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NON-IDEAL GAS EFFECTS ON SHOCK WAVES IN WEAKLY IONIZED GASES

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Abstract. A shock wave in a weakly ionized gas can be preceded by a charge separation region if the Debye length is larger than the shock width. It has been proposed that electrostatic contributions to pressure in the charge separation region can increase the sound speed ahead of the shock well above the sound speed in a neutral gas at the same temperature and therefore increase the shock propagation speed. This proposal is investigated numerically and theoretically. It is concluded that although the ion gas becomes strongly non-ideal in the charge separation region, there is no appreciable effect on the neutral shock.

Key words. plasma drag reduction, weakly ionized gases

Subject classification. Fluid Mechanics

1. Introduction. Ionized fluid systems are effectively electrically neutral at length scales larger than the *Debye length*, the scale at which charge screening becomes important [1]. In shock waves in weakly ionized gases, the great mass disparity between electrons and ions promotes the formation of a charge separation region near the neutral shock front. The size of this region is of the order of the Debye length; it might modify the shock dynamics if the Debye length is larger than the shock width [2].

Experimental observations [3] suggest that shock propagation in weakly ionized gases is ‘anomalous’ in some respects; for example, shocks appear to travel faster than they would in a completely neutral gas at the same temperature and pressure. It has been proposed [4] that the unscreened electrostatic forces in the charge separation region could cause a significant increase in pressure ahead of the shock, leading to the sound speed increase thought to be responsible for these effects. This pressure increase was at first estimated to be quite large [4]; however, it cannot simply be added to the neutral pressure. Only a weaker conclusion is possible, namely that electrostatic effects cause the ion and electron gases to become strongly non-ideal in the charge separation region.

Any mechanism which increases shock propagation speeds requires an energy source in the medium into which the shock propagates. In this proposal, the energy source is the electrostatic potential energy in the charge separation region. This possibility is attractive, because the existence of the charge separation region requires a large Debye length, hence a relatively low density of charged particles. If the ‘anomalous effects’ are real and are associated with the existence of the charge separation region, then the effects must vanish both in the limit of very high charged particle density, corresponding to a small Debye length, and in the limit of very low density, corresponding to a large Debye length but no charged particles, hence to an effectively purely neutral shock. This would explain the absence of ‘anomalous effects’ on re-entry vehicles and suggests that an optimal density might exist at which the anomalous effects are most pronounced.

This possibility is investigated numerically and theoretically. Increased ion pressure alone is found to have no effect on the neutral shock, but the shock can be accelerated by an increased neutral pressure, even if the increase only occurs in the charge separation region. However, because the electrostatic force

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responsible for the increased ion pressure does not act on the neutral particles, this pressure increase cannot be transferred to the neutrals. Consequently, the increased ion pressure in the charge separation region ahead of the shock front in a weakly ionized gas will not significantly enhance the neutral shock propagation speed.

2. Governing Equations for a Shock in a Weakly Ionized Gas. Following previous work on shock waves in weakly ionized gases [2], [5], [6], the weakly ionized gas will be treated as a three-component system consisting of neutrals, ions, and electrons. The continuity equations are

$$(2.1) \quad \frac{\partial n}{\partial t} + \frac{\partial(nu)}{\partial x} = 0$$

$$(2.2) \quad \frac{\partial n_i}{\partial t} + \frac{\partial(n_i u_i)}{\partial x} = R$$

$$(2.3) \quad \frac{\partial n_e}{\partial t} + \frac{\partial(n_e u_e)}{\partial x} = R$$

where R denotes ionization and recombination. The momentum equations are

$$(2.4) \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{mn} \frac{\partial p}{\partial x} + \nu_{in}(u_i - u) \frac{n_i}{n} + \nu_{en}(u_e - u) \frac{m_e n_e}{mn} + V^u$$

$$(2.5) \quad \frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{1}{mn_i} \frac{\partial p_i}{\partial x} + \frac{e}{m} E - \nu_{in}(u_i - u) \frac{n_i}{n} + V_i^u$$

$$(2.6) \quad \frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = -\frac{1}{m_e n_e} \frac{\partial p_e}{\partial x} - \frac{e}{m_e} E - \nu_{en}(u_e - u) \frac{n_e}{n} + V_e^u$$

and the energy equations are

$$(2.7) \quad \frac{3}{2} \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right) = -T \frac{\partial u}{\partial x} + \nu_{in}(T_i - T) + V^T$$

