

# Radiative energy transfer in compressible turbulence

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## Abstract

This *Letter* investigates the compressible energy transfer process. We extend a methodology developed originally for incompressible turbulence and use databases from numerical simulations of a weak compressible turbulence based on Eddy-Damped-Quasi-Normal-Markovian (EDQNM) closure. In order to analyze the compressible mode directly, the well known Helmholtz decomposition is used. While the compressible component has very little influence on the solenoidal part, we found that almost all of compressible turbulence energy is received from its solenoidal counterpart. We focus on the most fundamental building block of the energy transfer process, the triadic interactions. This analysis leads us to conclude that, at low turbulent Mach number, the compressible energy transfer process is dominated by a local radiative transfer (absorption) in both inertial and energy containing ranges.

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# 1 Introduction

Compressible turbulence research, especially using direct numerical simulations<sup>1-5</sup>, was invigorated recently by its engineering applications such as the high speed civil transport and supersonic combustions ram-jet engines. It is well known that direct numerical simulation is restricted to very low Reynolds numbers. The Reynolds average and large-eddy simulation are two alternative methods important for high Reynolds number scientific and engineering calculations. The accuracy of these computations relies on their respective turbulence models. Model development, in turn, requires a fundamental understanding of the energy transfer process. This *Letter* reports a first in-depth investigation of the energy transfer process in homogeneous isotropic compressible turbulence. We expect that the results of this work will find use in future model developments for Reynolds averaged models and large-eddy simulations.

# 2 Analysis

To analyze the compressibility effects, we use the Helmholtz decomposition to split the velocity vector into a solenoidal part  $u^S(\mathbf{K}, t)$ , which corresponds to the velocity fluctuations perpendicular to the wave vector  $\mathbf{K}$  in the Fourier space, and a compressible part  $u^C(\mathbf{K}, t)$ , which corresponds to fluctuations in the direction of the wave vector. We assume that the fluid is barotropic and that compressibility of the turbulence is weak. Moyal<sup>6</sup> pointed out that in the absence of mean velocity gradient, the interactions between these two components are exclusively due to the nonlinear terms and are very important. To obtain high Reynolds number flow fields, we shall use the energy transfer equations for weak compressible turbulence based on the EDQNM closure<sup>7-9</sup>, a two-point statistical theory. A forcing is applied in the large scales of the solenoidal velocity component. We extend the methodology developed originally for incompressible turbulence<sup>10-16</sup> and carry out our analysis when both solenoidal

and compressible modes have reached their asymptotic states<sup>17</sup>.

In the framework of our hypothesis, the governing evolution equations for a weakly compressible turbulence ( $M_t \ll 1$ ) are as follows:

- (1) The spectrum ( $E^{SS}$ ) of the “solenoidal” velocity correlation:

$$\frac{\partial}{\partial t} E^{SS}(K, t) = -2\nu K^2 E^{SS}(K, t) + T^{SS}(K, t). \quad (1)$$

- (2) the spectrum ( $E^{CC}$ ) of the “compressible” velocity correlation:

$$\frac{\partial}{\partial t} E^{CC}(K, t) = -2\nu' K^2 E^{CC}(K, t) + T^{CC}(K, t) - 2 \frac{K}{\langle \rho \rangle} E^{CP}(K, t). \quad (2)$$

Here  $\langle \rho \rangle$  is the mean density. In the case of a Stokes fluid,  $\nu' = \frac{\lambda+2\mu}{\langle \rho \rangle} = \frac{4}{3} \nu$ , where  $\mu$  and  $\lambda$  are the two dynamic viscosities. The evolution equation for  $E^{CC}$  requires the velocity-pressure correlation term,  $E^{CP}(K, t)$ , which is given by a separate transport equation that depends on the potential energy. This term, along with the viscous terms, transfers the compressible kinetic energy into internal energy. We will not investigate it here since it is not relevant to the nonlinear energy transfer of compressible turbulence.

In this letter, we report only results for a turbulent Mach number of  $10^{-2}$  and a Taylor micro-scale Reynolds number of 140. The turbulent Mach number,  $M_t = \sqrt{q^2}/c_0$ , is a dimensionless number characterizing the effects of compressibility and the Reynolds number is defined as  $R_e = \frac{q^2}{\frac{2}{3}\sqrt{\frac{15}{\nu\epsilon}}}$ , where  $q^2$  is twice the turbulent kinetic energy per unit of mass,  $c_0$  the sound speed and  $\epsilon$  the dissipation. In the next figures, the spectra of double correlations will be given in  $m^3 s^{-2}$  and  $K$  is in  $m^{-1}$ .

Figure 1 demonstrates the existence of an extended solenoidal inertial range spectrum, the same as that of Kolmogorov for incompressible turbulence. In the wave-number range corresponding to the inertial range of the solenoidal mode, the compressible spectrum shows a slope<sup>17</sup> of  $-11/3$ .

