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USE OF CAD GEOMETRY IN MDO

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1 Abstract

The purpose of this paper is to discuss the use of Computer-Aided Design (CAD) geometry in a Multi-Disciplinary Design Optimization (MDO) environment. Two techniques are presented to facilitate the use of CAD geometry by different disciplines, such as Computational Fluid Dynamics (CFD) and Computational Structural Mechanics (CSM). One method is to transfer the load from a CFD grid to a CSM grid. The second method is to update the CAD geometry for CSM deflection.

2 Introduction

The process of aircraft design can be broken into three phases[1]: (1) conceptual design, (2) preliminary design, and (3) detail design. The conceptual design process focuses on the basic design optimization of features, such as weights, sizes, and overall performance. During the preliminary design, the focus is on the mathematical modeling of the outside skin of an aircraft with sufficient accuracy. After this phase, the geometry is frozen, and any change could be costly. Detail design concentrates on the actual design of pieces to be fabricated.

Often an aircraft is represented by a simple model during the conceptual and preliminary designs. Because simple models are neither accurate nor complete, optimization of these models could lead to an impractical design [2, 3]. This shortcoming can be alleviated by using a high fidelity model, and the interaction among various disciplines must be modeled accurately. These interactions are very complicated and important piece of MDO.

The strong interactions of CSM and CFD are very common in an MDO environment. Such interactions can prompt physically important phenomena such as those occurring in aircraft due to aeroelasticity. Cor-

rect modeling of these complex aeroelastic phenomena requires direct coupling of CSM and CFD. During optimization of a flexible structure (e.g., wing), the geometry changes due to the aeroelastic effect. All disciplines share the same geometry, and must be able to consistently communicate and share information (e.g. deflection and load). The geometry representation for MDO must be accurate and suitable for grid generation for various disciplines such as CFD and CSM. To further complicate the process, modeling of complicated geometrical models requires use of CAD systems.

The interactions among various disciplines require the manipulation of the original CAD geometry that is stored as a set of NonUniform Rational B-Splines (NURBS). This paper describes two techniques to manipulate the NURBS geometry. In the next following sections there are brief discussions on NURBS, parameterization of aircraft geometry, NURBS-based optimization, load transfer, deflection transfer, results, and conclusions.

3 NURBS

This section contains a brief overview of the NURBS, and readers should consult [4] for a detailed discussion. A NURBS curve, $\vec{R}(u)$, can be represented as

$$\vec{R}(u) = \frac{\sum_{i=0}^n N_{i,p}(u)W_i\vec{P}_i}{\sum_{i=0}^n N_{i,p}(u)W_i}. \quad (1)$$

The parameter, u , is bounded by $u_{min} \leq u \leq u_{max}$. The \vec{P}_i are the control points (forming a control polygon), and W_i are the weights. The $N_{i,p}$ are the p-th degree B-spline basis functions defined on the non-periodic and nonuniform knot vector (u)

$$u \in [u_{min} = u_0 \leq u_1 \leq \dots u_j \leq \dots \leq u_k = u_{max}], \quad (2)$$

where k is the number of knots. This completes the mapping between the one-dimensional parameter space, u , and the three-dimensional Euclidean space, \vec{R} . A NURBS curve has five important properties:

- It is invariant under linear transformation.

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- A NURBS curve of order p , having no multiple interior knots, is $p - 2$ differentiable.
- The approximation is local in nature.
- A NURBS curve is contained in the convex hull of its control points.
- The NURBS approximation is variation diminishing.

To evaluate, the three-dimensional curve NURBS is commonly represented in homogeneous form as

$$\vec{R}(X, Y, Z) \iff \vec{R}^W(WX, WY, WZ, W). \quad (3)$$

So, the NURBS curve can conveniently be defined as a perspective map of its nonrational counterpart in four-dimensional space as

$$\vec{R}^W = \sum_{i=0}^n N_{i,p}(u) \vec{P}_i^W, \quad (4)$$

where \vec{P}_i^W is defined as

$$\vec{P}_i^W = \{X_i W_i, Y_i W_i, Z_i W_i, W_i\}. \quad (5)$$

The basis functions can be efficiently computed by using DeBoor algorithm [5] as

$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \leq u < u_{i+1}, u_i < u_{i+1}, \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

$$N_{i,p}(u) = L_i(u)N_{i,p-1}(u) + M_i(u)N_{i+1,p-1}(u), \quad (7)$$

where

$$L_i(u) = \frac{u - u_i}{u_{i+p} - u_i}, \quad (8)$$

$$M_i(u) = \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}}. \quad (9)$$

It is agreed that $0/0 = 0$.

