NASA/CR-2002-211637
ICASE Report No. 2002-12

Air Traffic Conflict Resolution and Recovery

Alfons Geser and César Muñoz
ICASE, Hampton, Virginia

Gilles Dowek
INRIA, Le Chesnay Cedex, France

Florent Kirchner
ENAC, Toulouse, France

May 2002

The NASA STI Program Office . . . in Profile

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the lead center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

- TECHNICAL PUBLICATION. Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA's counterpart of peer-reviewed formal professional papers, but having less stringent limitations on manuscript length and extent of graphic presentations.
- TECHNICAL MEMORANDUM. Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.
- CONTRACTOR REPORT. Scientific and technical findings by NASA-sponsored contractors and grantees.
- CONFERENCE PUBLICATIONS. Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or cosponsored by NASA.
- SPECIAL PUBLICATION. Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.
- TECHNICAL TRANSLATION. English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services that complement the STI Program Office's diverse offerings include creating custom thesauri, building customized data bases, organizing and publishing research results . . . even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI Program Home Page at ***http://www.sti.nasa.gov***
- Email your question via the Internet to help@sti.nasa.gov
- Fax your question to the NASA STI Help Desk at (301) 621-0134
- Telephone the NASA STI Help Desk at (301) 621-0390
- Write to:
  NASA STI Help Desk
  NASA Center for AeroSpace Information
  7121 Standard Drive
  Hanover, MD 21076-1320

NASA/CR-2002-211637
ICASE Report No. 2002-12

Air Traffic Conflict Resolution and Recovery

Alfons Geser and César Muñoz
ICASE, Hampton, Virginia

Gilles Dowek
INRIA, Le Chesnay Cedex, France

Florent Kirchner
ENAC, Toulouse, France

ICASE
NASA Langley Research Center
Hampton, Virginia

Operated by Universities Space Research Association

Prepared for Langley Research Center
under Contract NAS1-97046

May 2002

Available from the following:

NASA Center for AeroSpace Information (CASI)
7121 Standard Drive
Hanover, MD 21076-1320
(301) 621-0390

National Technical Information Service (NTIS)
5285 Port Royal Road
Springfield, VA 22161-2171
(703) 487-4650

AIR TRAFFIC CONFLICT RESOLUTION AND RECOVERY*

ALFONS GESER†, CÉSAR MUÑOZ‡, GILLES DOWEK§, AND FLORENT KIRCHNER¶

Abstract. An essential element in the Free Flight concept is the detection and avoidance of air traffic conflicts. A conflict occurs when the required separation between two aircraft, namely the ownship and the intruder, is lost. Conflict detection and resolution systems predict loss of separation and output conflict avoidance maneuvers that divert the ownship from its original trajectory. In this paper, we address the problem of redirecting the ownship to its original path, in a geometric optimal way, without introducing new conflicts. We call this concept *Resolution and Recovery*. Given the current 3-dimensional position and velocity vectors of two aircraft in predicted conflict and the ownship's required time of arrival at the target point, the resolution and recovery algorithm outputs a choice of *maneuvers*. Each maneuver comprises an *escape course* and a *recovery course* to be followed by the ownship. The escape course brings the ownship off the predicted conflict and the recovery course returns it to the original target point. We provide a rigorous mathematical description of the problem and show that the algorithm is *correct*, i.e., no matter which of the proposed maneuvers the ownship picks, it will arrive at the target point at the scheduled time while maintaining the minimum required separation to the intruder at all times.

Key words. conflict detection and resolution, 3-dimensional airspace, traffic avoidance, free flight

Subject classification. Computer Science

1. Introduction. Conflict Detection and Resolution (CD&R) systems are designed to warn air traffic controllers and/or pilots about an imminent loss of separation between aircraft, and to assist them in a corrective maneuver. Algorithms for CD&R have been largely studied over the last decade (for a survey on CD&R methods see [11]) and new algorithms are proposed every day.

With the emerging of more reliable surveillance and communication technologies, air borne CD&R capability is becoming a fundamental feature of new concepts for air traffic management such as Free Flight [14] and DAG-TM (Distributed Air/Ground Traffic concept) [1]. These concepts address the expected increase in air traffic density in the next decades, by distributing among the different actors of the airspace system the responsibility for keeping minimum traffic separation. In contrast to ground-based systems, on board systems have a limited access to computational resources and, by their nature, are distributed. To target the complexity of a free-flight environment, new approaches for CD&R have been proposed based on non-standard programming techniques such as genetic algorithms [6, 12, 9], neural networks [5], game theory [15], graph theory [3], or semidefinite programming [8]. These approaches deal with issues such as multiple aircraft conflicts and uncertainties in the prediction of aircraft trajectories. Given the computational complexity of some of these approaches, they may require time and space discretizations.

A more classical approach to CD&R is the so-called *geometric* approach [7, 10, 2, 4]. In this approach, aircraft trajectory predictions are based on linear projections of current aircraft states. This approach exploits the facts that linear projections can be computed efficiently and that prediction errors are negligible

*This work was supported by the National Aeronautics and Space Administration under NASA Contract No. NAS1-97046 while the authors were in residence at ICASE, NASA Langley Research Center, Hampton, VA 23681-2199, USA.

†ICASE, Mail Stop 132C, NASA Langley Research Center, Hampton, VA 23681-2199, USA, e-mail: geser@icase.edu.

‡ICASE, Mail Stop 132C, NASA Langley Research Center, Hampton, VA 23681-2199, USA, e-mail: munoz@icase.edu.

§INRIA, Domaine de Voluceau - Rocquencourt - B.P. 105, 78153 Le Chesnay Cedex, France, e-mail: Gilles.Dowek@inria.fr.

¶ENAC, 7 Avenue Edouard Belin, 31000 Toulouse, France, e-mail: fkirchne@mis.enac.fr.

during short look-ahead times. For this reason, this approach is also referred as *tactical*. For large look-ahead times a more *strategic* approach that looks at the *(pilot) intent information*, i.e., the flight plan, is in order. While tactical approaches have well-understood geometric descriptions that allow for efficient and clear algorithms, they fall short on pilots' expectations in some field studies [16]. Strategic approaches seem to be more appreciated by pilots, but their theory is far less understood.

In a previous work [4], we have proposed a geometric optimization algorithm for CD&R in a 3-D airspace. In this paper, we address the *recovery* problem, that is, redirecting the ownship to the original path while maintaining the minimum required separation at all times. The new proposed algorithm is on the border between tactical and strategic algorithms. Its inputs are the three-dimensional position and velocity vectors of two aircraft, which we call the *ownship* and the *intruder* aircraft, and the time where the ownship is required to arrive at its target point ("Required Time of Arrival", RTA). The RTA defines the position of the target point by the ownship's constant movement. RTA is a limited form of intent information: the target point may be the next trajectory change point in the ownship flight plan. Assuming a loss of separation between two aircraft, the algorithm outputs a choice of maneuvers for the ownship. Each maneuver consists of an *escape course* that brings the ownship off the predicted conflict, and a subsequent *recovery course* that leads the ownship back to its original path. If the ownship follows any proposed maneuver then it arrives at RTA at the scheduled position without having experienced a loss of separation at any time. In the mean time, the intruder aircraft is assumed to continue its current trajectory, that is, ownship resolution and recovery maneuvers take place assuming no cooperation of the intruder aircraft.

The various maneuvers that our algorithm proposes differ in the constraints they satisfy. For example, one constraint requires that during the maneuver, the ownship may only change its ground speed, but not its heading or its vertical speed. Another constraint requires that only the vertical speed of the ownship may change. Imposing such constraints restricts the number of choices to finitely many, simplifies the calculations performed by the algorithm, is simple to conceive and to perform by the crew, and enhances passenger comfort. The maneuvers that our algorithm outputs may be rendered at a display for the air traffic controller or the pilot who may select among the proposed solutions. Our algorithm is also suitable for use underneath a trajectory planner which may perform the selection. However, we do not address the question of physical implementability of the maneuvers proposed by the algorithm. In particular, the algorithm does not check for minimum/maximum altitude/airspeed. Nor does it implement cost-based analysis such as fuel consumption. It depends on the capabilities of the ownship whether a proposed maneuver can be implemented or not. Since aircraft performance data are not available to the algorithm, we assume that these kinds of analysis are implemented in an external module.

The algorithm is computationally efficient, suitable for embedding in a flight-deck computer, and appropriate for formal verification. In particular, we have a rigorous analysis, by pencil and paper, of the algorithm. Mechanically checked proofs are currently under development. We strongly believe that given the criticality of CD&R, rigorous techniques and well-understood mathematical models are required to guarantee the overall safety of the new, and more autonomous, air traffic systems.

Notation. Variables denoting positions are named "s", velocities are named "v", and times are named "t"; ownship variables are subscripted by "o"; intruder variables are subscripted by "i"; relative and generic variables have neither subscript. Coordinate names (x, y, or z) are appended as a subscript to the name. Variables assigned to the escape course receive a prime; variables assigned to the recovery course receive a double prime. For instance v'_{ox} denotes the x coordinate of the ownship escape speed, v_y denotes the y coordinate of the original relative speed, and t' denotes the time the escape course ends.

