.3	1400.0	55	-0.3420
0.	530.8	1600.0	54	-0.3420
0.	511.4	1800.0	53	-0.3420
0.	490.8	2000.0	52	-0.3420
0.	469.1	2200.0	51	-0.3420
0.	446.0	2400.0	49	-0.3420
0.	421.3	2600.0	47	-0.3420
0.	399.9	2765.0	46	-0.3420
0.	375.0	2942.0	44	-0.3420
0.	350.1	3102.0	42	-0.3420
0.	324.9	3249.0	39	-0.3420
0.	300.0	3376.0	37	-0.3420
0.	275.0	3486.0	34	-0.3420
0.	250.0	3580.0	32	-0.3420
0.	225.0	3664.0	30	-0.3420
0.	200.0	3734.0	28	-0.3420
0.	175.0	3797.0	27	-0.3420
0.	150.0	3853.0	26	-0.3420
0.	125.0	3905.0	25	-0.3420
0.	99.99	3956.0	24	-0.3420
0.	75.00	4000.0	24	-0.3420
0.	50.01	4043.0	23	-0.3420
0.	25.00	4083.0	23	-0.3420
0.	0.00	4122.0	23	-0.3420

*In BOSOR4, an eccentricity of 0.312 inches is input. BOSOR4 measures from the shell wall surface.

Effect of Boundary Conditions

The boundary conditions significantly affect the response of a stiffened shell.[5,6] A ring section or repeating element model of a ring- and stringer-stiffened shell is created with only one ring frame. In Fig. 12, a picture of the model in its undeformed shape is presented. Two models are created to investigate the behavior of the shell in the interior, away from edge effects. One has internal stiffening and the other has external stiffening. The boundary conditions for the models has symmetry conditions at each end, Table 4. One end is allowed to move axially to introduce the axial load into the shell. The opposite end is not allowed to move in the axial direction.

When loaded under axial compression the externally stiffened shell carries a higher load ($Nx_{int.}$ 3816 lbs/in. vs. $Nx_{ext.}$ = 4972 lbs/in.), but for temperature load, it tolerates a similar amount of temperature change ($T_{int.}$ 486°F vs. $T_{ext.}$ = 499°F). The two buckling interaction curves are similar and actually come together as the temperature rises, Fig. 13. Clamping of the boundaries plays an important role on the behavior of the shells. The clamping causes the two models to have the same mode shape along the entire buckling interaction curve, Fig. 12.

A comparison of the results for the single-ring models with the full-length model suggests that the critical axial loads are less dependent on the boundary conditions. However, a comparison of the thermal buckling results indicates that boundary conditions are critical.

IV. Concluding Remarks

A numerical study of the buckling behavior of a generic stiffened shell has been presented. Analysis of buckling under combined mechanical load and thermal load was conducted using a finite difference shell-of-revolution code. The thermal buckling analysis corresponds to the case where the shell skin was heated, but the rings, stringers and ends of the shell remained cool. Longitudinally stiffened shells with and without ring frames were considered.

Results for the shell with only longitudinal stiffening indicated a large difference in buckling effects from the positioning of eccentric stiffening. Under thermal load alone, internally stiffened shells withstood a temperature change of almost three times that of externally stiffened shells. The trend reversed when there is only mechanical load. The internally stiffened shell carried roughly half the axial load of the externally stiffened shell.

The stiffener eccentricity effects were more moderate for the ring- and stringer-stiffened shell. Under only thermal loading, the internally stiffened shell withstood twice the temperature change of the comparable shell with external longitudinal stiffening. The shells with external longitudinal stringers withstood a greater mechanical load than shells with internal stringers and rings.

A comparison of results for single-ring (interior) stiffened shells with results for a six ring-stiffened shell indicated that for axial compressive load, the buckling results were similar. The eccentricity effects were quite different for thermal buckling, with virtually no effect on the one-ring shells. As noted by previous investigators (e.g., ref. 5), thermal buckling was strongly related to boundary conditions. The present paper showed that eccentricity effects in thermal buckling were also greatly affected by edge effects.