$$(2.8) \quad \frac{3}{2} \left(\frac{\partial T_i}{\partial t} + u_i \frac{\partial T_i}{\partial x} \right) = -T_i \frac{\partial u_i}{\partial x} + \nu_{in}(u_i - u)^2 - \nu'_{in}(T_i - T) + V_i^T$$

$$(2.9) \quad \frac{3}{2} \left(\frac{\partial T_e}{\partial t} + u_e \frac{\partial T_e}{\partial x} \right) = -T_e \frac{\partial u_e}{\partial x} + \nu_{en}(u_e - u)^2 + V_e^T$$

If each component were an ideal gas, then the equations of state would be

$$(2.10) \quad p = nkT$$

$$(2.11) \quad p_i = n_i k T_i$$

$$(2.12) \quad p_e = n_e k T_e$$

Modified equations of state corresponding to non-ideal gas effects will be considered later. The system of governing equations is completed by the one-dimensional relation

$$(2.13) \quad \frac{\partial E}{\partial x} = \frac{e}{\epsilon_0} (n_i - n_e)$$

between the electric field and the density of charged particles; this equation is equivalent to Poisson's equation for the electrostatic potential.

In Eqs. (2.1)–(2.13), variables without a subscript pertain to the neutral particles, the subscripts i and e denote ions and electrons respectively, n is the number of particles, u the velocity, p the pressure, T the temperature, E the electric field, and m the mass of a particle, so that the product mn is the mass density. The masses of ions and neutrals have been assumed equal. Viscous and other additional terms are included in the final V terms in Eqs. (2.4)–(2.9); all of these terms are given explicitly in [6].

The experimental conditions under which anomalous effects have been reported are conditions of thermal non-equilibrium with $T_e \gg T_i = T$ ahead of the shock. Maintaining this condition in the presence of collisional coupling requires a source term in the electron energy equation [7]. Behind the shock, the system relaxes to a state of thermal equilibrium with $T_e = T_i = T$. For the analogous problem of relaxation behind shocks in vibrationally excited media, see [8]. The possible role of thermal nonequilibrium ahead of the shock on the decay of anomalous properties has been considered earlier by us [9].

The most important property of these equations is that the coupling between neutrals and charged species is through the collisional terms ν_{in} and ν_{en} only. Numerical solution of these equations [5] do not reveal any anomalous effects whatsoever because of the weakness of these couplings. The present work will investigate the possibility raised in [4] that by replacing the equations of state Eqs. (2.10)–(2.12) by modified equations of state which reflect the non-ideal gas behavior in the charge separation region, anomalous shock propagation speed can be predicted.

3. Numerical Simulations. The governing equations are integrated using a code developed by Adamovich et al. [6]. This code is based on a general-purpose solver for second-order systems of partial differential equations: pdecol [10] which integrates systems of the form

$$(3.1) \quad \frac{\partial \mathbf{U}}{\partial t} = \mathbf{F}(t, \mathbf{U}, \frac{\partial \mathbf{U}}{\partial x}, \frac{\partial^2 \mathbf{U}}{\partial x^2})$$

Fourth-order spatial spline interpolation is used with continuity of second-order derivatives required at each node. Because the solution method requires two boundary conditions for each equation, artificial viscous terms are added to the continuity equations Eqs. (2.1)–(2.3). The gridpoints are chosen nonuniformly with *tanh* stretching to place the finest spatial resolution inside the shock. The solution domain is the finite normalized region $0 \leq x \leq 1$; boundary conditions at the endpoints are assigned based on a Mach 2 neutral shock with conditions on the charged species set for representative experimental conditions [6]. Ahead of the shock, a state of thermal nonequilibrium $T_e \gg T_i = T_e$ exists, which relaxes to thermal equilibrium with $T_e = T_i = T$ behind the shock.

The shock motion is computed over a predetermined time step, then the spatial coordinates are recalculated by setting the shock center, identified by the position of maximum gradient of the neutral density, to the reference value $x = 0.5$. Consequently, if the shock begins to move too quickly, the calculation becomes inaccurate because the boundary conditions are contaminated by the quantities near the shock.

In order to obtain results rapidly for a large number of test cases, the code was run at low spatial resolution, with 200 space points.