2. Definition of the Problem. For conflict detection purposes, aircraft are assumed to be surrounded by an *avoidance region*, which is typically a cylinder of diameter 5 nautical miles and height 1000 feet. Two aircraft are said to be in conflict when their avoidance regions overlap. In this paper, we take an alternative, but equivalent view, where aircraft are surrounded by *protected zones* twice as big as the individual avoidance regions. In this view, a conflict is the incursion of one aircraft in the protected zone of another one.

We assume the airspace given as a three-dimensional Cartesian coordinate system, where the z-axis points upward in the vertical direction. We consider two aircraft, namely ownship and intruder. The ownship's initial position (i.e., its position at time $t = 0$) is given by the vector $\vec{s_o} = (s_{ox}, s_{oy}, s_{oz})$. The ownship's original velocity vector is given by $\vec{v_o} = (v_{ox}, v_{oy}, v_{oz})$. Likewise the intruder's initial position, $\vec{s_i}$, and the intruder's velocity vector, $\vec{v_i}$, are given. It is convenient to consider the ownship's motion relative to the intruder. For this purpose, we introduce a relative coordinate system where the intruder's position is at the origin, and we consider the relative position vector $\vec{s} = (s_x, s_y, s_z) = \vec{s_o} - \vec{s_i}$ and the relative velocity vector $\vec{v} = (v_x, v_y, v_z) = \vec{v_o} - \vec{v_i}$. In this coordinate system, the protected zone is a cylinder P around the intruder defined as follows:

$$P = \{(x, y, z) \mid x^2 + y^2 < D^2 \quad \text{and} \quad |z| < H\}.$$

In the usual view, D and H denote the diameter and height of the avoidance region, respectively.

The aircraft are said to be *in conflict at time* t when $\vec{s} + t\vec{v} \in P$. They are in *predicted conflict* if they are in conflict at some time $0 < t$.

Given a velocity vector $\vec{v} = (v_x, v_y, v_z)$, we define the following concepts.

- *Ground speed:* Length of the horizontal projection of $\vec{v}$, i.e., $\sqrt{v_x^2 + v_y^2}$.
- *Vertical speed:* Vertical component of $\vec{v}$, i.e., v_z.
- *Heading:* Direction of the horizontal projection of $\vec{v}$, i.e., angle α such that $v_x = v\cos(\alpha)$ and $v_y = v\sin(\alpha)$, where v is the ground speed of $\vec{v}$. We avoid explicit references to α in our analytical description of the escape-recovery maneuvers.

The task of the resolution and recovery algorithm (RR3D) is defined as follows:

Inputs:

- Initial relative ownship's position $\vec{s}$.
- Absolute velocity vectors $\vec{v_o}, \vec{v_i}$ of ownship and intruder aircraft, respectively. The relative velocity vector is given by $\vec{v} = \vec{v_o} - \vec{v_i}$.
- Required Time of Arrival (RTA) or target time $t'' > 0$, which determines the target point

$$\vec{s''} = \vec{s} + t''\vec{v}. \tag{2.1}$$

Assumptions:

- *Courses*, i.e., trajectories between way-points, are line segments. Hence, courses are described by a position, a velocity vector, and a time interval. Moreover, we assume that changes of course or speed are implemented in zero time by an aircraft.
- Absolute ground speeds are not zero, i.e., $v_{ox}^2 + v_{oy}^2 \neq 0$ and $v_{ix}^2 + v_{iy}^2 \neq 0$.
- Neither at initial time nor at target time are the aircraft in conflict, i.e., $\vec{s}, \vec{s''} \notin P$.
- Neither at initial time nor at target time is the ownship at the boundary of the intruder's protected zone, i.e., $\vec{s}, \vec{s''} \in \{(x, y, z) \mid x^2 + y^2 \neq D^2 \quad \text{or} \quad |z| > H\}$.
- The aircraft are in predicted conflict before t'', i.e., $\vec{s} + t\vec{v} \in P$ for some time $0 < t < t''$.

Outputs: A list of *maneuvers* each one a triple, $(t', \vec{v_o'}, \vec{v_o''})$, composed of

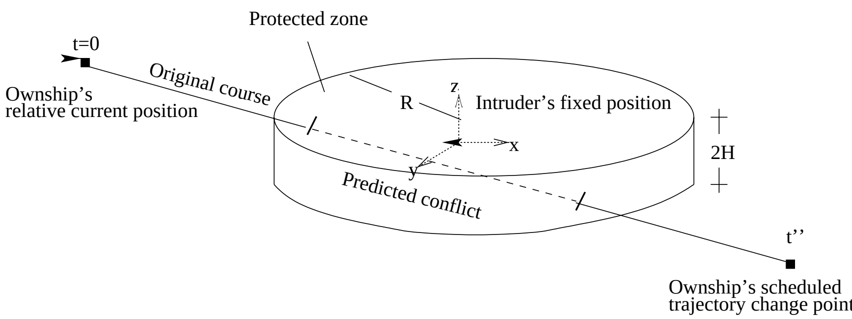

FIG. 2.1. *Predicted conflict scenario*

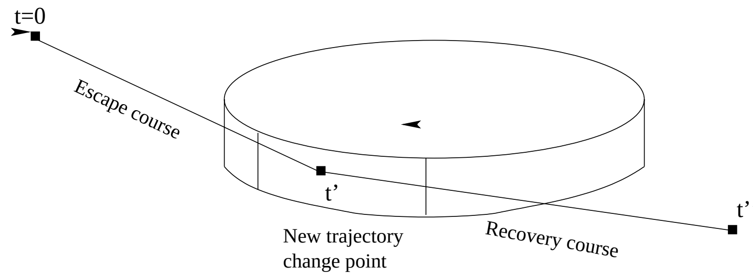

FIG. 2.2. *Escape-recovery maneuver*

- A *time of switch* t' such that $0 < t' < t''$.
- An *escape velocity vector* $\vec{v'_o}$ that determines a conflict-free *escape course* for the ownship, i.e., let $\vec{v'} = \vec{v'_o} - \vec{v_i}$, then

$$\vec{s} + t\vec{v'} \notin P \quad \text{for all times } 0 \leq t \leq t'. \tag{2.2}$$

- A *recovery velocity vector* $\vec{v''_o}$ that determines a conflict-free and on-time *recovery course* for the ownship, i.e., let $\vec{v''} = \vec{v''_o} - \vec{v_i}$, then

$$\vec{s} + t'\vec{v'} + (t - t')\vec{v''} \notin P \quad \text{for all times } t' \leq t \leq t'', \text{ and} \tag{2.3}$$

$$\vec{s} + t'\vec{v'} + (t'' - t')\vec{v''} = \vec{s''}. \tag{2.4}$$

In other words, if the ownship flies the escape course from time 0 to t', and the recovery course from time t' to t'', then (1) it shall not be in conflict at any time (by (2.2) and (2.3)) and (2) it arrives at s'' at time t'' (by (2.4)). We assume no cooperation from the intruder aircraft, i.e., the intruder does not need to maneuver. The original situation is depicted in Figure 2.1, and the escape and recovery courses are depicted in Figure 2.2. Henceforth, we call the ownship's change of the velocity vector from $\vec{v_o}$ to $\vec{v'_o}$ the *escape step*, and its change from $\vec{v'_o}$ to $\vec{v''_o}$ the *recovery step*.

The ownship's maneuvers shall be constrained in such a way that both $\vec{v'_o}$ and $\vec{v''_o}$ satisfy one of the following conditions:

1. *Change of vertical speed only.* The ownship's vertical speed may change but neither its heading nor its ground speed, i.e., $v'_{ox} = v_{ox} = v''_{ox}$ and $v'_{oy} = v_{oy} = v''_{oy}$.
2. *Change of ground speed only.* The ownship's ground speed may change but neither its heading nor its vertical speed. Formally, there is a $k > 0$ such that $v'_{ox} = kv_{ox}$, $v'_{oy} = kv_{oy}$, and $v'_{oz} = v_{oz}$, and there is a $j > 0$ such that $v''_{ox} = jv_{ox}$, $v''_{oy} = jv_{oy}$, and $v''_{oz} = v_{oz}$.

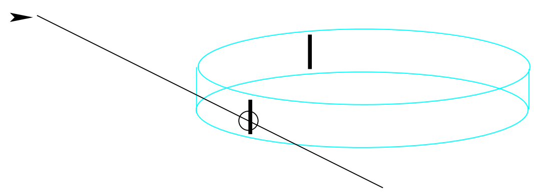

FIG. 3.1. *Escape/Recovery courses: Line case*

3. *Change of heading.* In the two dimensional projection, the escape course and the recovery course (each in absolute coordinates) form a triangle. By the triangle inequality, the escape course and the recovery course together are longer than the original course. To arrive at the target point at time t'', the ownship has to compensate the longer way by a greater average ground speed as opposed to its original ground speed. Hence, maneuvers where only heading changes are allowed cannot reach the target point in time. In this case, we propose a change of heading combined with a change of ground speed at time t'. For the escape step, the ownship's heading may change, but neither its ground speed nor its vertical speed; for the recovery step in addition to a heading change, one must allow for a change of ground speed as well. Formally, $v_{ox}'^2 + v_{oy}'^2 = v_{ox}^2 + v_{oy}^2$, $v_{oz}' = v_{oz}$, and $v_{oz}'' = v_{oz}$.