Finally, the eccentricity trends in the present paper need more investigation. The effects of pre-buckling deformations and loads, buckling boundary conditions, modeling techniques, and in-depth studies of the underlying equations and solutions used in analyzing thermal buckling of shells could all be subjects for future investigations. A physical experiment should be conducted to verify the trends discovered in this research.

V. Acknowledgment

The authors express their appreciation to David Bushnell for running some cases and helpful discussions on thermal buckling phenomena.

VI. References

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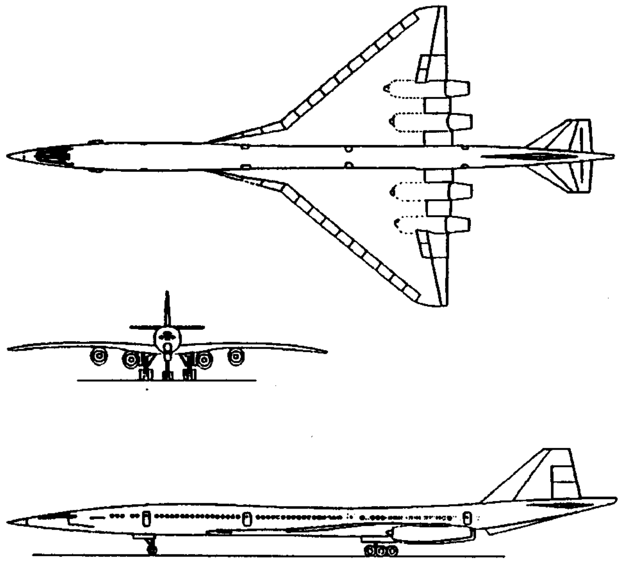

Fig. 1 Sketch of the early (1970) U.S. Supersonic Transport (SST).

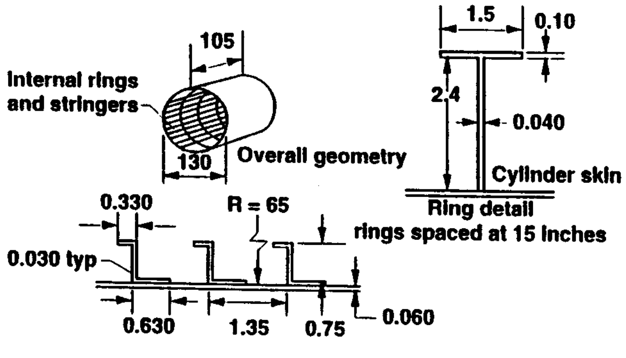

Fig. 2 Geometry of a typical ring- and stringer-stiffened shell analyzed in BOSOR4. Dimensions are in inches.

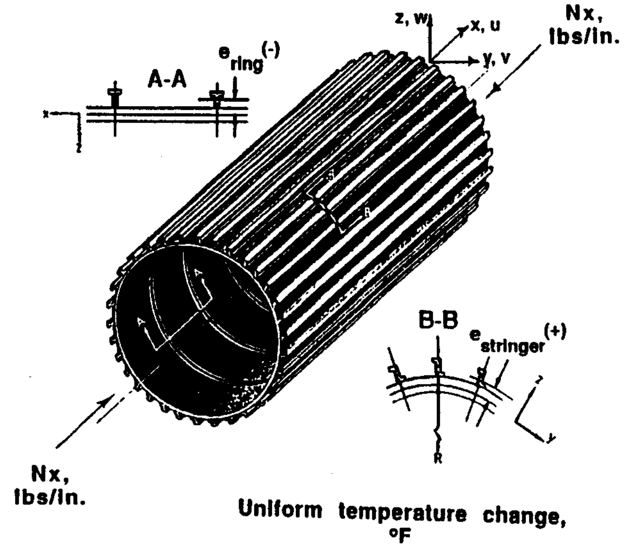

Fig. 3 Coordinate system of the shell model, placement of loads and depiction of stiffener eccentricities.

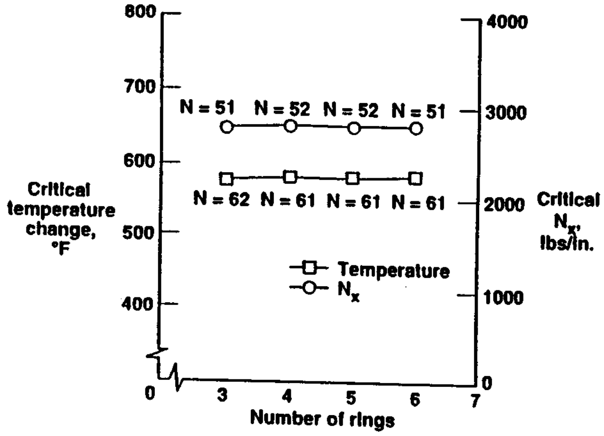

Fig. 4 Effect of number of rings on the critical buckling temperature and critical axial load.

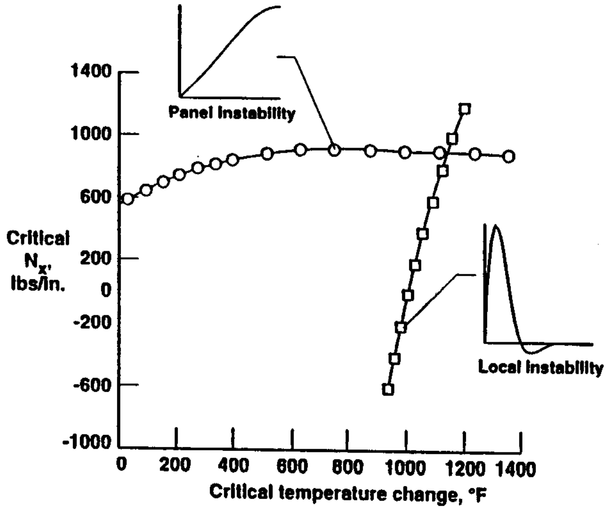

Fig. 5 Buckling interaction curve and associated mode shapes for a shell with stringers at an eccentricity of -0.342 inches (internal).

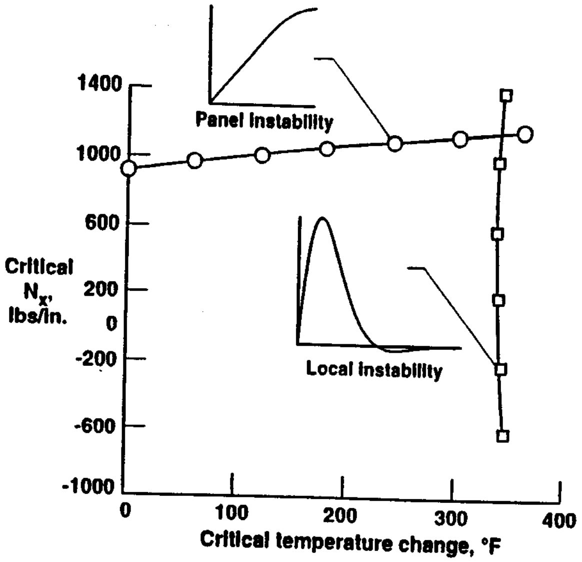

Fig. 6 Buckling interaction curve and associated mode shapes for a shell with stringers at an eccentricity of 0.342 inches (external).

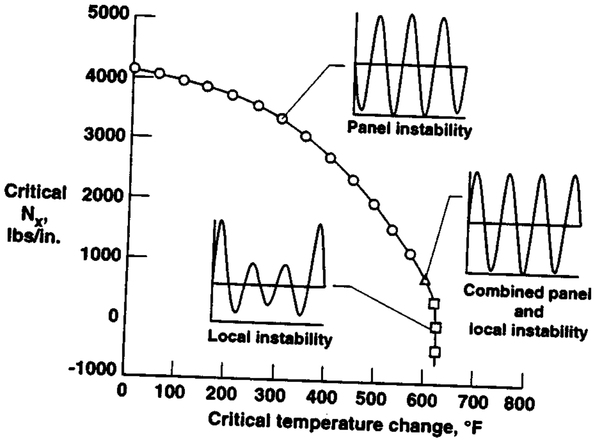

Fig. 7 Buckling interaction curve and associated mode shapes for a shell with internal ring frames and stringers at an eccentricity of -0.342 inches (internal).

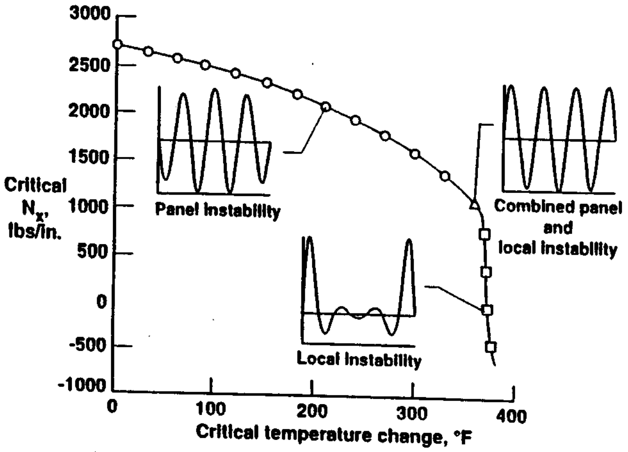

Fig. 8 Buckling interaction curve and associated mode shapes for a shell with internal ring frames and stringers at an eccentricity of 0.000 inches.

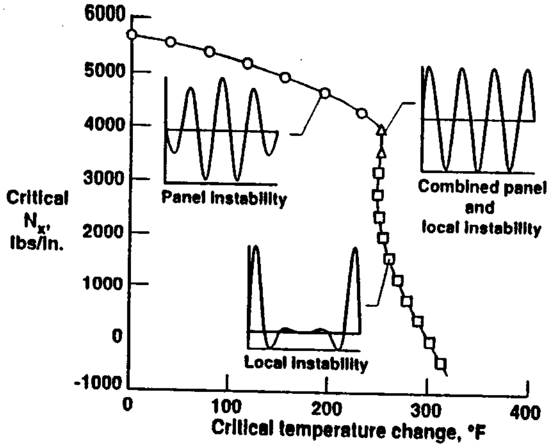

Fig. 9 Buckling interaction curve and associated mode shapes for a shell with internal ring frames and stringers at an eccentricity of 0.342 inches (external).

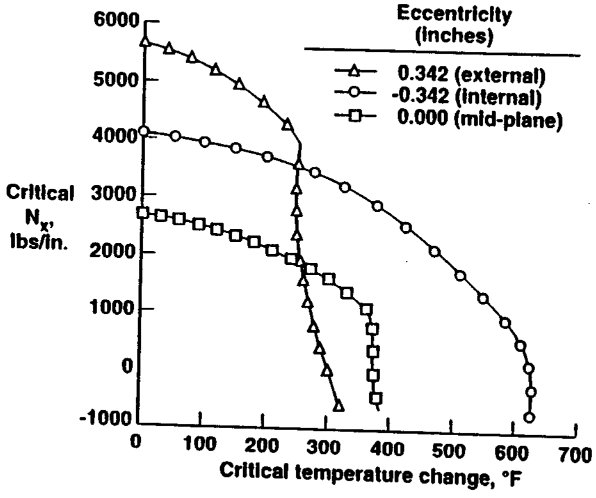

Fig. 10 Comparison of buckling interaction curves for three eccentricities.

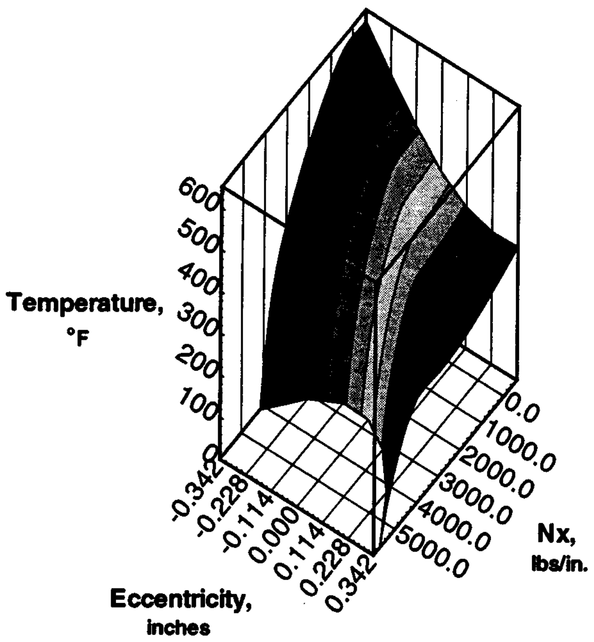

Fig. 11 Buckling interaction surface for a ring- and stringer-stiffened shell.

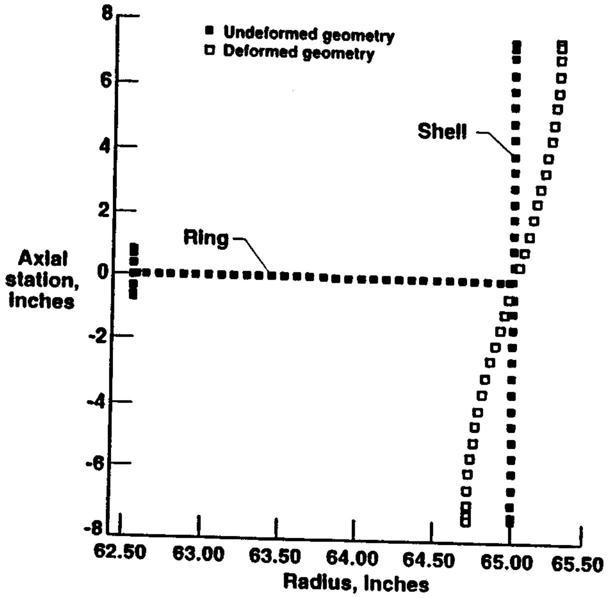

Fig. 12 Mode shape (panel instability) for a stiffened single-ring model (repeating element model) with either internal- or external-longitudinal stiffening at an eccentricity of 0.342 inches.

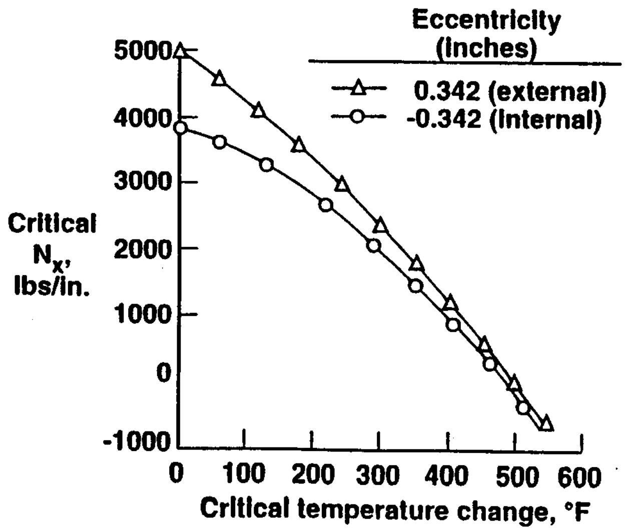

Fig. 13 Buckling interaction curves a stiffened single-ring model (repeating element model) with either internal- or external-longitudinal stiffening at an eccentricity of 0.342 inches.