A preliminary code verification was performed by computing a neutral shock, obtained by setting the collisional couplings to zero. It was found that the predicted motion is consistent with the postulated boundary conditions for a Mach 2 shock. A simple test of the inability of the equations of motion Eqs. (2.1)–(2.13) to capture anomalous effects is provided by restoring the collisional couplings. It was found that the effect on the shock path is negligible. Computational results illustrating this point will be discussed below.

4. Simulation Results.

4.1. The charge separation region. The density profiles across the shock are shown in Fig. (4.1) for the charge densities $\alpha = 10^{-5}, 10^{-6}$ and 10^{-8} in the left, center, and right graphs respectively. The value $\alpha = 10^{-6}$ is believed to be typical of experimental conditions.

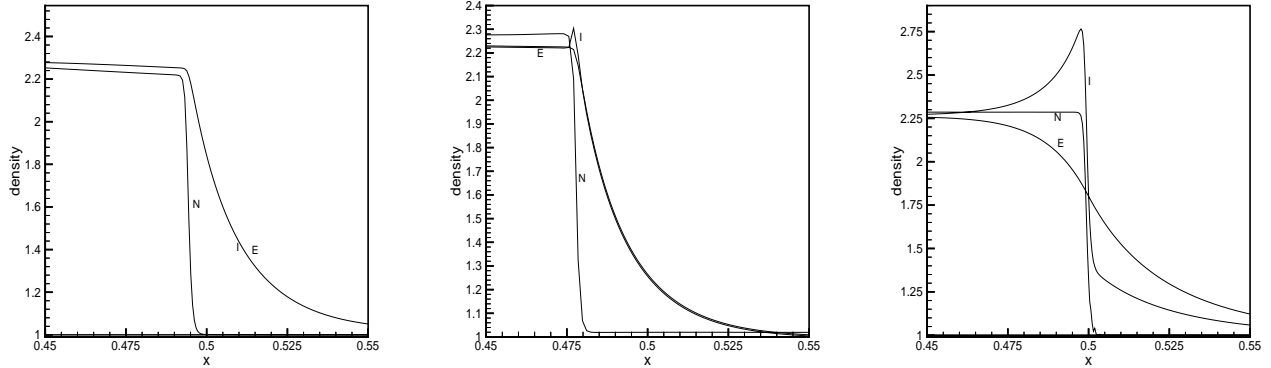


FIG. 4.1. neutral (N), ion (I), and electron (E) densities in shock with $\alpha =$ (left) 10^{-5} ; (center) 10^{-6} ; (right) 10^{-8} . The size of the charge separation region varies as $\alpha^{-1/2}$. The conditions in the center graph are believed typical of experimental conditions.

The calculations show a small but significant charge separation region when $\alpha = 10^{-6}$. In the units used in the calculation, the Debye length $l_D = 0.0025$, which is consistent with the width $\approx 0.005 = 2l_D$ of the charge separation region in Fig. (4.1). The charge separation region disappears completely when $\alpha = 10^{-5}$, but becomes much larger when $\alpha = 10^{-8}$. Again, since $l_d \sim \alpha^{-1/2}$, $l_d = 0.025$ in these units, and the region of unbalanced positive charge has width $\approx 2l_D$.

Note that the densities are all normalized in these figures to have the same dimensionless values ahead of and behind the shock; thus, although the charge separation region is very large when $\alpha = 10^{-8}$, it must be understood that the density of charged particles is extremely small in this case.

4.2. The effect of ion pressure. We next investigate whether an increased ion pressure alone can change the shock path. In the $\alpha = 10^{-6}$ case, multiplying the ion pressure gradient by factors as large as 50 has no effect on the shock path. The explanation appears to lie in the very weak collisional coupling and the low ion density. But increasing the neutral pressure gradient by as little as 25% causes a significant increase in shock propagation speed. These results are illustrated in Fig. (4.2) which compares the shock paths for a neutral shock, the shock with increased ion pressure gradient, and the shock with increased neutral pressure gradient. The plotted paths are the changes about the path of a neutral Mach 2 shock. This figure illustrates the point made above, that coupling to the charged species has no effect on the shock path, even if the ion pressure gradient is greatly enhanced. Thus, shock path modification requires an increase in the neutral pressure.