Furthermore, we require that the escape-recovery maneuvers are tangential to the lateral surface of the protected zone. We conjecture that tangential maneuvers are *optimal*, among all maneuvers that satisfy the same set of constraints, in the following sense. The amount of change in one step is expressed by the velocity change vectors $\Delta\vec{v'} = \vec{v_o'} - \vec{v_o}$ and $\Delta\vec{v''} = \vec{v_o''} - \vec{v_o}'$. The *effort* of a maneuver is the sum of squares of its velocity change vectors, i.e., $|\Delta\vec{v'}|^2 + |\Delta\vec{v''}|^2$. A maneuver is called *optimal* if its effort is less than, or equal to, the effort of any other maneuver.

3. Correctness Criteria. In this section we use $\vec{s}, \vec{v}, t$ in an generic way, i.e., they do not necessarily refer to the relative variables. We define the *infinite cylinder* as the set of points

$$P_\infty = \{(x, y, z) \mid x^2 + y^2 < D^2\},$$

and we define the *infinite slice* as the set of points

$$S_\infty = \{(x, y, z) \mid |z| < H\}.$$

The algorithm only considers two kinds of escape and recovery courses:

- *Line case*: Courses that are tangential to the lateral surface of the protected zone (Figure 3.1). That is, for some point $\vec{s}$, $\vec{s}$ is tangent to P_∞.
- *Circle case*: Courses that are incident to one of the disks of the protected zone without intersecting its interior (Figure 3.2). That is, for some point $\vec{s}$, $s_x^2 + s_y^2 = D^2$, $|s_z| = H$, and the moving point either enters the infinite cylinder upon leaving the infinite slice or leaves the infinite cylinder upon entering the infinite slice.

Note that the case where the lateral surface is touched and the touching point is incident with a circle is counted as a line case, and that the case $v_z = 0, |s_z| = H$ is counted as a circle case.

Below we provide a rigorous proof that line and circle cases are *correct*, i.e., they describe conflict-free courses. Since escape and recovery courses touch P, we state without proof that they are also *optimal* from a geometric point of view. In order to provide the correctness proof, we first take a time reference relative to the time of the tangent point in the line case and relative to the time of the incident point in the circle case, and define vertical and horizontal separation criteria.

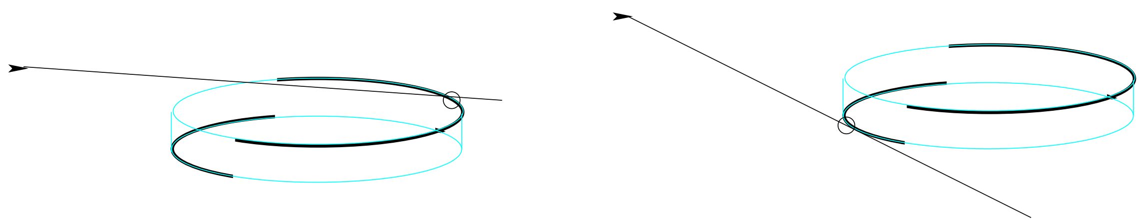

FIG. 3.2. *Escape/Recovery courses: Circle cases*

For the first criterion we consider only the vertical coordinate.

LEMMA 3.1 (Vertical separation). *Let a moving point $\vec{s}+t\vec{v}$ be at time 0 at the surface of S_∞, i.e., $|s_z| = H$. (1) If $s_z v_z \geq 0$ holds then $|s_z + tv_z| \geq H$ for all $t \geq 0$. (2) If $s_z v_z \leq 0$ holds then $|s_z + tv_z| \geq H$ for all $t \leq 0$.*

Proof.

1. We prove that $(s_z + tv_z)^2 \geq H^2$, and so $|s_z + tv_z| \geq H$, for all $t \geq 0$. Since $s_z^2 = H^2$, $(s_z + tv_z)^2 = H^2 + 2ts_z v_z + t^2 v_z^2$. By hypothesis, $s_z v_z \geq 0$. Thus, $(s_z + tv_z)^2 \geq H^2$ for $t \geq 0$.
2. By substitution of $-v_z$ for v_z and $-t$ for t in (1).

□

REMARK 3.2. *If $v_z = 0$ then we have vertical separation for all times t.*

For the second criterion we consider only the horizontal plane. We are going to need a technical definition.

DEFINITION 3.3 (Entry point, exit point). *We call $\vec{s}, s_x^2 + s_y^2 = D^2$ an* entry point *to the infinite cylinder (for $\vec{v}$), if the derivative, with respect to t, of the squared horizontal distance $(s_x + tv_x)^2 + (s_y + tv_y)^2$ at time 0 is non-positive; and an* exit point *from the infinite cylinder if it is non-negative (see Figure 3.3). If the derivative is equal to zero then we have a* tangent point*, which is both an entry point and an exit point. Formally, $\vec{s}$ is an entry point if*

$$s_x v_x + s_y v_y \leq 0, \tag{3.1}$$

an exit point if

$$s_x v_x + s_y v_y \geq 0, \tag{3.2}$$

and a tangent point if

$$s_x v_x + s_y v_y = 0. \tag{3.3}$$

LEMMA 3.4 (Horizontal separation). *Let a moving point $\vec{s}+t\vec{v}$ be at time 0 at the lateral surface of P, i.e., $s_x^2 + s_y^2 = D^2$. (1) If $\vec{s}$ is an exit point then $(s_x + tv_x)^2 + (s_y + tv_y)^2 \geq D^2$ for $t \geq 0$. (2) If $\vec{s}$ is an entry point then $(s_x + tv_x)^2 + (s_y + tv_y)^2 \geq D^2$ for $t \leq 0$.*

Proof.

1. The point $\vec{s}$ is an exit point, i.e., $s_x v_x + s_y v_y \geq 0$. Since $s_x^2 + s_y^2 = D^2$, $(s_x + tv_x)^2 + (s_y + tv_y)^2 = D^2 + 2t(s_x v_x + s_y v_y) + t^2(v_x^2 + v_y^2)$. By hypothesis, $(s_x v_x + s_y v_y) \geq 0$. Thus, $(s_x + tv_x)^2 + (s_y + tv_y)^2 \geq D^2$ for $t \geq 0$.
2. By substitution of $-\vec{v}$ for $\vec{v}$ and $-t$ for t in (1).

□

REMARK 3.5. *If $\vec{s}$ is a tangent point we have horizontal separation for all times t.*

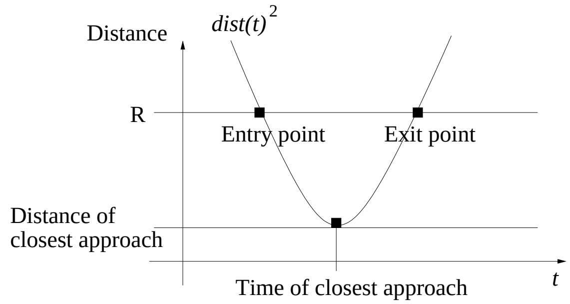

FIG. 3.3. *Entry and exit points to/from an infinite cylinder*

Finally, we provide correctness proofs for the line and circle cases.

THEOREM 3.6 (Line Case Correctness). *Let $\vec{s} + t\vec{v}$ be a moving point such that $\vec{s}$ is tangent to P_∞. Then, $(s_x + tv_x)^2 + (s_y + tv_y)^2 \geq D^2$ for all times t.*

Proof. At time 0 we have a tangent point to the lateral surface of the infinite cylinder. Using the horizontal separation criterion twice, we conclude that horizontal separation holds for all times. ∎

THEOREM 3.7 (Circle Case Correctness). *Let $\vec{s}+t\vec{v}$ be a moving point such that $s_x^2 + s_y^2 = D^2$, $|s_z| = H$, and either (1) $s_x v_x + s_y v_y \leq 0$ and $s_z v_z \geq 0$ or (2) $s_x v_x + s_y v_y \geq 0$ and $s_z v_z \leq 0$. Then, for all times t, either (a) horizontal separation: $(s_x + tv_x)^2 + (s_y + tv_y)^2 \geq D^2$ or (b) vertical separation: $|s_z + tv_z| \geq H$.*

Proof. In case (1) we get vertical separation for all $t \geq 0$ and horizontal separation for all $t \leq 0$. In case (2) we get vertical separation for all $t \leq 0$ and horizontal separation for all $t \geq 0$. ∎

4. Resolution and Recovery Algorithm. In this section we develop the algorithm for conflict resolution and recovery, namely RR3D. We present the algorithm as a set of formulas that describe escape-recovery maneuvers, i.e., triples $(t', \vec{v_o'}, \vec{v_o''})$, where t' is a time of switch, $\vec{v_o'}$ is a velocity vector that determines an escape course, and $\vec{v_o''}$ is a velocity vector that determines a recovery course. The formulas are organized according to the constraints that the escape-recovery maneuvers satisfy. As explained in Section 2, we consider three constraints: change of vertical speed only, change of ground speed only, and change of heading combined with a change of ground speed at time t'. For each of these constraints we distinguish several cases according to the part of the surface of P that is touched during the escape and recovery courses. We identify the following cases: line/line (Figure 4.1), line/circle (Figure 4.2), circle/line (Figure 4.3), one-circle (Figure 4.4), circle/circle (Figure 4.5), escape-circle (Figure 4.6), and recovery-circle (Figure 4.7). As we will see below, not all the cases are possible in all situations.