In the equations (1) and (2),  $T^{SS}(k)$  and  $T^{CC}(k)$  are the nonlinear energy transfer

terms and they are composed of different contributions (details given in Appendix):

$$T^{SS}(k) = \sum_{i=1}^5 T_i^{SS}(k), \quad (3)$$

$$T^{CC}(k) = \sum_{i=1}^6 T_i^{CC}(k). \quad (4)$$

It is interesting to note that  $T_S^{SS}(k) = T_1^{SS}(k) + T_3^{SS}(k)$  is the contribution from the purely solenoidal velocity component and is the same as the incompressible turbulence transfer function. In fact,  $T^{SS}$  is dominated by  $T_S^{SS}$  for turbulent flows at low Mach numbers and it is the same as that found in incompressible studies<sup>10-16</sup> (not shown). Other terms in  $T^{SS}(k)$  are the energy transfer resulting from the interactions between the solenoidal and compressible modes. Therefore, the remaining terms in  $T^{SS}$ , denoted as  $T_C^{SS}$  hereafter, are negligible when one examines the solenoidal velocity transfer<sup>8,18</sup>.

We now focus on the energy transfer process in  $T^{CC}(k)$ , which is a pure compressible transfer term. We stress that  $T_C^{SS}$  term plays important and fundamental roles in exchanging energy between the solenoidal and compressible modes when our focus is the compressible energy transfer. Indeed, the compressible component energy,  $T^{CC}(K)$ , has the same magnitude, but opposite sign from that of  $T_C^{SS}(K)$ . Note that the compressible transfer function is positive for all resolved wavenumbers (Figure 2). This, in turn, indicates that all of the compressible energy is transferred from the solenoidal mode. To distinguish it from the well known ‘‘cascade’’ picture and in analogy with the radiation transfer, we shall call the positive peak absorption while the negative peak is called emission.

The most fundamental building block of energy transfer process is the triadic interactions. Specifically, we are interested in the energy transfer for a given mode  $\mathbf{K}$  due to its interactions with all the pairs of modes  $\mathbf{P}$  and  $\mathbf{Q} = \mathbf{K} - \mathbf{P}$  that form a triangle with  $\mathbf{K}$ . For this reason, we introduce the triadic energy transfer function,  $T(K, P, Q)$ , according to

$$T^{SS}(K) = \sum_{P,Q=|\mathbf{K}-\mathbf{P}|} T^{SS}(K, P, Q). \quad (5)$$

$$T^{CC}(K) = \sum_{P,Q=|\mathbf{K}-\mathbf{P}|} T^{CC}(K, P, Q). \quad (6)$$

Here  $T(K, P, Q)$  is defined as energy transfer to  $K$  due to triads with one leg in  $Q$  and the other in  $P$ . An examination of the purely incompressible contributions ( $T_S^{SS}(K, P, Q)$ ) reproduces the results of incompressible turbulence<sup>10-16</sup> and indicates again that the solenoidal triadic energy transfer is not affected by compressible effects. The total triadic solenoidal transfer  $T^{SS}(K, P, Q)$  (with the compressible terms  $T_C^{SS}(K, P, Q)$  included) is essentially the same<sup>19</sup>. As a result, the compressibility has very little influence on the solenoidal triadic interactions.

We now turn our attention to the triadic interactions in compressible energy transfer,  $T^{CC}(K, P, Q)$ . In Figure 3, we present  $T^{CC}(K, P, Q)$  for various  $Q$  values when  $P$  is in the inertial range ( $P = 512$ ). We first observe that the structures of  $T^{CC}(K, P, Q)$  are rather similar for different  $Q$  values. All of them show the *radiative* (absorption) type of energy transfer. Figure 4 illustrates the same kind of behavior for  $P$  in the energy containing range ( $P = 128$ ). Recall that there is only a direct energy *cascade* in incompressible energy transfer.

We now examine the behavior of the function  $T^{CC}(K, P)$  which is given by

$$T^{CC}(K, P) = \sum_Q T^{CC}(K, P, Q).$$

Here  $T^{CC}(K, P)$  is defined as the transfer of compressible energy to wavenumber  $K$  due to triads with at least one leg in a wavenumber  $P$ . Of particular importance, it indicates the direction as well as the locality of the energy transfer. First of all,  $T^{CC}(K, P)$  is dominated by energy absorption when  $P$  is either in the energy containing or in the inertial range. Since most of the energy transfer is concentrated about  $P = K$  (within a decade), we conclude that the compressible energy transfer

is relatively local. We note that there is no further cancelation when summing over all  $Q$  in  $T^{CC}(K, P, Q)$ . Moreover, at this low turbulent Mach number, the compressible energy input is dominated by the radiative energy transfer from the solenoidal modes; there is no significant compressible energy cascade from large scales to the small scales. The dominance of energy absorption clearly establishes the different physical mechanism of compressible energy transfer compared to its incompressible counterpart.

### 3 Conclusions

This *letter* focuses on the influence of the compressibility effects on compressible energy transfer. Another important issues, the detailed analysis of the individual terms in Eqs. (5) and (6) as well as the locality of the interacting scales, are out of scope of present *letter* and the reader is referred to our full paper<sup>19</sup> for details. At low turbulent Mach number, our observation suggests that the compressible energy transfer process is completely different from its incompressible counterpart. The well known “cascade” picture of the incompressible local energy transfer is no longer dominant here. We believe that this result should be reflected in all models for compressible turbulence.

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## APPENDIX

The transfer terms  $T^{SS}$  and  $T^{CC}$  are given by :

$$T^{SS} = T_1^{SS} + T_2^{SS} + T_3^{SS} + T_4^{SS} + T_5^{SS} \quad (7)$$

$$T^{CC} = T_1^{CC} + T_2^{CC} + T_3^{CC} + T_4^{CC} + T_5^{CC} + T_6^{CC} \quad (8)$$

The different contributions to  $T^{SS}$  are :

$$T_1^{SS} = \int_{\Delta} \frac{K^3}{PQ} \frac{1 - xyz - 2y^2z^2}{2} \theta_{KPQ}^{SS-SS-SS} E^{SS}(P, t) E^{SS}(Q, t) dPdQ \quad (9)$$

$$T_2^{SS} = \int_{\Delta} \frac{K^3}{PQ} \frac{(1 - y^2)(x^2 + y^2)}{1 - x^2} \theta_{KPQ}^{SS-SS-CC} E^{SS}(P, t) E^{CC}(Q, t) dPdQ \quad (10)$$

$$T_3^{SS} = - \int_{\Delta} \frac{P^2}{Q} (xy + z^3) \theta_{PQK}^{SS-SS-SS} E^{SS}(K, t) E^{SS}(Q, t) dPdQ \quad (11)$$

$$T_4^{SS} = \int_{\Delta} \frac{P^2}{Q} (2xy) \theta_{PQK}^{SS-CC-SS} E^{SS}(K, t) E^{CC}(Q, t) dPdQ \quad (12)$$

$$T_5^{SS} = - \int_{\Delta} \frac{P^2}{Q} (z(1 - z^2)) \theta_{PQK}^{CC-SS-SS} E^{SS}(K, t) E^{SS}(Q, t) dPdQ \quad (13)$$

The different contributions of  $T^{CC}$  are :

$$T_1^{CC}(K, t) = \int_{\Delta} \frac{K^3}{PQ} ((x + yz)^2) \theta_{KPQ}^{CC-SS-SS} E^{SS}(P, t) E^{SS}(Q, t) dPdQ \quad (14)$$

$$T_2^{CC}(K, t) = \int_{\Delta} \frac{K^3}{PQ} \frac{(x^2 - y^2)^2}{(1 - x^2)} \theta_{KPQ}^{CC-SS-CC} E^{SS}(P, t) E^{CC}(Q, t) dPdQ \quad (15)$$

$$T_3^{CC}(K, t) = \int_{\Delta} \frac{K^3}{PQ} (x^2) \theta_{K PQ}^{CC-CC-CC} E^{CC}(P, t) E^{CC}(Q, t) dP dQ \quad (16)$$

$$T_4^{CC}(K, t) = - \int_{\Delta} \frac{P^2}{Q} 2z(1-z^2) \theta_{P Q K}^{SS-SS-CC} E^{CC}(K, t) E^{SS}(Q, t) dP dQ \quad (17)$$

$$T_5^{CC}(K, t) = - \int_{\Delta} \frac{P^2}{Q} (2z^3 - z + xy) \theta_{P Q K}^{CC-SS-CC} E^{SS}(Q, t) E^{CC}(K, t) dP dQ \quad (18)$$

$$T_6^{CC}(K, t) = \int_{\Delta} \frac{P^2}{Q} (2xy) \theta_{P Q K}^{CC-CC-CC} E^{CC}(K, t) E^{CC}(Q, t) dP dQ \quad (19)$$

The integration in the P, Q plane extends over a domain such that  $\mathbf{K}$ ,  $\mathbf{P}$  and  $\mathbf{Q}$  can be the three sides of a triangle.

In the equations, the polynomial expressions in x, y, z are coefficients associated with the geometry of the triad. x, y, and z are the cosines of the angles respectively opposite to  $\mathbf{K}$ ,  $\mathbf{P}$ ,  $\mathbf{Q}$  in the triad  $\mathbf{K}$ ,  $\mathbf{P}$  and  $\mathbf{Q}$ .  $\theta$  are the decorrelation times involved in the model. Their expressions can be found in Bataille [8].

We should remark that the transfer equations are not completely given since we have omitted the  $E^{CP}$  contributions. The complete equations can be found in Bataille [8].