4.3. The effect of neutral pressure. To test the effect of modifying the neutral equation of state in the charge separation region, the neutral equation of state Eq. (2.10) was replaced by

$$(4.1) \quad p = nkT\{1 + C \max[n_i - n_e, 0]\}$$

where the term proportional to C represents an enhancement of the neutral pressure due to charge imbalance. The specific mechanism for this enhancement will be considered later; the goal of this section is to evaluate the possibility of shock propagation speed enhancement through a modified neutral equation of state. The constant C was given the values $C = 1.001, 1.050, 1.100$.

The effect of the modified neutral equation of state Eq. (4.1) on the shock paths for the values of α used previously are shown in Fig. (4.3) along with the result for $C = 0$. The apparent increased roughness of the

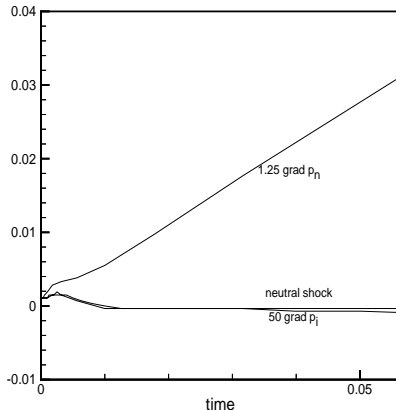


FIG. 4.2. *shock path for neutral shock, shock with enhanced ion pressure gradient, and shock with enhanced neutral pressure gradient.*

shock paths in Fig. (4.3) for $C = 0$ is an effect of the y -axis scale, which is much finer in this graph than the others because the shock is almost unperturbed by coupling to the charged particles alone.

In these examples, the pressure gradient is only enhanced in the charge separation region. The break in the curves when the shock propagation speed increase is large is explained by a feature of the numerical method noted earlier: if the shock moves too quickly, the boundary conditions become contaminated by shock quantities, and the numerical solution is unable to converge.

The figures exhibit no increase of shock propagation speed when $\alpha = 10^{-5}$, consistent with the absence of a charge separation region in these cases. In fact, a small decrease in shock propagation speed is consistently found, but this decrease is too small to be significant. The figures all show that even if the neutral pressure enhancement is localized in a small region, the shock propagation speed can be increased. Since the pressure increase has been chosen independently of the density α , the increase in energy when $\alpha = 10^{-8}$ is very large. A more useful way to present the results is to plot the shock speed increase as a function of some measure of the energy addition per particle. One possible measure is

$$(4.2) \quad c = \frac{C}{l_c \alpha} \int dx \max\{n_i(x) - n_e(x), 0\}$$

where the length l_c , defined by

$$(4.3) \quad l_c = \int_{n_i(x) \geq n_e(x)} dx$$

is the width of the charge separation region. As noted above, $l_c \sim l_D$. The results are shown in Fig. (4.4) where the ‘added pressure factor’ is the quantity c defined in Eq. (4.2).

5. Discussion. Given that the shock propagation speed can be increased if the neutral pressure can be increased, even if only in the charge separation region, it is necessary to ask how the neutral equation of state could be modified by non-ideal gas effects on the ions; that is, how can the electrostatic potential energy in the charge separation region be transferred to the neutral particles?

Consider the model system of an Enskog non-ideal gas collisionally coupled to an ideal gas. The equation of state for this system has been computed by Thorne ([11] Sect. 16.6). The pressure in the ideal gas is