In all the cases, we use the following approach:

1. We find candidates to maneuvers by solving the equations given by the constraints and the case analysis.
2. We disregard candidates that contradict the specification. This may require *sanity checks* with the implicit meaning that only solutions satisfying them are considered any further.
3. Candidates that survive the last check are the maneuvers returned by the algorithm.

Correctness is guaranteed by construction. Furthermore, we aim for a set of candidates that is a superset of the set of solutions.

In the algorithm, when we solve a quadratic equation with non-null coefficients, we assume that the discriminant is non-negative. If the discriminant happens to be negative, we tacitly infer that there are no

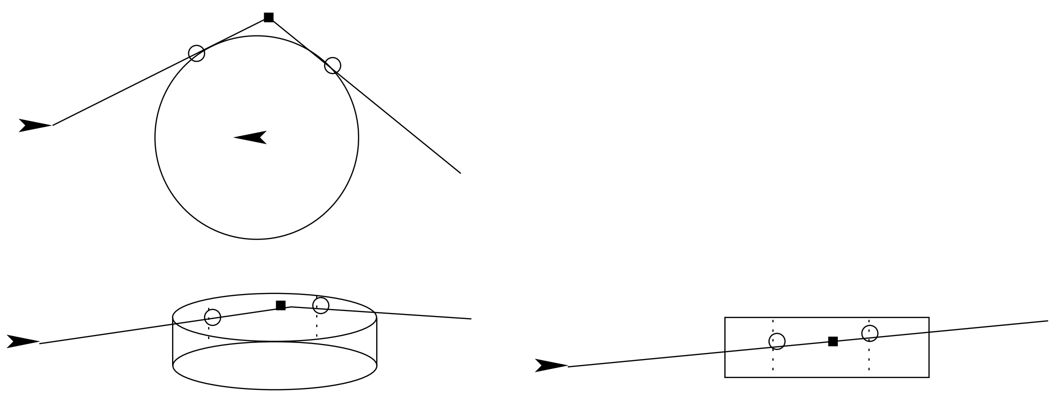

Fig. 4.1. *Line/line (top view, perspective view, and side view)*

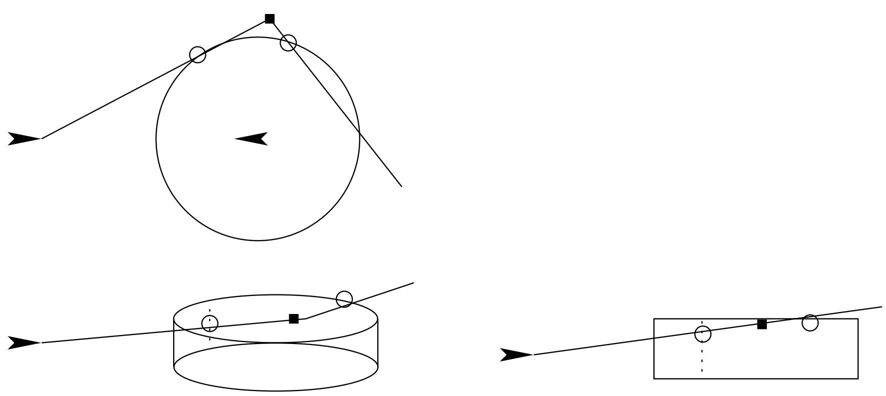

Fig. 4.2. *Line/circle (top view, perspective view, and side view)*

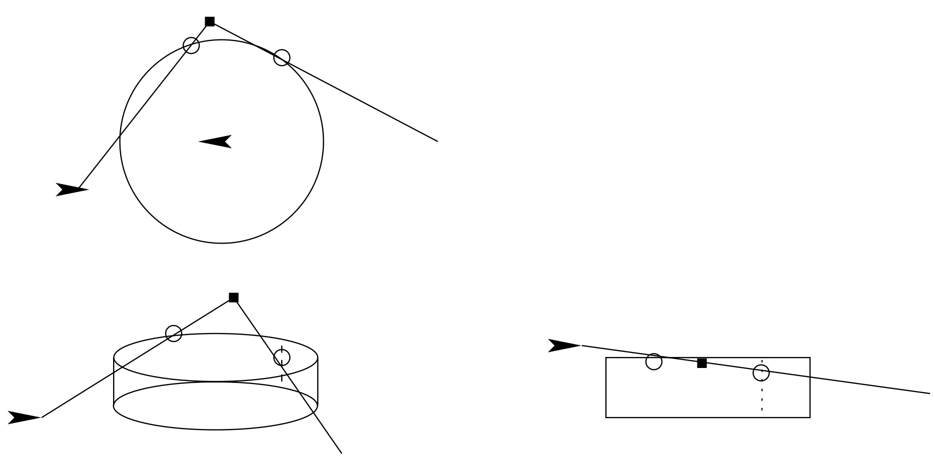

Fig. 4.3. *Circle/line (top view, perspective view, and side view)*

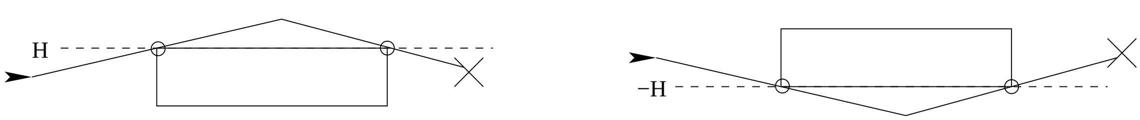

Fig. 4.4. *One-circle cases (side views)*

FIG. 4.5. *Circle-circle cases (side views)*

FIG. 4.6. *Escape-circle cases (side views)*

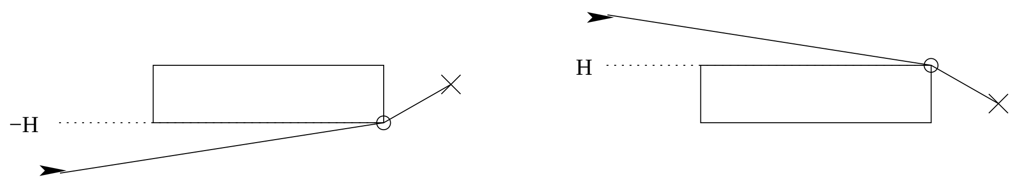

FIG. 4.7. *Recovery-circle cases (side views)*

solutions to the equation.

4.1. Common Definitions. We first introduce a set of technical definitions that can be used as procedures in an actual implementation of RR3D.

The times at which a moving point $\vec{s} + t\vec{v}$ intersects the lateral surface of the infinite cylinder are determined by

$$(s_x + tv_x)^2 + (s_y + tv_y)^2 = D^2. \tag{4.1}$$

If the ground speed is different from zero, i.e., $v_x^2 + v_y^2 > 0$, then (4.1) reduces to a quadratic equation in t:

$$t^2(v_x^2 + v_y^2) + 2t(s_x v_x + s_y v_y) + s_x^2 + s_y^2 - D^2 = 0. \tag{4.2}$$

The discriminant Δ is defined as

$$\begin{aligned} \Delta &= (s_x v_x + s_y v_y)^2 - (v_x^2 + v_y^2)(s_x^2 + s_y^2 - D^2) \\ &= D^2(v_x^2 + v_y^2) - (s_x v_y - s_y v_x)^2. \end{aligned} \tag{4.3}$$

If $\Delta \leq 0$ then the moving point does not intersect P_∞. In particular, if $\Delta = 0$ we have the tangent case.

DEFINITION 4.1 (Tangent condition). *A moving point $\vec{s} + t\vec{v}$ is* tangent *to P_∞ if and only if*

$$D^2(v_x^2 + v_y^2) = (s_x v_y - s_y v_x)^2. \tag{4.4}$$

For a moving point that is tangent to P_∞, the time τ of closest approach in the horizontal plane is the one solution of (4.2)*:*

$$\tau = -\frac{s_x v_x + s_y v_y}{v_x^2 + v_y^2}. \tag{4.5}$$

DEFINITION 4.2 (Entering and leaving P_∞). *If $\Delta > 0$, we get two solutions for* (4.2)

$$\Theta' = \frac{-s_x v_x - s_y v_y - \sqrt{\Delta}}{v_x^2 + v_y^2}, \tag{4.6}$$

$$\Theta'' = \frac{-s_x v_x - s_y v_y + \sqrt{\Delta}}{v_x^2 + v_y^2}. \tag{4.7}$$

By definition, $\Theta' < \Theta''$. It is easy to check that Θ' is an entry point and that Θ'' is an exit point.