A NURBS surface is a parametric surface and is defined as a function of two parameters as

$$\vec{R}(\vec{U}) = \{X(\vec{U}), Y(\vec{U}), Z(\vec{U})\}^T, \quad (10)$$

$$\vec{U} = \{u_1, u_2\}^T \in [(a, b), (c, d)],$$

where the components of vector, \vec{U} , are the surface parameters and have no geometrical significance. However, for a constant u_2 , as u_1 increases the point, $\vec{R}(\vec{U})$ moves always from one side of the surface to the other side. The NURBS definition for the surface is defined as

$$\vec{R}(\vec{U}) = \frac{\sum_{i=0}^n \sum_{j=0}^m N_{i,p}(u_1) N_{j,q}(u_2) W_{i,j} \vec{P}_{i,j}}{\sum_{i=0}^n \sum_{j=0}^m N_{i,p}(u_1) N_{j,q}(u_2) W_{i,j}}. \quad (11)$$

where $\vec{P}_{i,j}$ are control points (forming a control surface), $W_{i,j}$ are the weights, and $N_{i,p}$ and $N_{j,q}$ are the p -th and q -th degree B-spline basis functions defined on the non-periodic and nonuniform knot vector. The evaluation process is very similar to the NURBS curve evaluation.

4 Aircraft Parameterization

To use complex shapes in an MDO environment, the parameterization and geometry modeling must be compatible with existing CAD systems, and it must be adaptable to CFD (i.e., block-structured or unstructured grids) and CSM. The four approaches for parameterization of an aircraft geometry are based on: analytical, semi-analytical, discrete, and CAD representation.

Analytical methods for optimization have been used for a long time. These methods convert a set of design variables (e.g., wing sweep, thickness ratios) into a set of surfaces. Then, these surfaces can be used to analyze and compute the objective function (e.g., [6, 7, 8]). This approach is very simple and requires a few design variables. The geometry based on this approach is very smooth. On the other hand, the geometry can take a limited form, and it is hard to optimize existing and free-form geometry. Interactions among disciplines are very difficult to model. Blair and Reich [8] have implemented a Virtual Design Process (VDM) that is integrated with full associativity within Pro/Engineer CAD/CAM software [9].

In the second approach, semi-analytical, a set of points can describe the initial geometry, and a polynomial can model the perturbation of geometry [10]. Then, the coefficients of this polynomial are used as a set of design variables. Again, this method is simple, and it allows the designer to use existing geometry. This approach requires a few design variables, and the smoothness of the geometry depends on the baseline geometry. This approach is very difficult to generalize and use in an MDO environment for a complex geometry.

The third approach, discrete, is based on a discrete representation of the geometry. The baseline geometry creates the grids, and the position of each grid point becomes a design variable for the optimizer.

This is very easy to implement, and the geometry changes don't have a limited form. The latter could create a problem in which the optimum design may not be practical to manufacture. The number of design variables often becomes very expensive which, leads to high costs and a difficult optimization problem to solve. Also it is difficult to maintain a smooth geometry.

The fourth approach, CAD, is based on the optimization of a NURBS representation of geometry (e.g., [11]). The large number of design variables often becomes very expensive which leads to high costs and a difficult optimization problem to solve. The geometry continuity and smoothness are guaranteed. Also, the geometry can change locally without affecting everything else. This type of parameterization is flexible enough to represent a wide range of geometries. Existing complicated CAD models can be used as the baseline model, but modeling the interaction among disciplines is very difficult.

These four approaches are summarized and listed in Table 1. In the next section, a NURBS-based optimization that is a hybrid approach based on first and fourth approaches is discussed.