$$(5.1) \quad p = knT \left\{ 1 + \frac{2}{3} \pi n' \sigma^3 \chi \right\}$$

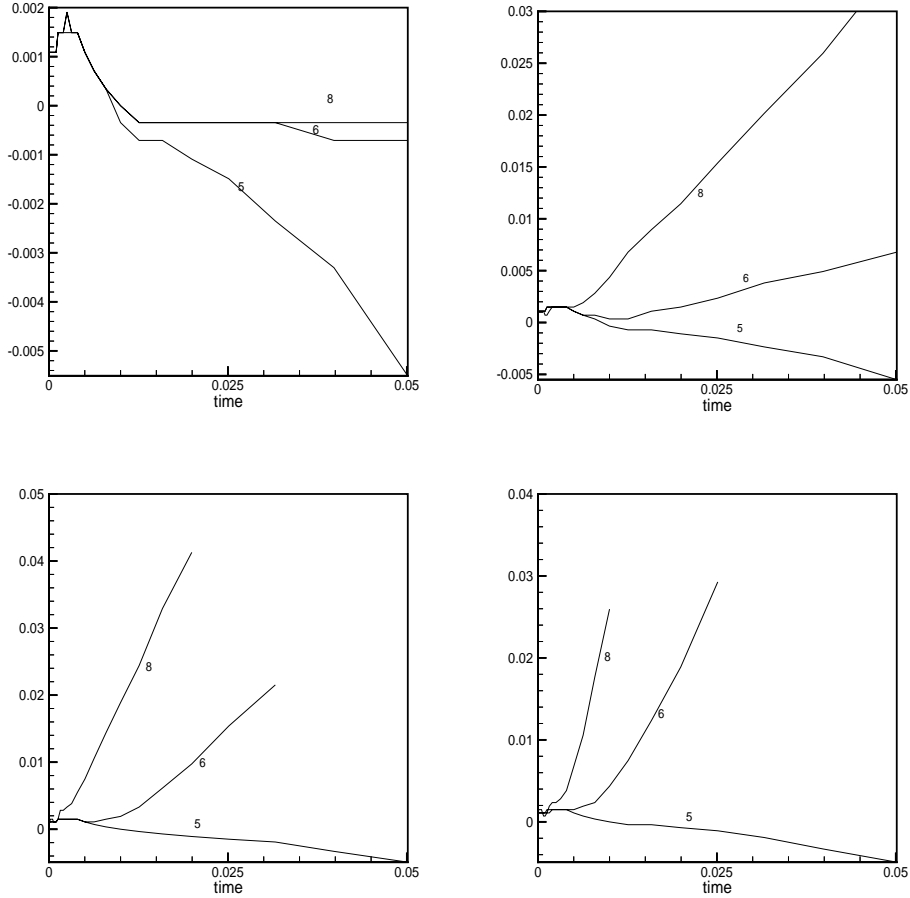


FIG. 4.3. *shock paths for $\alpha = 10^{-5}$ [5], 10^{-6} [6], and 10^{-8} [8] with (upper left) $C = 0$; (upper right) $C = 1.001$; (lower left) $C = 1.050$; (lower right) $C = 1.100$. The shock is accelerated as the strength of the neutral pressure gradient enhancement C increases, although the pressure enhancement is confined to the charge separation region alone.*

where n' is the number density of Enskog particles and σ is their radius, so that $n'\sigma^3$ is the volume fraction occupied by the Enskog particles. The factor χ is given by

$$(5.2) \quad \chi = 1 + \frac{5\pi}{12}n'\sigma^3 + \dots$$

Eq. (5.1) shows that the pressure in the ideal gas is enhanced by the presence of the non-ideal gas. Since long-range forces like the Coulomb force correspond to large σ , the effect of the coupling could be large, even if the Enskog gas is relatively rarified.

Although this result is suggestive, its applicability to weakly ionized gases is doubtful because the electrostatic force does not act on the neutral particles. To make the model system apply to a weakly ionized gas, the radius σ should take a nonzero value only for ion-ion or electron-electron collisions, but in ion-neutral collisions, σ must be set to zero. In this case, the increased pressure predicted in Eq. (5.1) disappears.

Higher order interactions, like ion-ion-neutral collisions will certainly be modified by the presence of the electrostatic forces, but these will not significantly change the neutral equation of state. To conclude, although there is considerable electrostatic energy stored in the charge separation region, there is no obvious

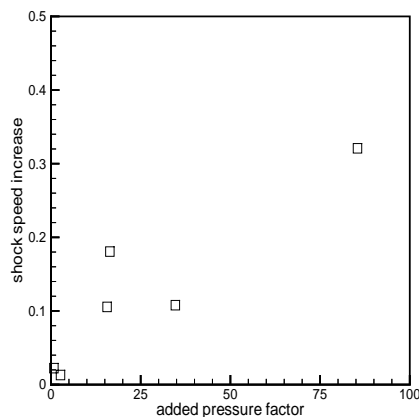


FIG. 4.4. *shock propagation speed increase as a function of energy added per particle*

collisional mechanism which will transfer this energy to the neutral particles.

6. Conclusion. The increased ion pressure due to non-ideal gas effects in the charge separation region ahead of the shock front in a weakly ionized gas will not significantly enhance the neutral shock propagation speed. Other non-thermal mechanisms which might lead to shock propagation speed enhancement include production of molecules in excited states [12] and the polarizability of the neutral species [2], [13].

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