We may need (4.4) and (4.5) instantiated with the parameters of the escape and the recovery courses. For the escape course we get the tangent condition

$$D^2(v_x'^2 + v_y'^2) = (s_x v_y' - s_y v_x')^2, \tag{4.8}$$

and the time of closest approach in the horizontal plane

$$\tau' = -\frac{s_x v_x' + s_y v_y'}{v_x'^2 + v_y'^2}. \tag{4.9}$$

The moving point $\vec{s''} + (t - t'')\vec{v''}$ describes the recovery course in a translated time $t - t''$. Therefore, for the recovery course we get the tangent condition

$$D^2(v_x''^2 + v_y''^2) = (s_x'' v_y'' - s_y'' v_x'')^2, \tag{4.10}$$

and the time of closest approach in the horizontal plane

$$\tau'' = -\frac{s_x'' v_x'' + s_y'' v_y''}{v_x''^2 + v_y''^2} + t''. \tag{4.11}$$

DEFINITION 4.3 (Reaching altitude H or $-H$). *If $v_z \neq 0$ then the times when the ownship reaches altitude H or $-H$ are the solutions of $|s_z + tv_z| = H$ for t:*

$$\theta' = \frac{-\operatorname{sign}(v_z)H - s_z}{v_z}, \tag{4.12}$$

$$\theta'' = \frac{\operatorname{sign}(v_z)H - s_z}{v_z}. \tag{4.13}$$

By definition, $\theta' < \theta''$.

DEFINITION 4.4 (Time of switch). *The time t' of switch from the escape course to the recovery course is given by* (2.4) *and $\vec{s''} = \vec{s} + t''\vec{v}$. This time satisfies*

$$t'(\vec{v'} - \vec{v''}) = t''(\vec{v} - \vec{v''}),$$

or in coordinate notation

$$t'(v_x' - v_x'') = t''(v_x - v_x''), \tag{4.14}$$

$$t'(v_y' - v_y'') = t''(v_y - v_y''), \tag{4.15}$$

$$t'(v_z' - v_z'') = t''(v_z - v_z''). \tag{4.16}$$

The equations (4.14) and (4.15) allow us to express v_y'' and v_x'' in terms of t', v_x', v_x'':

$$v_x'' = \frac{t''v_x - t'v_x'}{t'' - t'}, \tag{4.17}$$

$$v_y'' = \frac{t''v_y - t'v_y'}{t'' - t'}. \tag{4.18}$$

4.2. Change of vertical speed only. We impose the constraint that only the vertical component of the velocity vector may change. Formally,

$$v_{ox}' = v_{ox} = v_{ox}'' \qquad \text{and} \qquad v_{oy}' = v_{oy} = v_{oy}''. \tag{4.19}$$

If the relative ground speed is zero ($v_x^2 + v_y^2 = 0$) then either the ownship is inside the infinite cylinder ($s_x^2 + s_y^2 < D^2$), and there is no vertical solution, or else there is no predicted conflict. Otherwise, Θ' and Θ'' are defined as in equations (4.6) and (4.7), and we may have the following independent solutions.

- **Escape-circle.** If $0 < \Theta' < t''$ and $|s_z''| \geq H$ then there is an escape-circle solution (Figure 4.6). It is given by $t' = \Theta'$,

$$\begin{aligned} v_{oz}'' &= v_{iz} + \frac{-\operatorname{sign}(v_z'')H - s_z''}{\Theta' - t''}, \\ v_{oz}' &= v_{iz} + \frac{t''(v_{oz} - v_{iz}) - (t'' - \Theta')(v_{oz}'' - v_{iz})}{\Theta'} \\ &= \frac{t''(v_{oz} - v_{oz}'')}{\Theta'} + v_{oz}''. \end{aligned}$$

- **Recovery-circle.** If $0 < \Theta'' < t''$ and $|s_z| \geq H$ then there is a recovery-circle solution (Figure 4.7). It is given by $t' = \Theta''$,

$$\begin{aligned} v_{oz}' &= v_{iz} + \frac{-\operatorname{sign}(v_z)H - s_z}{\Theta''}, \\ v_{oz}'' &= v_{iz} + \frac{t''(v_{oz} - v_{iz}) - \Theta''(v_{oz}' - v_{iz})}{t'' - \Theta''} \\ &= \frac{t''v_{oz} - \Theta''v_{oz}'}{t'' - \Theta''}. \end{aligned}$$

- **One-circle.** If $0 < \Theta'$ and $\Theta'' < t''$ then for both $\varepsilon \in \{-1, 1\}$ there may be a one-circle solution. Figure 4.4 shows the case where a one-circle solution exists for each $\varepsilon = 1$ (left) and $\varepsilon = -1$ (right). If $\varepsilon s_z < H$ and $\varepsilon s_z'' < H$, then we compute the vertical speeds

$$\begin{aligned} v_{oz}' &= v_{iz} + \frac{\varepsilon H - s_z}{\Theta'}, \\ v_{oz}'' &= v_{iz} + \frac{\varepsilon H - s_z''}{\Theta'' - t''}. \end{aligned}$$

  If $v_{oz}' \neq v_{oz}''$, then t' is given by (4.16)

$$t' = t''(v_{oz} - v_{oz}'')/(v_{oz}' - v_{oz}'').$$

  In this case, there is a one-circle solution for ε given by v_{oz}', v_{oz}'', and t'.

We remark that there are no vertical solutions that touch the lines, neither circle-circle solutions.

4.3. Change of ground speed only. We impose the constraint that only the ground speed of the ownship changes in each step. Formally, there are factors $k, j > 0$, such that

$$v'_{ox} = kv_{ox}, \qquad v'_{oy} = kv_{oy}, \qquad v'_{oz} = v_{oz}, \tag{4.20}$$
$$v''_{ox} = jv_{ox}, \qquad v''_{oy} = jv_{oy}, \qquad v''_{oz} = v_{oz}. \tag{4.21}$$

Since absolute ground speeds are different from zero, either $v_{ox} \neq 0$ or $v_{oy} \neq 0$. In the first case we get

$$t'(k-j) = t''(1-j) \tag{4.22}$$

from (4.14). In the second case, i.e., $v_{oy} \neq 0$, we get the same formula (4.22) from (4.15).

If $k = j$ then from (4.22), $0 = t''(1-j)$. Since $t'' > 0$, we have $j = 1$ and so $k = 1$. So there is no predicted conflict at time 0. Since this contradicts the premise, we must have $k \neq j$. Therefore, the time t' of switch can be obtained from (4.22) as

$$t' = \frac{t''(1-j)}{k-j}. \tag{4.23}$$

We have the following independent solutions. For the cases involving an escape line course, we check for sanity that $0 < \tau' < t'$, where τ' is defined as in (4.9). For the cases involving a recovery line course, we check for sanity that $t' < \tau'' < t''$, where τ'' is defined as in (4.11). Furthermore, for the cases involving a circle course, we assume that relative vertical speed is not zero, i.e., $v_z \neq 0$; otherwise, there is no solution. In all the cases, we check for sanity that $k, j > 0$.

- **Line/line.** During the escape course and during the recovery course, the lateral surface of P is touched (Figure 4.1).
  In the tangent condition for the escape course (4.8), we instantiate using $\vec{v'} = \vec{v'_o} - \vec{v_i}$ and (4.20):

  $$D^2((kv_{ox} - v_{ix})^2 + (kv_{oy} - v_{iy})^2) = (s_x(kv_{oy} - v_{iy}) - s_y(kv_{ox} - v_{ix}))^2. \tag{4.24}$$

  This reduces to a quadratic equation in k:

  $$\begin{aligned} &k^2[D^2(v_{ox}^2 + v_{oy}^2) - (s_x v_{oy} - s_y v_{ox})^2] + \\ &2k[-D^2(v_{ox}v_{ix} + v_{oy}v_{iy}) + (s_x v_{oy} - s_y v_{ox})(s_x v_{iy} - s_y v_{ix})] + \\ &D^2(v_{ix}^2 + v_{iy}^2) - (s_x v_{iy} - s_y v_{ix})^2 = 0. \end{aligned}$$

  This quadratic equation in k can be solved to obtain v'_{ox} and v'_{oy}. If all coefficients in the quadratic equation for k are zero then the aircraft are not in predicted conflict at time 0.
  The tangent condition for the recovery course (4.10), instantiated using $\vec{v''} = \vec{v''_o} - \vec{v_i}$ and (4.21),

  $$D^2((jv_{ox} - v_{ix})^2 + (jv_{oy} - v_{iy})^2) = (s''_x(jv_{oy} - v_{iy}) - s''_y(jv_{ox} - v_{ix}))^2, \tag{4.25}$$

  can be reduced likewise to a quadratic equation in j. We solve it to obtain v''_{ox} and v''_{oy}.
  The time t' of switch is obtained from (4.23).
- **Line/circle.** During the escape course, the lateral surface of P is touched. During the recovery course, one of the two circles is intersected (Figure 4.2).
  For the line case, we have to solve the tangent condition for the escape course (4.24), which we can solve for k as in the line/line case.