5 NURBS-Based Optimization

CAD systems have been developed very rapidly and integrated into the design process (e.g., see [8]). Use of CAD systems for geometry modeling in an MDO environment could potentially save development time. However, there are two drawbacks: (1) initial investment (software and training), and (2) inability to calculate analytical sensitivity. The geometry representation in these systems is complicated due to a large number of entity representations. In a traditional CAD system, the geometry is represented as one of many possible mathematical forms such as Bezier, Coons patch, B-Spline, surface of revolution, etc. However, one can use NURBS equations to represent most parametric and implicit surfaces without loss of accuracy [4]. NURBS can represent quadric primitives (e.g., cylinder, cones), as well as free form geometry [4]. There are some CAD surfaces (e.g., helix and helicoidal[12]) that cannot be directly converted to NURBS. These surfaces are not common in most CAD system. The NURBS representation must be used in such a way that it should be possible to alter it automatically to accommodate the changes due to: (1) design variable changes, or (2) structural

and control surface deflections.

Calculating of the sensitivity of geometry with respect to the design variables could prove to be very difficult. In some instances, it is possible to relate the NURBS control points to the design variables. Then the analytical sensitivity can be calculated outside of the CAD system. Another way to calculate the sensitivity is to use finite difference, as long as the perturbed geometry has the same topology as the unperturbed one. Both methods, the analytical and finite difference, have their pitfalls and limitations.

Implementation described here is based on the Framework for Interdisciplinary Design Optimization (FIDO) [13, 14] developed at NASA Langley Research Center (LaRC). The process for geometry creation, integration, and manipulation are designed around the NURBS representation of the complete geometry. To embed this process into an optimization process such as FIDO, the model (High-Speed Civil Transport) has to be parameterized with a set design variables (DVs).

For each optimization cycle, the run starts with a definition for a set of design variables (see Figure 1), to build a NURBS-based geometry. Pro-Engineer ([9]) is proposed for this step as the geometry builder. Once the geometry is built, the NURBS geometry will be deposited into a NURBS database, which will be shared among different disciplines. This database will be maintained as the baseline geometry changes. During the optimization process, CFD and CSM disciplines will need the complete geometry definition.

For each iteration of the aeroelastic loop, the CAD geometry is used to create CFD and CSM grids. This requires that both disciplines use and modify the same unique geometry in the geometry database. In each loop, the CFD grid is used to compute the aerodynamic load, which is transferred to the CSM grid. This load will be converted to a NURBS definition and deposited into the database. At this point, the CSM grid will be generated based on the NURBS database. The NURBS CFD load will be mapped to the CSM grid, which along with the CSM grid is used to compute the resulting deflection. The next critical step is to modify the original CAD geometry to accommodate and reflect the aeroelastic deflection.

Each optimization loop can be summarized as shown in Figure 1:

- 1. The CAD system, Pro/Engineer, will convert a set of design variables (D) into a NURBS geometry (G). Because the source for Pro/Engineer code is not available, the sensitivity has to be

calculated outside of the system.

- 2. At this point, the geometry, G , and its sensitivity, GD , will be stored in the NURBS database.
- 3. The Coordinates and Sensitivity Calculator for Multidisciplinary Design Optimization (CSCMDO) ([15]) will calculate the CFD mesh and its sensitivity M , MD .
- 4. The CFD code, with the capability to calculate the sensitivity, will obtain the load, which will be stored in the NURBS database.
- 5. Similarly, the CSM grid generator can create the finite element grid and its sensitivity. The load stored in the NURBS database will be mapped onto the finite element model.
- 6. The CSM code, with the capability to calculate the sensitivity, will obtain the deflection which will be stored in the NURBS database.
- 2'. The final step is to modify the NURBS database to accommodate the new structural deflection.

The two steps that are defined and discussed in this paper in detail are: (1) converting CFD load to a NURBS definition, and (2) modify the NURBS geometry to accommodate the deflection.

6 Load Transfer

To transfer the load to CSM elements, one needs to be able to compute the load at any point on the surface. Initially, the load, $F(\vec{R})_{CFD}$, is computed on the CFD grid, ($\vec{R}_{CFD} = \{X, Y, Z\}^T$), which could be a set of structured quadrangles or unstructured triangles with the appropriate connectivity. There are two basic problems in fitting this data with NURBS: (1) the data has four dimensions, (X, Y, Z, F) , and (2) the CFD grid could be an unstructured grid.

The first problem can be solved by mapping the CFD grid to the original NURBS surfaces, hence, reducing the dimension from four, (X, Y, Z, F) , to three, (U, V, F) . The U, V are the parametric coordinates of the original NURBS surface. This information may be available from a CFD grid generation process. If not, the CFD grid points can be projected onto the original NURBS surface [16]. The process of projecting a point, $\vec{r} = \{X, Y, Z\}^T$, on a surface, $\vec{R}(\vec{U})$, can be performed by finding a \vec{w} such that the

distance, d , between the \vec{r} and $\vec{R}(\vec{w})$ is minimal and \vec{w} is constrained to $\in [(a, b), (c, d)]$. The distance, d , can be written in terms of parameters \vec{w} as

$$d^2(\vec{w}) = f(\vec{w}) = |\vec{R}(\vec{w}) - \vec{r}| \cdot |\vec{R}(\vec{w}) - \vec{r}|. \quad (12)$$

The next step is to fitting a single-value three-dimensional surface, $F = F(U, V)$. This surface can be fitted based on a least-squares approximation [17, 18] that minimizes the approximation error. A three-dimensional curve is used as an example of the least-squares fitting.

A set of points in three space, $\vec{r}(u)$, can be fitted by a B-Spline curve, $\vec{R}(u)$. The B-Spline equation can be expressed at each parameter, u , as

$$\vec{r}_j = N_{1,k}(u_j)\vec{R}_1 + N_{2,k}(u_j)\vec{R}_2 + \dots + N_{n,k}(u_j)\vec{R}_n \quad (13)$$

The above equation can be expressed as

$$[\vec{r}] = [N][\vec{R}] \quad (14)$$

where

$$[\vec{r}]^T = [\vec{r}_1, \vec{r}_2, \dots, \vec{r}_{jmax}]^T \quad (15)$$

$$[\vec{R}]^T = [\vec{R}_1, \vec{R}_2, \dots, \vec{R}_n]^T \quad (16)$$

$$[N] = \begin{bmatrix} N_{1,k}(u_1) & \dots & N_{n,k}(u_1) \\ N_{1,k}(u_j) & \dots & N_{n,k}(u_j) \\ N_{1,k}(u_{jmax}) & \dots & N_{n,k}(u_{jmax}) \end{bmatrix} \quad (17)$$

where $jmax$ is the maximum number of data points, \vec{r}_j . \vec{R}_n is the n th B-Spline control points. If $jmax = n$, the matrix $[N]$ is a square matrix and the control points can be calculated directly by matrix inversion,

$$[\vec{R}] = [N]^{-1}[\vec{r}]. \quad (18)$$

In this case, the resulting B-Spline curve passes through each data point. However, if the number of data points, $jmax$, is greater than the number of control points, n , the problem is over-specified. A least-square method can solve the problem as,

$$[\vec{R}] = [[N]^T[N]]^{-1}[N]^T[\vec{r}]. \quad (19)$$

The least-squares approximation for surfaces is very similar to the least-squares approximation for the curves. The minimization error can be written as

8 Results and Discussions

$$\text{Error} = \sum_0^N E_N, \\ E_N = \left[F(\vec{U}_N) - \frac{\sum_i^n \sum_j^m N_{i,p}(U)N_{j,q}(V)W_{i,j}F_{i,j}}{\sum_i^n \sum_j^m N_{i,p}(U)N_{j,q}(V)W_{i,j}} \right]^2 \quad (20)$$

where $F_{i,j}$ are control points for the NURBS surface representing the CFD load, $W_{i,j}$ are the weights, and $N_{i,p}$ and $N_{j,q}$ are the p-th and q-th degree B-spline basis functions defined on the non-periodic and nonuniform knot vector. N is the number of points in the CSM grid. This forms a system of linear equations that can be solved for control points of a NURBS surface representing the load.

7 Deflection Transfer

As described in the previous section, the load is defined on the CFD grid, $\vec{R}_{CFD} = \{X, Y, Z\}^T$. On the other hand, the deflection, $\Delta\vec{R}_{CSM}$, is defined on the CSM grid, $\vec{R}_{CSM} = \{X, Y, Z\}^T$, which is represented by a set of polygons (e.g., triangles and quadrangles) with appropriate connectivity. The goal is to modify the CAD geometry definition, $\vec{R}(\vec{U})$, such that it reflects the deflection produced by CSM. The algorithm for deflection transfer has four steps:

1. Project the \vec{R}_{CSM} onto the original NURBS surface.
2. Create a NURBS surface based on the deflection, $\Delta\vec{R}_{CSM}(U, V)$, which has the same degree as the original NURBS surface.
3. Add/remove knots from the new surface to make it compatible with the original NURBS surface.
4. Add the control points to the original NURBS surface to form the new surface.

This algorithm has following properties:

1. As $\Delta\vec{R}_{CSM}$ approaches zero, the method will reproduce the original NURBS surface, $\vec{R}(\vec{U})$.
2. Smoothness is controlled on the resulting NURBS surface.
3. The results surface is a NURBS surface with the same degree as the original NURBS surface.
4. It is possible to maintain the same knot vector as the original NURBS surface.

The results are presented for the load and deflection transfers. For the load transfer, a generic pathfinder geometry (see Figure 2) is used. The geometry is a single cubic NURBS surface with 53 by 24 control points. There are two test cases for the load transfer. For the first test case, a Sin function is defined over a triangular mesh (see Figure 3), which covers the surface of the pathfinder. The data is fitted with a NURBS surface. The original and interpolated contours are shown in Figure 4. The Root Mean Squares (RMS) error for interpolation is less than two percent, and the resulting cubic NURBS surface has 15 by 15 control points. For the second test case, the pressure distribution on the surface is fitted with a cubic NURBS surface (see Figure 5). The resulting NURBS surface has 35 by 24 control points, and the RMS error is one percent. Figure 6 shows a close-up view near the inboard leading edge.

The next two test cases demonstrate the results of deflection transfer. A generic High-Speed Civil Transport (HSCT) geometry is used for both test cases. This geometry is made of three surfaces: fuselage, inboard wing, and outboard wing. Figure 7 shows the original NURBS surface, the deflected CSM grid, and the deflected NURBS surface. To test the limits of the deflection-transfer algorithm, the CSM grid has a large and unrealistic deflection. Figure 8 shows a side view of the same test case. For the last test case, the HSCT model was deflected by using a Sin function. Figure 9 shows the original NURBS surface, the deflected CSM grid, and the deflected NURBS surface. Figure 10 shows a side view of the same test case. The resulting NURBS surfaces have an RMS error of less than one percent. For the last two test cases, even though the inboard and outboard were fitted separately, no gaps or overlaps were observed. But this could be a potential problem.

Using the techniques described here, it is possible to include CAD geometry within an MDO environment with strong interdisciplinary interactions. The potential drawbacks for these algorithms are: (1) calculating sensitivity, and (2) potential gaps and overlaps among multiple surfaces. Both need to be studied further.

9 Acknowledgments

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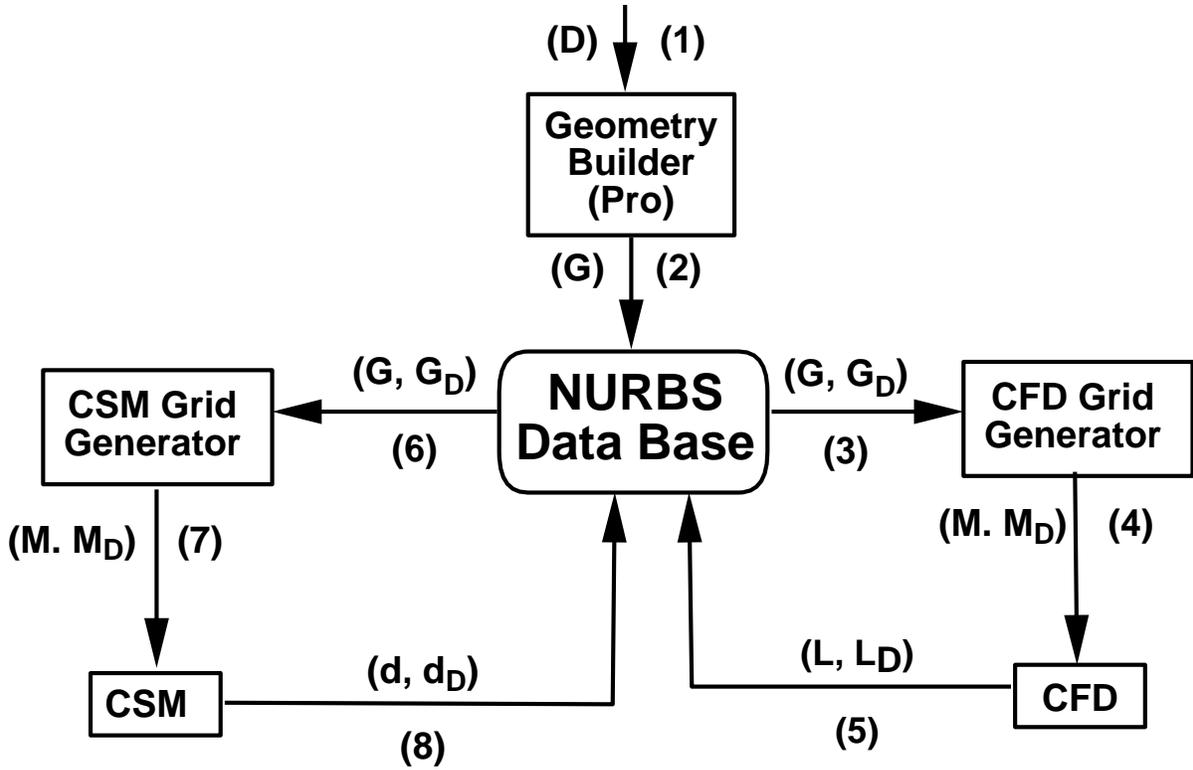


Figure 1: NURBS Based Optimization

Criteria	Analytical	Semi-Analytical	Discrete	CAD
Design Variables	Few	Few	Many	Many
Level of Complex Geometry	Low	Low	Medium	High
Discipline Interaction	Difficult	Difficult	Fair	Difficult
Smoothness of Geometry	Easy	Easy	Difficult	Easy
Shape Limitation	Very Limited	Limited	No Limits	No Limits
Local Control	None	Some	Good	Excellent
Analytical Sensitivity	Easy	Easy	Difficult	Very Difficult

Table 1: Summary of Parameterization Techniques

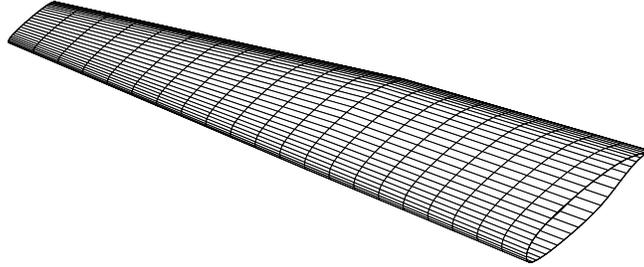


Figure 2: A Generic Pathfinder

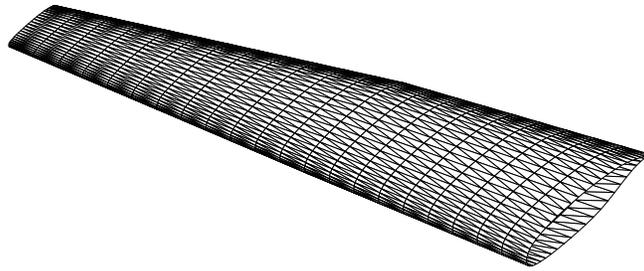


Figure 3: Triangular Mesh for the Generic Pathfinder

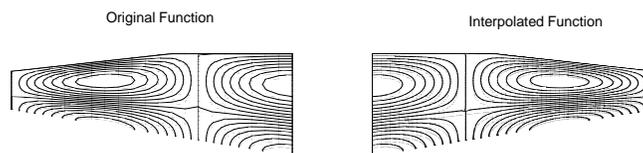


Figure 4: Transfer of Sin Function for a Generic Pathfinder

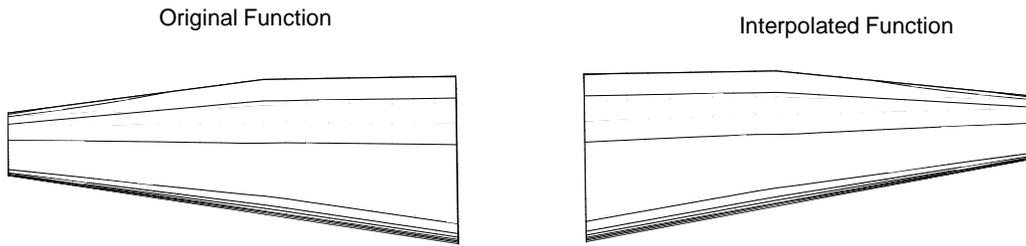


Figure 5: Load Transfer for a Generic Pathfinder

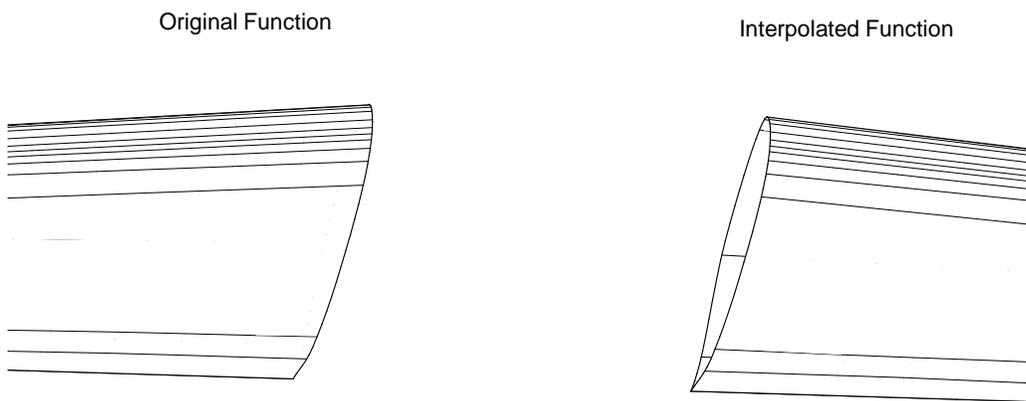


Figure 6: Load Transfer for a Generic Pathfinder (Close Up)

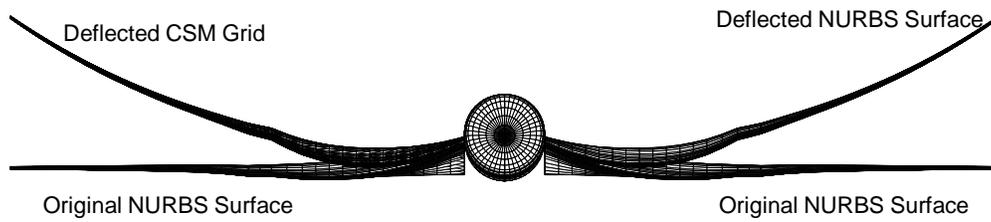


Figure 7: Deflection of a Generic HSCT Configurations (Front View)

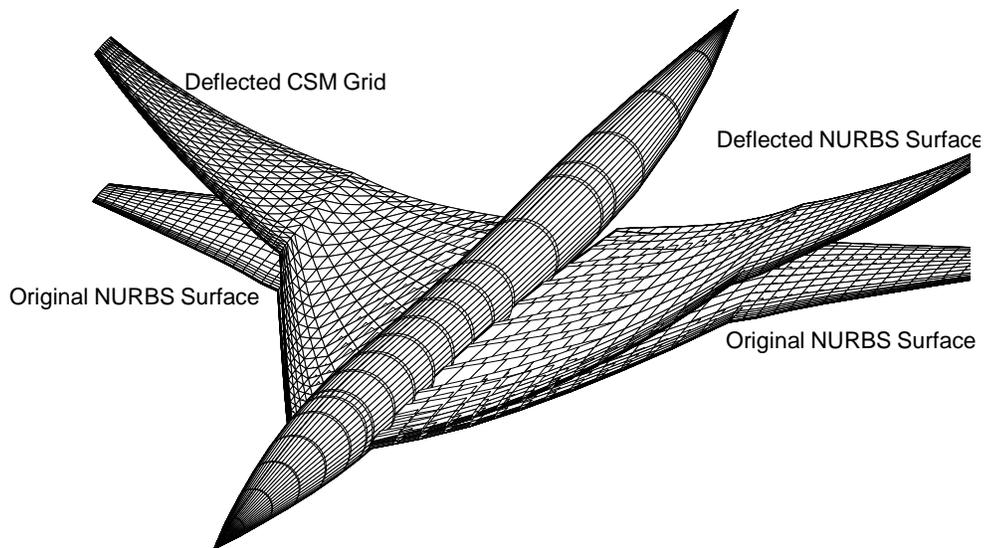


Figure 8: Deflection of a Generic HSCT Configurations

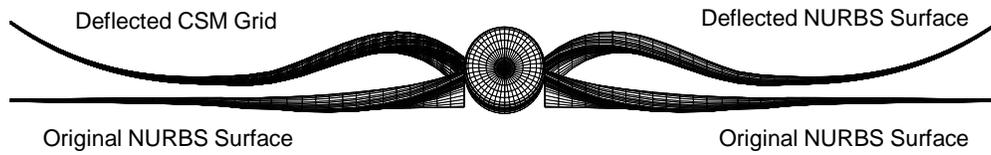


Figure 9: Deflection of a Generic HSCT Configurations Based on a Large Deflection (Front View)

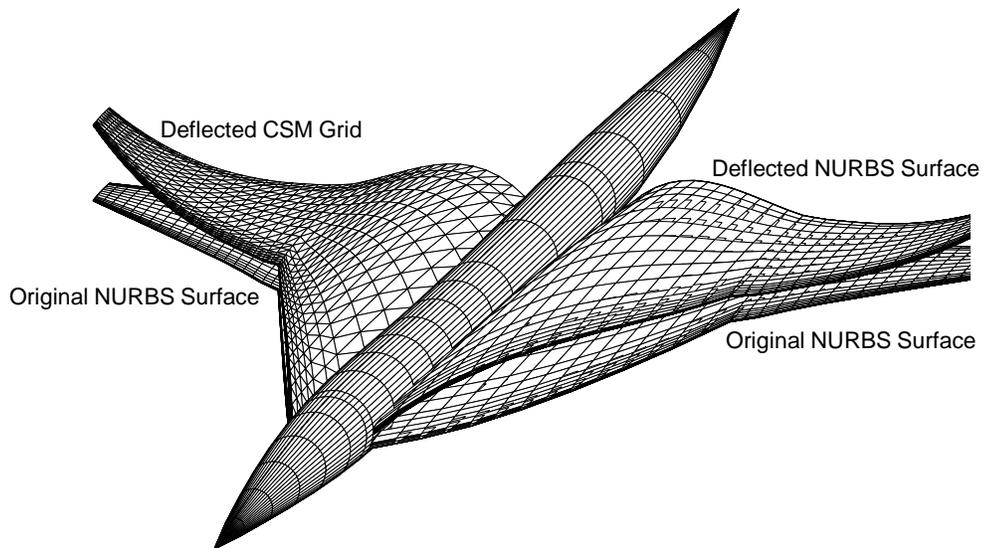


Figure 10: Deflection of a Generic HSCT Configurations Based on a Large Deflection