For the circle case, we get θ'' by (4.13). We check for sanity that $0 < \theta'' < t''$. At time θ'' the circle is intersected:

$$(s''_x + (\theta'' - t'')(jv_{ox} - v_{ix}))^2 + (s''_y + (\theta'' - t'')(jv_{oy} - v_{iy}))^2 = D^2. \tag{4.26}$$

This equation yields a quadratic equation in j that can be solved:

$$\begin{aligned} &j^2(\theta'' - t'')^2(v_{ox}^2 + v_{oy}^2)^2 + \\ &2j(\theta'' - t'')(s''_x v_{ox} - (\theta'' - t'')v_{ix}v_{ox} + s''_y v_{oy} - (\theta'' - t'')v_{iy}v_{oy}) + \\ &(s''_x - (\theta'' - t'')v_{ix})^2 + (s''_y - (\theta'' - t'')v_{iy})^2 - D^2 = 0. \end{aligned} \tag{4.27}$$

Using (3.1), we check for sanity that θ'' is an entry point to the infinite cylinder:

$$(s''_x + (\theta'' - t'')(jv_{ox} - v_{ix}))(jv_{ox} - v_{ix}) + (s''_y + (\theta'' - t'')(jv_{oy} - v_{iy}))(jv_{oy} - v_{iy}) \leq 0. \tag{4.28}$$

The time t' of switch is obtained from (4.23). We check for sanity that $t' < \theta''$.

- **Circle/line.** The escape course intersects one of the circles, and the recovery course touches the lateral surface, of P. This case is depicted in Figure 4.3.
  For the circle, we get θ' by (4.12). We check for sanity that $0 < \theta' < t''$.
  The circle is intersected at time θ':

$$(s_x + \theta'(kv_{ox} - v_{ix}))^2 + (s_y + \theta'(kv_{oy} - v_{iy}))^2 = D^2. \tag{4.29}$$

This equation yields a quadratic equation in k that can be solved:

$$\begin{aligned} &k^2\theta'^2(v_{ox}^2 + v_{oy}^2) + \\ &2k\theta'(s_x v_{ox} - \theta' v_{ix}v_{ox} + s_y v_{oy} - \theta' v_{iy}v_{oy}) + \\ &(s_x - \theta' v_{ix})^2 + (s_y - \theta' v_{iy})^2 - D^2 = 0. \end{aligned} \tag{4.30}$$

Using (3.2), we check for sanity that θ' is an exit point from the infinite cylinder:

$$(s_x + \theta'(kv_{ox} - v_{ix}))(kv_{ox} - v_{ix}) + (s_y + \theta'(kv_{oy} - v_{iy}))(kv_{oy} - v_{iy}) \geq 0. \tag{4.31}$$

For the line, we have to satisfy the tangent condition for the recovery course (4.25), which can be solved for j as in the line/line case.
The time t' of switch is obtained from (4.23). For sanity, we check that $\theta' < t'$.

- **Circle/circle.** We may have a circle-circle solution (Figure 4.5) when times θ', θ'', given by (4.12) and (4.13), satisfy $0 < \theta'$ and $\theta'' < t''$ — or, equivalently, when either ($s_z < -H$ and $H < s''_z$) or ($s''_z < -H$ and $H < s_z$). We solve the two quadratic equations (4.27) and (4.30) for j and k respectively. We check for sanity that time θ' defines an exit point from the infinite cylinder (4.31) and that at time θ'' there is an entry point (4.28).
  The time t' of switch is obtained from (4.23). We check for sanity that $\theta' < t' < \theta''$.
- **Escape-circle.** We have an escape-circle solution if there is only one intersection with a circle at time θ'' given by (4.13). This case is shown in side view in Figure 4.6. We check for sanity that $0 < \theta'' < t''$. At time θ'' the circle is intersected (4.26). We solve the quadratic equation (4.27) for j and check for sanity that at time θ'' there is an entry point to the infinite cylinder (4.28).
  The time of switch is given by $t' = \theta''$. The speed factor k of the escape step is obtained from (4.22) as

$$k = \frac{j(\theta'' - t'') + t''}{\theta''}.$$

- **Recovery-circle.** We have a recovery-circle case if there is only one intersection with a circle at time θ' given by (4.12). This case is shown in side view in Figure 4.7. We check for sanity that $0 < \theta' < t''$. At time θ' the circle is intersected (4.29). We solve the quadratic equation (4.30) for k and check for sanity that at time θ' there is an exit point from the infinite cylinder (4.31).
  The time of switch is given by $t' = \theta'$. The speed factor j of the recovery step is obtained from (4.22) as
$$j = \frac{k\theta' - t''}{\theta' - t''}.$$

In this case there are no one-circle solutions.

4.4. Change of heading. We impose the constraint that for the escape step only the heading of the velocity vector may change. For the recovery step the heading and the horizontal speed may change. Formally,

$$v'^2_{ox} + v'^2_{oy} = v^2_{ox} + v^2_{oy} \qquad \text{and} \qquad v'_{oz} = v_{oz} = v''_{oz}. \tag{4.32}$$

For the cases involving an escape line course, we assume that s is not in the infinite cylinder neither at its boundary, i.e., $s_x^2 + s_y^2 > D^2$. We also check for sanity that $0 < \tau' < t'$, where τ' is defined as in (4.9). Symmetrically, for the cases involving a recovery line course, we assume that s'' is not in the infinite cylinder neither at its boundary, i.e., ${s''_x}^2 + {s''_y}^2 > D^2$ and we check for sanity that $t' < \tau'' < t''$, where τ'' is defined as in (4.11). Furthermore, for the cases involving a circle course, we assume that relative vertical speed is not zero, i.e., $v_z \neq 0$; otherwise, there is no solution. We have the following independent solutions.

- **Line/line.** The situation is shown in Figure 4.1. For the escape step we consider cases:
  - Case $v'_x = 0$. Then the tangent condition (4.8) specializes to $D^2 v'^2_y = s_x^2 {v'_y}^2$. So there are escape courses
    - $v'_{ox} = v_{ix}$, $v'_{oy} = \varepsilon\sqrt{v^2_{ox} + v^2_{oy} - v^2_{ix}}$ by (4.32), for each $\varepsilon \in \{-1, 1\}$, when $v^2_{ox} + v^2_{oy} \geq v^2_{ix}$ (sanity check) and $D^2 = s_x^2$, and
    - $v'_{ox} = v_{ix}$, $v'_{oy} = v_{iy}$, when $v^2_{ox} + v^2_{oy} = v^2_{ix} + v^2_{iy}$.
  - Case $v'_x \neq 0$. We may define $\alpha' = v'_y / v'_x$. In this case the tangent condition (4.8) reduces to
$$D^2(1 + \alpha'^2) = (s_x\alpha' - s_y)^2,$$
    and further to a quadratic equation in α':
$$\alpha'^2(D^2 - s_x^2) + 2\alpha' s_x s_y + D^2 - s_y^2 = 0. \tag{4.33}$$
    If $D^2 = s_x^2$ then $s_y \neq 0$, because s is not at the boundary of the infinite cylinder. In this case,
$$\alpha' = -\frac{D^2 - s_y^2}{2 s_x s_y}.$$
    Otherwise, Equation (4.33) has solutions
$$\alpha' = \frac{-s_x s_y + \varepsilon' D\sqrt{s_x^2 + s_y^2 - D^2}}{D^2 - s_x^2}$$
    where $\varepsilon' \in \{-1, 1\}$.
    Equation (4.32) and $\vec{v'} = \vec{v'_o} - \vec{v_i}$ yields a quadratic equation in v'_x:
$$v'^2_x(1 + \alpha'^2) + 2v'_x(v_{ix} + \alpha' v_{iy}) + v^2_{ix} + v^2_{iy} - v^2_{ox} - v^2_{oy} = 0. \tag{4.34}$$
    We get v'_x by solving (4.34) and v'_y by $v'_y = v'_x\alpha'$.

For the recovery step we consider cases:

- Case $v''_x = 0$. We check for sanity that $v'_x \neq 0$. In order to satisfy the tangent condition (4.10) we must satisfy $D^2 v''^2_y = s''^2_x v''^2_y$. This gives two cases:
  - $v''_{ox} = v_{ix}$, $v''_{oy} = v_{iy}$.
  - $v''_{ox} = v_{ix}$, $v''_{oy} \in \{y | {v''_{ox}}^2 + y^2 \neq 0\}$, when $D^2 = s''^2_x$.

  From the time t' of switch we get the equation system

$$t' v'_x = t'' v_x, \tag{4.35}$$

$$t'(v'_y - v''_y) = t''(v_y - v''_y). \tag{4.36}$$

  From (4.35) we get t' as $t' = t'' v_x / v'_x$. We finally get v''_y by (4.18).

- Case $v''_x \neq 0$. We define $\alpha'' = v''_y / v''_x$. The tangent condition (4.10) then reduces to

$$D^2(1 + \alpha''^2) = (s''_x \alpha'' - s''_y)^2. \tag{4.37}$$

  If $D^2 \neq s''^2_x$, Equation (4.37) yields a quadratic equation in α'' with solutions:

$$\alpha'' = \frac{-s''_x s''_y + \varepsilon'' D \sqrt{s''^2_x + s''^2_y - D^2}}{D^2 - s''^2_x}$$

  where $\varepsilon'' \in \{-1, 1\}$. If $D^2 = s''^2_x$ then $s''_y \neq 0$, because s'' is not at the boundary of the infinite cylinder. In this case,

$$\alpha'' = -\frac{D^2 - {s''_y}^2}{2 s''_x s''_y}.$$

  The time t' of switch is given by the equation system (4.14) and (4.15). Replacing v''_y with $v''_x \alpha''$, and subtracting from (4.15) the α''-multiple of (4.14), we get

$$t'(v'_y - \alpha'' v'_x) = t''(v_y - \alpha'' v_x). \tag{4.38}$$

  We check for sanity that $v'_y \neq \alpha'' v'_x$. From (4.38), t' is obtained as

$$t' = \frac{t''(v_y - \alpha'' v_x)}{v'_y - \alpha'' v'_x}. \tag{4.39}$$

  We get v''_x by (4.17) and v''_y by $v''_y = v''_x \alpha''$. The absolute ownship's velocity is obtained from the relative one.

- **Line/circle.** The situation is depicted in Figure 4.2. The line case of the escape step is solved exactly as in the case line/line. Thus, we get v'_x and v'_y. For the circle case of the recovery step, we determine the time of contact with the circle θ'', given by (4.13). For sanity, we check that $\tau' < \theta'' < t''$, where τ' is defined as in (4.9).

  The horizontal distance to the origin at time θ'' is D:

$$(s_x + t' v'_x + (\theta'' - t') v''_x)^2 + (s_y + t' v'_y + (\theta'' - t') v''_y)^2 = D^2. \tag{4.40}$$

  This equation reduces via (4.17), (4.18), and (2.1) to the quadratic equation in t'

$$\begin{aligned}
&t'^2[(s''_x + (\theta'' - t'') v'_x)^2 + (s''_y + (\theta'' - t'') v'_y)^2 - D^2] + \\
&2t'[-(s''_x + (\theta'' - t'') v'_x) t'' (s_x + \theta'' v_x) - (s''_y + (\theta'' - t'') v'_y) t'' (s_y + \theta'' v_y) + t'' D^2] + \\
&t''^2((s_x + \theta'' v_x)^2 + (s_y + \theta'' v_y)^2 - D^2) = 0.
\end{aligned}$$

By construction of the line case and $\tau' \neq \theta''$ we get $(s''_x + (\theta'' - t'')v'_x)^2 + (s''_y + (\theta'' - t'')v'_y)^2 > D^2$. We solve the quadratic equation to yield t'. Speeds v''_{ox} and v''_{oy} are obtained as follows:

$$v''_{ox} = \frac{t''v_x - t'v'_x}{t'' - t'} + v_{ix},$$
$$v''_{oy} = \frac{t''v_y - t'v'_y}{t'' - t'} + v_{iy}.$$

Using (3.1), we check for sanity that θ'' is an entry point to the infinite cylinder:

$$(s''_x + (\theta'' - t'')(v''_{ox} - v_{ix}))(v''_{ox} - v_{ix}) + (s''_y + (\theta'' - t'')(v''_{oy} - v_{iy}))(v''_{oy} - v_{iy}) \leq 0. \tag{4.41}$$

- **Circle/line.** A circle/line maneuver is shown in Figure 4.3. For the circle case of the escape step, we get the time θ' of contact with the circle, given by by (4.12). We check for sanity that $0 < \theta' < t''$. The escape course satisfies

$$(s_x + \theta'(v'_{ox} - v_{ix}))^2 + (s_y + \theta'(v'_{oy} - v_{iy}))^2 = D^2, \tag{4.42}$$

which reduces to

$$-2(s_y - \theta' v_{iy})\theta' v'_{oy} = (s_x - \theta' v_{ix})^2 + (s_y - \theta' v_{iy})^2 + 2(s_x - \theta' v_{ix})\theta' v'_{ox} + \theta'^2 v'^2_{ox} + \theta'^2 v'^2_{oy} - D^2. \tag{4.43}$$

Squaring both sides and using

$$v'^2_{oy} = v^2_{ox} + v^2_{oy} - v'^2_{ox} \tag{4.44}$$

yields a quadratic equation in v'_{ox}

$$v'^2_{ox} A + v'_{ox} B + C = 0, \tag{4.45}$$

where

$$A = 4\theta'^2((s_x - \theta' v_{ix})^2 + (s_y - \theta' v_{iy})^2),$$
$$B = 4(s_x - \theta' v_{ix})\theta' E,$$
$$C = E^2 - 4(s_y - \theta' v_{iy})\theta'^2(v^2_{ox} + v^2_{oy}),$$
$$E = (s_x - \theta' v_{ix})^2 + (s_y - \theta' v_{iy})^2 + \theta'^2 v^2_{ox} + \theta'^2 v^2_{oy} - D^2.$$

If $A = 0$, then $s_x = \theta' v_{ix}$ and $s_y = \theta' v_{iy}$, and so $B = 0$ and $C = E^2$. If $E = 0$, i.e., $\theta'^2 v^2_{ox} + \theta'^2 v^2_{oy} = D^2$, then

$$(s_x + \theta'(v_{ox} - v_{ix}))^2 + (s_y + \theta'(v_{oy} - v_{iy}))^2 = D^2,$$

so there is no predicted conflict. If $E \neq 0$ then there is no solution.
If $A \neq 0$, we get two solutions for v'_{ox} which yields solutions for v'_{oy}, via (4.44). We check for sanity that $v^2_{ox} + v^2_{oy} \geq v'^2_{ox}$.
Using (3.2), we check for sanity that θ' is an exit point from the infinite cylinder:

$$(s_x + \theta'(v'_{ox} - v_{ix}))(v'_{ox} - v_{ix}) + (s_y + \theta'(v'_{oy} - v_{iy}))(v'_{oy} - v_{iy}) \geq 0. \tag{4.46}$$

For the line case of the recovery step, we have to satisfy the tangent condition

$$D^2(v''^2_x + v''^2_y) = (s''_x v''_y - s''_y v''_x)^2.$$

We consider the following cases:

– Case $v''_x \neq 0$. We substitute $\alpha'' = v''_y / v''_x$ and get

$$D^2(1 + \alpha''^2) = (s''_x \alpha'' - s''_y)^2.$$

This equation reduces the quadratic equation in α''

$$\alpha''^2(D^2 - s''^2_x) + \alpha'' 2 s''_x s''_y + D^2 - s''^2_y = 0.$$

If $D^2 = s''^2_x$ then $s''_y \neq 0$, because s'' is not at the boundary of the infinite cylinder. In this case

$$\alpha'' = (D^2 - s''^2_y)/(2 s''_x s''_y).$$

Otherwise, we solve the quadratic equation for α''.

The time of switch t' yields (4.39) and so t', v''_x, v''_y as in the line/line case. We check for sanity that $\theta' < t'$.

– Case $v'_x \neq 0$ and $v''_x = 0$. From (4.14) we get $t' = t'' v_x / v'_x$, and finally we get v''_y by (4.18).

- **Circle/circle.** Circle-circle solutions (Figure 4.5) may exist if the times θ', θ'', given by (4.12) and (4.13), satisfy $0 < \theta'$ and $\theta'' < t''$ — or, equivalently, if either ($s_z < -H$ and $H < s''_z$) or ($s''_z < -H$ and $H < s_z$).

  From (4.42) and (4.32) we get v'_{ox} and v'_{oy} as in the circle/line case.

  At time θ'' the circle is intersected:

$$(s''_x + (\theta'' - t'')v''_x)^2 + (s''_y + (\theta'' - t'')v''_y)^2 = D^2.$$

  By (4.17) and (4.18) this reduces to the quadratic equation in t':

$$\begin{aligned}
&t'^2[(s''_x + (\theta'' - t'')v'_x)^2 + (s''_y + (\theta'' - t'')v'_y)^2] + \\
&t'[-2t''(s''_x + (\theta'' - t'')v'_x)(s''_x + (\theta'' - t'')v_x) - 2t''(s''_y + (\theta'' - t'')v'_y)(s''_y + (\theta'' - t'')v_y)] + \\
&t''^2(s''_x + (\theta'' - t'')v_x)^2 + t''^2(s''_y + (\theta'' - t'')v_y)^2 - D^2(t'' - t')^2 = 0.
\end{aligned}$$

  If $v'_x = -s''_x/(\theta'' - t'')$ and $v'_y = -s''_y/(\theta'' - t'')$ then this equation has no solution. Otherwise, we solve this equation for t' and we get v''_x and v''_y from (4.17) and (4.18), respectively.

  We check for sanity that $\theta' < t' < \theta''$, that time θ' defines an exit point from the infinite cylinder (4.46), and that at time θ'' there is an entry point (4.41).

- **Escape-circle.** Escape-circle solutions (Figure 4.6) may exist if there is only one intersection with a circle at time θ'' defined by (4.13). We check for sanity that $0 < \theta'' < t''$.

  At time θ'' the circle is intersected:

$$(s_x + \theta''(v'_{ox} - v_{ix}))^2 + (s_y + \theta''(v'_{oy} - v_{iy}))^2 = D^2.$$

  Speeds v'_{ox} and v'_{oy} are derived as in the circle/line case, but for θ'' instead of θ'. The time of switch t' is given by $t' = \theta''$. The velocity vector of the recovery case is given by (4.14) and (4.15) as follows:

$$v''_{ox} = \frac{t' v'_{ox} - t'' v_{ox}}{\theta'' - t''}, \tag{4.47}$$

$$v''_{oy} = \frac{t' v'_{oy} - t'' v_{oy}}{\theta'' - t''}. \tag{4.48}$$

  We check for sanity that at time θ'' there is an entry point to the infinite cylinder (4.41).

- **Recovery-circle.** Recovery-circle solutions (Figure 4.7) may exist if there is only one intersection with a circle at time θ' defined by (4.12). We check for sanity of θ' that $0 < \theta' < t''$.
  At time $t' = \theta'$ the circle is intersected:
  $$(s_x + \theta'(v'_{ox} - v_{ix}))^2 + (s_y + \theta'(v'_{oy} - v_{iy}))^2 = D^2.$$
  We derive solutions for v'_{ox} and v'_{oy} exactly as in the circle/line case. We check for sanity that at time θ' there is an exit point from the infinite cylinder (4.46). The values for v''_{ox} and v''_{oy} are obtained from (4.47) and (4.48), respectively.

In this case there are no one-circle solutions.

5. Conclusion and Future Work. We presented a *resolution and recovery* algorithm for two aircraft, namely ownship and intruder. The algorithm outputs a choice of maneuvers for the ownship, which is predicted to be in conflict with an intruder aircraft when flying towards a target point. Each maneuver consists of an escape velocity vector, a recovery velocity vector, and a time of trajectory change t'. Together they determine an escape course, a recovery course, and a new trajectory change point. If the ownship flies the escape course from time 0 to time t' and then switches to the recovery course at time t', it arrives at the original target point at the scheduled time, while maintaining minimum separation at all times.

We gave a rigorous mathematical description of the problem and proved that the recovery and resolution algorithm (RR3D) is *correct*: given some initial assumptions, e.g., there is a predicted conflict and the aircraft are not in violation of the minimum separation, the algorithm outputs a choice of *conflict-free maneuvers* that lead the ownship to the target point at the scheduled time. We intend to check the correctness proof using the PVS theorem prover [13].

The maneuvers returned by the algorithm are optimal in a geometric way, i.e., escape and recovery courses are tangential to the intruder's protected zone. They differ in the constraints they satisfy. For instance, one constraint requires that only a change of heading is allowed for the escape course. These constraints, which are not limitations imposed by the aircraft, are there to reduce the number of solutions to the problem. Our set of constraints is by no means exhaustive or unique. Other constraints may, however, easily be added. Specifically, we are currently working on the following additional constraints:

- A change of heading and ground speed for the escape step such that the ground speed does not change during the recovery step, i.e., $v''^2_{ox} + v''^2_{oy} = v'^2_{ox} + v'^2_{oy}$ and $v'_{oz} = v_{oz} = v''_{oz}$. This case yields surprisingly difficult systems of equations, including polynomials of degree 3, 4, and 6.
- Optimal solutions satisfying no constraints. These solutions may be difficult to implement by a human pilot since they require a simultaneous change of several parameters of the aircraft (heading, vertical speed, and ground speed). However, in the future, they could be suitable for a flight guidance system.

A prototype of RR3 has been implemented in Java. Currently, we are conducting experiments and simulations to study the applicability of RR3D as the inner loop of a strategic conflict detection and resolution approach.

Acknowledgments. The idea to extend KB3D [4] with recovery courses is due to David Wing. We acknowledge stimulating discussions with the DAG-TM team, Hanne Gottliebsen, and Victor Carreño.

REFERENCES

[1] Advanced Air Transportation Technologies (AATT) Project, *Concept definition for distributed air/ground traffic management (DAG-TM), version 1.0.* NASA Ames Research Center - NASA Langley Research Center, 1999.

[2] K. Bilimoria, *A geometric optimization approach to aircraft conflict resolution*, in Guidance, Navigation, and Control Conference, vol. AIAA 2000-4265, Denver, CO, August 2000.

[3] Y.-J. Chiang, J. Klosowsky, C. Lee, and J. Mitchell, *Geometric algorithms for conflict detection/resolution in air traffic management*, in 36th IEEE Conference on Decision and Control, 1997.

[4] G. Dowek, C. Muñoz, and A. Geser, *Tactical conflict detection and resolution in 3-D airspace*, in 4th USA/Europe Air Traffic Management R&D Seminar (ATM-2001), Santa Fe, New Mexico, 2001.

[5] N. Durand, J.-M. Alliot, and F. Medioni, *Neural nets trained by genetic algorithms for collision avoidance*, in Applied Artificial Intelligence, no. 3, 2000.

[6] N. Durand, J.-M. Alliot, and J. Noailles, *Automatic aircraft conflict resolution using genetic algorithms*, in Symposium on Applied Computing, Philadelphia, PA, 1996.

[7] M. Eby, *A self-organizational approach for resolving air traffic conflicts*, Lincoln Laboratory Journal, 7 (1994), pp. 239–254.

[8] E. Frazzoli, Z.-H. Mao, J.-H. Oh, and E. Feron, *Resolution of conflicts involving many aircraft via semidefinite programming*, Journal of Guidance, Control, and Dynamics, 24 (2001), pp. 79–86.

[9] G. Granger, N. Durand, and J.-M. Alliot, *Optimal resolution of en route conflicts*, in 4th USA/Europe Air Traffic Management R&D Seminar (ATM-2001), Santa Fe, New Mexico, 2001.

[10] J. Hoekstra, R. Ruigrok, R. van Gent, J. Visser, B. Gijsbers, M. Valenti, W. Heesbeen, B. Hilburn, J. Groeneweg, and F. Bussink, *Overview of NLR free flight project 1997-1999*, Tech. Report NLR-CR-2000-227, National Aerospace Laboratory (NLR), May 2000.

[11] J. Kuchar and L. Yang, *Survey of conflict detection and resolution modeling methods*, in AIAA Guidance, Navigation, and Control Conference, vol. AIAA-97-3732, New Orleans, LA, August 1997, pp. 1388–1397.

[12] J. McDonald and R. Vivona, *Strategic airborne conflict detection of air traffic and area hazards*, Tech. Report NASA Contract: NAS2-98005 RTO-29, TITAN Systems Corporation, SRC Division, November 2000.

[13] S. Owre, J. M. Rushby, and N. Shankar, *PVS: A prototype verification system*, in 11th International Conference on Automated Deduction (CADE), D. Kapur, ed., vol. 607 of Lecture Notes in Artificial Intelligence, Saratoga, NY, June 1992, Springer-Verlag, pp. 748–752.

[14] Radio Technical Commission for Aeronautics, *Final report of the RTCA board of directors' select committee on free flight*, Tech. Report Issued 1-18-95, RTCA, Washington, DC, 1995.

[15] C. Tomlin, G. Pappas, and S. Sastry, *Conflict resolution for air traffic management: A study in multi-agent hybrid systems*, IEEE Transactions on Automatic Control, (1998).

[16] D. Wing, R. Adams, B. Barmore, and D. Moses, *Airborne use of traffic intent information in a distributed air-ground traffic management concept: Experiment design and preliminary results*, in 4th USA/Europe Air Traffic Management R&D Seminar (ATM-2001), Santa Fe, New Mexico, 2001.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	May 2002	Contractor Report

4. TITLE AND SUBTITLE
Air traffic conflict resolution and recovery

5. FUNDING NUMBERS
C NAS1-97046
WU 505-90-52-01

6. AUTHOR(S)
Alfons Geser, César Muñoz, Gilles Dowek, and Florent Kirchner

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
ICASE
Mail Stop 132C
NASA Langley Research Center
Hampton, VA 23681-2199

8. PERFORMING ORGANIZATION REPORT NUMBER
ICASE Report No. 2002-12

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23681-2199

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA/CR-2002-211637
ICASE Report No. 2002-12

11. SUPPLEMENTARY NOTES
Langley Technical Monitor: Dennis M. Bushnell
Final Report
Submitted to the 21st Digital Avionics Systems Conference.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified–Unlimited
Subject Category 60, 61
Distribution: Nonstandard
Availability: NASA-CASI (301) 621-0390

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
An essential element in the Free Flight concept is the detection and avoidance of air traffic conflicts. A conflict occurs when the required separation between two aircraft, namely the ownship and the intruder, is lost. Conflict detection and resolution systems predict loss of separation and output conflict avoidance maneuvers that divert the ownship from its original trajectory. In this paper, we address the problem of redirecting the ownship to its original path, in a geometric optimal way, without introducing new conflicts. We call this concept Resolution and Recovery. Given the current 3-dimensional position and velocity vectors of two aircraft in predicted conflict and the ownship's required time of arrival at the target point, the resolution and recovery algorithm outputs a choice of maneuvers. Each maneuver comprises an escape course and a recovery course to be followed by the ownship. The escape course brings the ownship off the predicted conflict and the recovery course returns it to the original target point. We provide a rigorous mathematical description of the problem and show that the algorithm is correct, i.e., no matter which of the proposed maneuvers the ownship picks, it will arrive at the target point at the scheduled time while maintaining the minimum required separation to the intruder at all times.

14. SUBJECT TERMS
conflict detection and resolution, 3-dimensional airspace, traffic avoidance, free flight

15. NUMBER OF PAGES
24

16. PRICE CODE
A03

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified		