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# LINEAR PARAMETER VARYING CONTROL FOR ACTUATOR FAILURE

JONG-YEOB SHIN*, N. EVA WU†, AND CHRISTINE BELCASTRO‡

**Abstract.** A *robust* linear parameter varying (LPV) control synthesis is carried out for an HiMAT vehicle subject to loss of control effectiveness. The scheduling parameter is selected to be a function of the estimates of the control effectiveness factors. The estimates are provided on-line by a two-stage Kalman estimator. The inherent conservatism of the LPV design is reduced through the use of a scaling factor on the uncertainty block that represents the estimation errors of the effectiveness factors. Simulations of the controlled system with the on-line estimator show that a superior fault-tolerance can be achieved.

**Key words.** fault tolerant control system, fault parameter estimation, reconfigurable controller

**Subject classification.** Guidance and Control

**1. Introduction.** One of control schemes for a nonlinear system is a gain-scheduled linear parameter varying control technique [13, 1, 7, 16]. This approach is particularly appealing in that a nonlinear plant is treated as a linear parameter varying (LPV) system whose state-space matrices are functions of a scheduling parameter vector. This allows linear control techniques to be applied to a nonlinear system. Several researches on an LPV synthesis methodology allow the design of the global control law for an LPV system over a parameter set which is bounded and measurable [13, 1, 7, 16]. An LPV controller synthesis is formulated into a linear matrix inequality (LMI) optimization problem. There are LPV control synthesis methods according to a functional form of an LPV system. The polytopic LPV control synthesis method [2] is used for an LPV system which is a polytopic function of a scheduling parameter vector. The affine LPV control synthesis method [1] is applied to an affine LPV system, whose LMI constraints are evaluated at only vertex points of an LPV system. The grid LPV control synthesis method [7, 16] is for an LPV system which is a bounded function of a scheduling parameter vector. In the method, LMI constraints are evaluated at grid points over parameter spaces. These methods can be converted to each other by increasing conservatism to describe an LPV system. The grid LPV synthesis method has been successfully applied to synthesis controllers for the pitch-axis missile autopilots[17, 12], F-14 aircraft lateral-directional axis during powered approach[6, 4], turbofan engines [15, 5] and F-16 aircraft [14]. Scheduling parameters of these applications are physical parameters such as angle of attack, mach number, velocity, dynamic pressure, etc. Scheduling parameters in LPV control synthesis are required to be measurable and the variations of scheduling parameters should be in a bounded set.

In this paper, actuator failures are modeled as an LPV system as functions of actuator effectiveness parameters [9]. These parameters are estimated as biases using an augmented Kalman filter. A set of covariance-dependent forgetting factors is introduced into the filtering algorithm. As a result, the change in the actuator effectiveness is accentuated to help achieve a more accurate estimate more rapidly. The $H_\infty$ bounds on parameter estimation errors are assessed through simulations, which are then used as bounds of real parameter uncertainty in the construction of a robust LPV control law. Actuator faults can be parameterized as estimated fault effectiveness parameters. Thus, it is possible to formulate a fault tolerance control design problem as an LPV control synthesis problem based on estimated faults parameters.

Fault estimation errors and modeling uncertainties are represented by an uncertainty block in the construction of a robust LPV control law. The structure of an uncertainty block is not included in an conventional LPV control synthesis methodology [7, 16]. A scaling factor on a uncertainty block can reduce conservatism of the LPV synthesis [1]. In Ref.[1], it is formulated into a single optimization problem to find a scaling factor and a control law to achieve a certain level of performance. The optimization problem is not a convex problem, which has unknown positive matrices $X$ and $Y$ related with a control law and a scaling factor $S$ related with the uncertainty block structure. The problem is solved by an iteration method of fixing $X$ and $Y$ or a scaling factor $S$. In this paper, the problem is formulated into two LMI optimizations: one is to design

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an LPV controller with given a scaling factor and the other is to determine a scaling factor $S$ with a given control law. The problem is solved by an iteration method of fixing a control law instead of fixing variables $X$ and $Y$. This helps to find a scaling factor $S$ for minimizing an induced-$\mathcal{L}_2$ norm of the closed-loop system.

This paper contains the following sections. In Section 2, an LPV synthesis methodology used in this paper is summarized. In Section 3, fault parameter estimation methods are presented. In Section 4, an LPV controller for an aerospace vehicle is designed by using an LPV synthesis control methodology with a scaling factor. In Section 5, the simulation results of the closed-loop system are presented and this paper concludes with a brief summary in Section 6.

## 2. LPV Synthesis.

**2.1. Problem Statements.** In this section, a control synthesis problem is defined, based on an estimated parameter vector $\bar{\rho} \in \mathcal{R}^{n_s}$ such as actuator effectiveness [9]. Actuator effectiveness parameters represent actuator failure cases (actuator damage). Suppose actuator effectiveness parameter can be estimated using the estimation methods presented in Section 3. A system dynamics can be represented by an LPV system according to an estimated scheduling parameter vector and estimation error bounds.

An LPV system can be represented as functions of an estimated scheduling parameter vector $\bar{\rho}$ with an uncertainty block $\Delta$ which captures parameter estimation errors and unmodeled dynamics. An LPV system can be written as:

$$\begin{bmatrix} \dot{x} \\ e_\Delta \\ e_p \\ y \end{bmatrix} = \begin{bmatrix} A(\bar{\rho}) & B_\Delta(\bar{\rho}) & B_p(\bar{\rho}) & B_u(\bar{\rho}) \\ C_e(\bar{\rho}) & D_{\Delta\Delta}(\bar{\rho}) & D_{\Delta p}(\bar{\rho}) & D_{\Delta u}(\bar{\rho}) \\ C_p(\bar{\rho}) & D_{p\Delta}(\bar{\rho}) & D_{pp}(\bar{\rho}) & D_{pu}(\bar{\rho}) \\ C_y(\bar{\rho}) & D_{y\Delta}(\bar{\rho}) & D_{yp}(\bar{\rho}) & 0 \end{bmatrix} \begin{bmatrix} x \\ d_\Delta \\ d_p \\ u \end{bmatrix}, d_\Delta = \Delta e_\Delta, \tag{2.1}$$

where $x \in \mathcal{R}^{n_x}$, $e_\Delta \in \mathcal{R}^{n_\Delta}$, $e_p \in \mathcal{R}^{n_{e_p}}$, $y \in \mathcal{R}^{n_y}$, $d_\Delta \in \mathcal{R}^{n_\Delta}$, $d_p \in \mathcal{R}^{n_{d_p}}$, and $u \in \mathcal{R}^{n_u}$. All of the state-space matrices are of appropriate dimensions.

An uncertainty block set $\boldsymbol{\Delta}$ is defined as:

$$\boldsymbol{\Delta} = \{\Delta = diag(\delta_1 I_1, \cdots, \delta_n I_n, \Delta_{n+1}, \cdots, \Delta_{n+m}) : \delta_i \in R, \Delta_i \in R^{l \times l}, \bar{\sigma}(\Delta) \le \beta\}, \tag{2.2}$$

where $\beta$ is normalized to 1 without loss of generality. There exists a scaling factor set $\boldsymbol{S}$ such that

$$\boldsymbol{S} = \{S : S > 0,\ \ S\Delta = \Delta S, \quad S \in \mathcal{R}^{n_\Delta \times n_\Delta}\}. \tag{2.3}$$

The input/output scaling matrices $L^{-1/2}$ and $J^{1/2}$ are defined as

$$L^{-1/2} = \begin{bmatrix} S^{-1/2} & 0 \\ 0 & I_{n_{d_p}} \end{bmatrix} \in \mathcal{R}^{(n_\Delta + n_{d_p}) \times (n_\Delta + n_{d_p})}, \quad J^{1/2} = \begin{bmatrix} S^{1/2} & 0 \\ 0 & I_{n_{e_p}} \end{bmatrix} \in \mathcal{R}^{(n_\Delta + n_{e_p}) \times (n_\Delta + n_{e_p})}, \tag{2.4}$$

where $S^{1/2} \in \boldsymbol{S}$ and $S^{-1/2} \in \boldsymbol{S}$.

The induced $\mathcal{L}_2$-norm of a parameter dependent stable LPV system is defined as

$$||G||_{2\leftarrow 2} \equiv \sup_{\forall \bar{\rho},\ d \in \mathcal{L}_2,\ d \neq 0} \frac{||e||_2}{||d||_2},$$

for zero initial conditions $x(0) = 0$.

Suppose there exists an LPV controller $K(\bar{\rho})$ which stabilizes an LPV system $P(\bar{\rho})$. The control synthesis problem is:

$$\min_{K(\bar{\rho}),\ \ S \in \boldsymbol{S}} ||J^{1/2}(S) F_l(P(\bar{\rho}), K(\bar{\rho})) L^{-1/2}(S)||_{2\leftarrow 2}, \tag{2.5}$$

where $F_l(P(\bar{\rho}), K(\bar{\rho}))$ means a lower linear fraction transformation (LFT). The optimization problem of equation (2.5) is not convex in $K(\bar{\rho})$ and $S$. The problem is similar to a D-K iteration. In this paper, we approach to the problem in a similar manner of solving the D-K iteration problem[3].

**2.2. Control Synthesis Methodology.** In this section, a procedure of solving the problem is presented. There is an LPV control synthesis methodology in Ref. [1] with a scaling factor. In Ref. [1], an LMI optimization can be formulated with unknown matrices $X > 0$, $Y > 0$, and scaling factor matrices $J^{1/2}$ and $L^{-1/2}$. However, an LMI optimization problem in Ref. [1] has an equality constraint. In this paper, to avoid an equality constraint, the augmented LPV system with the scaling matrices $J^{1/2}$ and $L^{-1/2}$ is used to design an LPV controller.

Suppose a scaling factor $S$ is given. The augmented LPV open-loop system with the scaling matrices $J^{1/2}$ and $L^{-1/2}$ can be written as:

$$\begin{bmatrix} \dot{x} \\ e \\ y \end{bmatrix} = \begin{bmatrix} A(\bar{\rho}) & \bar{B}_1(\bar{\rho}) & B_2(\bar{\rho}) \\ \bar{C}_1(\bar{\rho}) & \bar{D}_{11}(\bar{\rho}) & \bar{D}_{12}(\bar{\rho}) \\ C_2(\bar{\rho}) & \bar{D}_{21}(\bar{\rho}) & 0 \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix} \equiv \begin{bmatrix} A(\bar{\rho}) & B_1(\bar{\rho})L^{-1/2} & B_2(\bar{\rho}) \\ J^{1/2}C_1(\bar{\rho}) & J^{1/2}D_{11}(\bar{\rho})L^{-1/2} & J^{1/2}D_{12}(\bar{\rho}) \\ C_2(\bar{\rho}) & D_{21}(\bar{\rho})L^{-1/2} & 0 \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix} \tag{2.6}$$

where $e = [e_\Delta^T \;\; e_p^T]^T$ and $d = [d_\Delta^T \;\; d_p^T]^T$. With assumption that $\bar{D}_{12}(\bar{\rho})$ and $\bar{D}_{21}(\bar{\rho})$ are full column and row rank for all $\bar{\rho}$, respectively, an LPV control synthesis methodology in Ref. [16, 7] can be used in this paper. For the sake of completeness, a brief summary of the LPV control synthesis methodology in Ref. [16] is presented in this section.

Using Q-R decompositions[7] of matrices $\bar{D}_{12}(\bar{\rho})$ and $\bar{D}_{21}(\bar{\rho})$, the augmented LPV system is rewritten as:

$$\begin{bmatrix} \dot{x} \\ e_1 \\ e_2 \\ y \end{bmatrix} = \begin{bmatrix} A(\bar{\rho}) & B_{11}(\bar{\rho}) & B_{12}(\bar{\rho}) & B_2(\bar{\rho}) \\ C_{11}(\bar{\rho}) & D_{1111}(\bar{\rho}) & D_{1112}(\bar{\rho}) & 0 \\ C_{12}(\bar{\rho}) & D_{1121}(\bar{\rho}) & D_{1122}(\bar{\rho}) & I_{n_{e_2}} \\ C_2(\bar{\rho}) & 0 & I_{n_{d_2}} & 0 \end{bmatrix} \begin{bmatrix} x \\ d_1 \\ d_2 \\ u \end{bmatrix}, \tag{2.7}$$

where $d_1 \in \mathcal{R}^{n_{d_1}}$, $d_2 \in \mathcal{R}^{n_{d_2}}$, $e_1 \in \mathcal{R}^{n_{e_1}}$, and $e_2 \in \mathcal{R}^{n_{d_2}}$. Matrices $D_{1111}$, $D_{1112}$, $D_{1121}$, $D_{1122}$, $C_{11}$, $C_{12}$ are of appropriate dimensions.

There exists an LPV controller $K(\bar{\rho})$ which leads to the induced-$\mathcal{L}_2$ norm of the closed-loop system being less than constant $\gamma$. The LPV controller $K(\bar{\rho})$ can be constructed from the solution matrices, $X(\bar{\rho}) \in \mathcal{R}^{n\times n}$ and $Y(\bar{\rho}) \in \mathcal{R}^{n\times n}$ which are calculated by solving the following LMI optimization.

$$\min_{X(\bar{\rho}),Y(\bar{\rho})} \gamma, \tag{2.8}$$

subject to

$$\begin{bmatrix} X(\bar{\rho})\hat{A}^T(\bar{\rho}) + \hat{A}(\bar{\rho})X(\bar{\rho}) + \sum_{i=1}^m (\underline{\nu}_i \frac{\partial X}{\partial \bar{\rho}_i}) - B_2(\bar{\rho})B_2^T(\bar{\rho}) & X(\bar{\rho})C_{11}^T(\bar{\rho}) & \gamma^{-1}\hat{B}(\bar{\rho}) \\ C_{11}(\bar{\rho})X(\bar{\rho}) & -I_{n_{e_1}} & \gamma^{-1}D_{111.}(\bar{\rho}) \\ \gamma^{-1}\hat{B}^T(\bar{\rho}) & \gamma^{-1}D_{111.}^T(\bar{\rho}) & -I_{n_d} \end{bmatrix} < 0, \tag{2.9}$$

$$\begin{bmatrix} \tilde{A}(\bar{\rho})Y(\bar{\rho}) + Y(\bar{\rho})\tilde{A}^T(\bar{\rho}) + \sum_{i=1}^m (\underline{\nu}_i \frac{\partial Y}{\partial \bar{\rho}_i}) - C_2^T(\bar{\rho})C_2(\bar{\rho}) & Y(\bar{\rho})B_{11}(\bar{\rho}) & \gamma^{-1}\tilde{C}^T(\bar{\rho}) \\ B_{11}^T(\bar{\rho})Y(\bar{\rho}) & -I_{n_{d_1}} & \gamma^{-1}D_{11.1}^T(\bar{\rho}) \\ \gamma^{-1}\tilde{C}(\bar{\rho}) & \gamma^{-1}D_{11.1}(\bar{\rho}) & -I_{n_e} \end{bmatrix} < 0, \tag{2.10}$$

$$\begin{bmatrix} X(\bar{\rho}) & \gamma^{-1}I_n \\ \gamma^{-1}I_n & Y(\bar{\rho}) \end{bmatrix} \geq 0, \tag{2.11}$$

$$X(\bar{\rho}) > 0, \;\; Y(\bar{\rho}) > 0,$$

where

$$\hat{A}(\bar{\rho}) \equiv A(\bar{\rho}) - B_2(\bar{\rho})C_{12}(\bar{\rho}), \;\; \hat{B}(\bar{\rho}) \equiv B_1(\bar{\rho}) - B_2(\bar{\rho})D_{112.}(\bar{\rho}), \tag{2.12}$$

$$\tilde{A}(\bar{\rho}) \equiv A(\bar{\rho}) - B_{12}(\bar{\rho})C_2(\bar{\rho}), \;\; \tilde{C}(\bar{\rho}) \equiv C_1(\bar{\rho}) - D_{11.2}(\bar{\rho})C_2(\bar{\rho}). \tag{2.13}$$

The definitions of matrices $D_{112.}$, $D_{11.1}$, $D_{111.}$ and $D_{11.2}$ are taken from Ref. [16]. Also, the realization of an LPV controller from the solution matrices $X$ and $Y$ are taken from Ref. [16].

The benefit of the LPV synthesis methodology is that there is no limitation of an affine functional form of LPV system state-space matrices. Since the LMI constraints in equations (2.9)-(2.10) are evaluated at grid points over all scheduling parameter spaces, an LPV system should be just a function of a scheduling parameter. In this paper, we consider actuators are failed one at a time. In the failure case, the system variations due to actuator failures can not be represented by an affine function of an actuator failure parameter vector. The disadvantage of the LPV control synthesis methodology is that robust stability over all parameter spaces is not guaranteed unless choosing appropriate number of grid points.

Suppose there exists an designed LPV controller $K(\bar{\rho})$ which stabilizes the augmented LPV system. The closed-loop LPV system with a given controller is written as:

$$\begin{bmatrix} \dot{x}_{cl} \\ e_\Delta \\ e_p \end{bmatrix} = \begin{bmatrix} A_{cl}(\rho) & B_{1_{cl}}(\rho)S^{-1/2} & B_{2_{cl}}(\rho) \\ S^{1/2}C_{1_{cl}}(\rho) & S^{1/2}D_{11_{cl}}(\rho)S^{-1/2} & S^{1/2}D_{21_{cl}}(\rho) \\ C_{2_{cl}}(\rho) & D_{21_{cl}}(\rho)S^{-1/2} & D_{22_{cl}}(\rho) \end{bmatrix} \begin{bmatrix} x_{cl} \\ d_\Delta \\ d_p \end{bmatrix}, \qquad d_\Delta = \Delta e_\Delta, \tag{2.14}$$

where $x_{cl} = [x^T \quad x_k^T]^T$.

Applying the Kalman-Yakubovich-Popov (KYP) Lemma [9], the LMI optimization is formulated to find a scaling factor $S$. There exists an scaling factor $S \in \boldsymbol{S}$ which leads to the induced-$\mathcal{L}_2$ norm of the closed-loop system being less than $\gamma_s$. The scaling factor $S$ can be determined solving the following LMI optimization.

$$\min_{P>0, S\in\boldsymbol{S}} \gamma_s, \tag{2.15}$$

$$\begin{bmatrix} M_{11} & PB_{1_{cl}} + \gamma_s^{-1}C_{2_{cl}}^T D_{21_{cl}} + C_{1_{cl}}^T SD_{11_{cl}} & PB_{2_{cl}} + \gamma_s^{-1}C_{2_{cl}}^T D_{22_{cl}} + C_{1_{cl}}^T SD_{12_{cl}} \\ \star & \gamma_s^{-1}D_{21_{cl}}^T D_{21_{cl}} + D_{11_{cl}}^T SD_{11_{cl}} - S & \gamma_s^{-1}D_{21_{cl}}^T D_{22_{cl}} + D_{11_{cl}}^T SD_{12_{cl}} \\ \star & \star & \gamma_s^{-1}D_{22_{cl}}^T D_{22_{cl}} + D_{12_{cl}}^T SD_{12_{cl}} - I \end{bmatrix} < 0 \tag{2.16}$$

where

$$M_{11} = A^T P + PA + \dot{P} + \gamma_s^{-1}C_{2_{cl}}^T C_{2_{cl}} + C_{1_{cl}}^T SC_{1_{cl}}.$$

In the LMI constraint, $\star$ denotes a symmetric component.

The iteration procedure to solve the problem in equation (2.5) is follows:

1. Design an LPV controller $K(\bar{\rho})$ for a system from the LMI optimization in equation (2.8) with fixed $S$. At the first iteration, $S$ is assumed as $I$.
2. Solve the LMI optimization problem in equation (2.15) over $P(\bar{\rho})$ and $S$ based on the closed-loop system with the designed LPV controller $K(\bar{\rho})$.
3. Generate an augmented LPV system with the scaling factor $S$

$$G_{i+1}(\bar{\rho}) = \begin{bmatrix} S^{1/2} & 0 \\ 0 & I \end{bmatrix} G_i(\bar{\rho}) \begin{bmatrix} S^{-1/2} & 0 \\ 0 & I \end{bmatrix}, \tag{2.17}$$

   where $G_i(\bar{\rho})$ is an LPV model at the $i^{th}$ iteration.
4. Iterate over step 1 to 3 until convergence or terminate iteration based on satisfaction with a designed LPV controller.

The iteration method can not guarantee finding global solutions of $K$ and $S$ since the problem in equation (2.5) is not convex in $K$ and $S$. Also, there is no guarantee of convergence in the iteration process. In the LPV synthesis methodology, the matrix $P(\bar{\rho})$ is related with the solution matrices $X$ and $Y$ of equations (2.9)-(2.10). When a designed controller is fixed to calculate the scaling factor $S$, the matrix $P(\bar{\rho})$ can be calculated from the solution matrices $X$ and $Y$ and fixed in the LMI optimization [16]. In this paper, the matrix $P(\bar{\rho})$ is also set as an unknown matrix in the LMI optimization in equation (2.15) to relax the constraints of fixing the LPV controller $K(\bar{\rho})$. Thus, there are two LMI optimizations in the iteration process.

**3. Parameter Estimation.** This section briefly describes the formulation of a real parameter estimation problem, which, when specialized to the actuator effectiveness estimation, transforms a fault (loss of actuator effectiveness) tolerant control problem to a robust LPV control problem. The development of this section follows that in Ref. [9].

The estimator is based on a linear discrete design model of the form:

$$
\begin{aligned}
x_{k+1} &= A_k^d x_k + \begin{bmatrix} b_1^d \gamma_k^1 & \cdots & b_{n_u}^d \gamma_k^{n_u} \end{bmatrix} \begin{bmatrix} u_k^1 \\ u_k^2 \\ \vdots \\ u_k^{n_u} \end{bmatrix} \\
&\quad + B_k^d u_k + w_k^x && (3.1) \\
&= A_k^d x_k + E_k^d \gamma_k + B_k^d u_k + w_k^x, && (3.2) \\
\gamma_{k+1} &= \gamma_k + w_k^\gamma, && (3.3) \\
y_k &= C_k^d x_k + v_k, && (3.4)
\end{aligned}
$$

where $x_k \in R^{n_x}$, $\gamma_k \in R^{n_u}$, $u_k \in R^{n_u}$ and $y_k \in R^{n_y}$ are the state, bias, input, and output variables, respectively. The discrete model can be obtained from a continuous model via, for example, the Euler's rule with a sampling period $T_s$, which preserves the functional dependence of the "$B$" matrix on $\gamma$. The bias vector $\gamma$ with component $-1 \le \gamma_k^i \le 0$ relates to a actuator failure parameter. It is obvious that $E_k^d = B_k^d \times diag\{u_k^1, \cdots, u_k^{n_u}\}$. $w_k^x$, $w_k^\gamma$ and $v_k$ denote the white noise sequences of uncorrelated Gaussian random vectors with zero means and covariance matrices $Q_k^x$, $Q_k^\gamma$ and $R_k$, respectively.

The minimum variance solution is obtained by a direct application of the two-stage Kalman filter algorithm of Keller and Darouach[8], with constant coefficient matrices in Ref. [8] replaced by time-varying matrices. The filter is decoupled into four sets of equations. They given as follows.

—Optimal bias estimator

$$
\begin{aligned}
\hat{\gamma}_{k+1|k} &= \hat{\gamma}_{k|k}, \\
P_{k+1|k}^\gamma &= P_{k|k}^\gamma + Q_k^\gamma, \\
\hat{\gamma}_{k+1|k+1} &= \hat{\gamma}_{k+1|k} + K_{k+1}^\gamma (\tilde{r}_{k+1} - H_{k+1|k} \hat{\gamma}_{k|k}), \\
K_{k+1}^\gamma &= P_{k+1|k}^\gamma H_{k+1|k}^T \\
&\quad (H_{k+1|k} P_{k+1|k}^\gamma H_{k+1|k}^T + \tilde{S}_{k+1})^{-1}, \\
P_{k+1|k+1}^\gamma &= (I - K_{k+1}^\gamma H_{k+1|k}) P_{k+1|k}^\gamma .
\end{aligned}
$$

—Bias-free state estimator

$$
\begin{aligned}
\tilde{x}_{k+1|k} &= A_k^d \tilde{x}_{k|k} + B_k^d u_k + W_k \hat{\gamma}_{k|k} - V_{k+1|k} \hat{\gamma}_{k|k}, \\
\tilde{P}_{k+1|k}^x &= A_k^d \tilde{P}_{k|k}^x (A_k^d)^T + Q_k^x + W_k P_{k|k}^\gamma W_k^T \\
&\quad - V_{k+1|k} P_{k+1|k}^\gamma V_{k+1|k}^T, \\
\tilde{x}_{k+1|k+1} &= \tilde{x}_{k+1|k} + \tilde{K}_{k+1}^x (y_{k+1} - C_{k+1}^d \tilde{x}_{k+1|k}), \\
\tilde{K}_{k+1}^x &= \tilde{P}_{k+1|k}^x (C_{k+1}^d)^T \{ C_{k+1}^d \tilde{P}_{k+1|k}^x (C_{k+1}^d)^T \\
&\quad + R_{k+1} \}^{-1}, \\
\tilde{P}_{k+1|k+1}^x &= (I - \tilde{K}_{k+1}^x C_{k+1}^d) \tilde{P}_{k+1|k}^x ,
\end{aligned}
$$

where the filter residual and its covariance are given as

$$
\begin{aligned}
\tilde{r}_{k+1} &= y_{k+1} - C_{k+1}^d \tilde{x}_{k+1|k}, \\
\tilde{S}_{k+1} &= C_{k+1}^d \tilde{P}_{k+1|k}^x (C_{k+1}^d)^T + R_{k+1}.
\end{aligned}
$$

—Coupling equations

$$
\begin{aligned}
W_k &= A_k^d V_{k|k} + E_k^d, \\
V_{k+1|k} &= W_k P_{k|k}^{\gamma} (P_{k+1|k}^{\gamma})^{-1}, \\
H_{k+1|k} &= C_{k+1}^d V_{k+1|k}, \\
V_{k+1|k+1} &= V_{k+1|k} - \tilde{K}_{k+1}^x H_{k+1|k}.
\end{aligned}
$$

—And finally the compensated state and error covariance estimates

$$
\begin{aligned}
\hat{x}_{k+1|k+1} &= \tilde{x}_{k+1|k+1} + V_{k+1|k+1}\hat{\gamma}_{k+1|k+1}, \\
P_{k+1|k+1} &= \tilde{P}_{k+1|k+1}^x + V_{k+1|k+1} P_{k+1|k+1}^{\gamma} V_{k+1|k+1}^T.
\end{aligned}
$$

A further measure is taken to modify the above filtering algorithm so that the estimates become more responsive to abrupt changes in the control effectiveness factors.

A well known technique for estimating time-varying parameters is the use of forgetting factors. The basic idea is to enable a recursive algorithm to discount the past information so that the filter is more apt to recognize the changes in the system. Since the time update of the bias estimate governed by $\hat{\gamma}_{k+1|k} = \hat{\gamma}_{k|k}$ is the dominant opposing force to acknowledge the abrupt changes in the biases, forgetting factors introduced into the time propagation equation $P_{k+1|k}^{\gamma} = P_{k|k}^{\gamma} + Q_k^{\gamma}$ of the bias covariance is likely to function most effectively.

Assume that covariance $P_{k|k}^{\gamma}$ adequately describes the bias estimation error along both temporal and spacial directions under the normal system operation condition. Then this covariance provides a basis for the selection of forgetting factors. The bias estimates should be prevented from being impetuous, as well as from being indifferent to the changes shown in the measurements. A technique suggested in Ref. [11] amounts to select forgetting factors that would force the adjusted covariance in $P_{k+1|k}^{\gamma} = P_{k|k}^{\gamma} + Q_k^{\gamma}$ to stay within some prescribed bounds

$$
\sigma_{\min} I \le P_{k+1|k}^{\gamma} \le \sigma_{\max} I, \tag{3.5}
$$

where $\sigma_{\min}$ and $\sigma_{\max}$ are positive constants with $0 < \sigma_{\min} < \sigma_{\max} < \infty$, and $I$ is the identity matrix. Let the dyadic expansion of $P_{k|k}^{\gamma}$ be given by

$$
P_{k|k}^{\gamma} = \sum_{i=1}^{n_u} \alpha_{k|k}^i e_k^i (e_k^i)^T, \tag{3.6}
$$

where $\alpha_{k|k}^1, ..., \alpha_{k|k}^{n_u}$ are the eigenvalues of $P_{k|k}^{\gamma}$ with $\alpha_{k|k}^1 \ge ... \ge \alpha_{k|k}^{n_u}$, and $e_k^1, ..., e_k^{n_u}$ are the corresponding eigenvectors with $\|e_k^1\| = ... = \|e_k^{n_u}\| = 1$. Equation (3.6) can then be expressed as

$$
P_{k+1|k}^{\gamma} = \sum_{i=1}^{n_u} \frac{\alpha_{k|k}^i}{\lambda_k^i} e_k^i (e_k^i)^T + Q_k^{\gamma}, \qquad 0 < \lambda_k^i \le 1.
$$

Following the argument in Ref. [11], the forgetting factor $\lambda_k^i$ can be chosen as a decreasing function of the amount of information received in the direction $e_k^i$. Since eigenvalue $\alpha_{k|k}^i$ of $P_{k|k}^{\gamma}$ is a measure of the uncertainty in the direction of $e_k^i$, a choice of forgetting factor $\lambda_k^i$ based on the above constraints can be

$$
\lambda_k^i = \begin{cases} \lambda_o, & \alpha_{k|k}^i > \alpha_{\max}, \\ \alpha_{k|k}^i \left[\alpha_{\min} + \frac{\alpha_{\max} - \alpha_{\min}}{\alpha_{\max}} \alpha_{k|k}^i\right]^{-1}, & \alpha_{k|k}^i \le \alpha_{\max}. \end{cases} \tag{3.7}
$$

The estimation algorithm discussed in this section will be seen to have been applied successfully to a HiMAT vehicle.

**4. Example.** In this section, the iteration process described in Section 2.2 is applied to control a HiMAT vehicle for actuator failure cases. Recall it is assumed that actuators are failed one at a time. Thus, the control reconfigurability of the HiMAT vehicle never goes to zero [10]. The system variations due to actuator failures can be modeled as an LPV system, a function of an estimated scheduling parameter.

**4.1. Linear Parameter Varying Model of HiMAT.** The model of the HiMAT vehicle taken from the $\mu$-synthesis Toolbox [3] has two inputs: elevons $\delta_e$ and canards $\delta_c$; two outputs: angle of attack $\alpha$ in radians and pitch angle $\theta$ in radians; and four states: velocity $V$ in ft/sec, angle of attack, pitch rate $q$ in rad/sec, and pitch angle. The open-loop model is

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}, \quad u = \begin{bmatrix} \delta_e \\ \delta_c \end{bmatrix}, \tag{4.1}$$

where

$$A = \begin{bmatrix} -0.0226 & -36.6 & -18.9 & -32.1 \\ 0 & -1.9 & 0.98 & 0 \\ 0.012 & -11.7 & -2.63 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ -0.414 & 0 \\ -77.8 & 22.4 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, D = 0_{2\times 2}. \tag{4.2}$$

A failure parameter vector $\tau = [\tau_1 \quad \tau_2]^T$ is determined by the two actuator effectiveness parameters $\tau_1$ and $\tau_2$ of elevon and canard actuators, respectively. Assume that the failure parameters linearly enter in the model. The state-space model of the HiMAT vehicle is written as

$$\dot{x} = Ax + B(\tau)u, \quad y = Cx,$$

where $A$ and $C$ are constant matrices and $B(\tau) = [b_1\tau_1 \quad b_2\tau_2]$. The vectors $b_1$ and $b_2$ are the columns of $B$. The actuator failure parameters can be estimated using the estimation method described in Section 3. However, there is estimation error $\delta_\tau = [\delta_{\tau_1} \quad \delta_{\tau_2}]^T$. The actuator failure parameter vector $\tau$ is written as

$$\tau = \bar{\tau} + \delta_\tau,$$

where $\bar{\tau}$ is an estimated value. The estimation error bound is assumed as $\sqrt{\delta_\tau^T \delta_\tau} \leq 0.05$ for each actuator failure case. The matrix $B(\tau)$ is rewritten as:

$$B(\tau) = B \begin{bmatrix} \bar{\tau}_1 + 0.05\delta_1 & 0 \\ 0 & \bar{\tau}_2 + 0.05\delta_2 \end{bmatrix}, \tag{4.3}$$

where the real uncertainty parameters $\delta_1$ and $\delta_2$ vary from -1 to 1, respectively.

The LPV model of HiMAT is

$$\dot{x} = Ax + B \begin{bmatrix} \bar{\tau}_1 & 0 \\ 0 & \bar{\tau}_2 \end{bmatrix} u + B \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix} w, \tag{4.4}$$

$$z = u, \quad y = Cx, \tag{4.5}$$

$$w = \Delta z, \quad \Delta = \begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix}. \tag{4.6}$$

The LPV model in equation (4.4) is a linear function of a parameter vector $\tau$. However, the parameter vector $\tau$ can not be chosen as a scheduling parameter since we consider that the actuators are failed one at a time. Thus, $\tau_1$ and $\tau_2$ can not be zero simultaneously. To describe the failure cases, a synthetic scheduling parameter $\bar{\rho}$ is introduced as $0 \leq \bar{\rho} \leq 2$.

$$\begin{array}{rcll} 0 \leq \bar{\rho} < 1 & : & 0 \leq \bar{\tau}_1 < 1, & \bar{\tau}_2 = 1 \\ \bar{\rho} = 1 & : & \bar{\tau}_1 = 1, & \bar{\tau}_2 = 1 \\ 1 < \bar{\rho} \leq 2 & : & \bar{\tau}_1 = 1, & 0 \leq \bar{\tau}_2 < 1 \end{array} \tag{4.7}$$

Note that the LPV model of the HiMAT vehicle is not an affine function of a scheduling parameter $\bar{\rho}$.

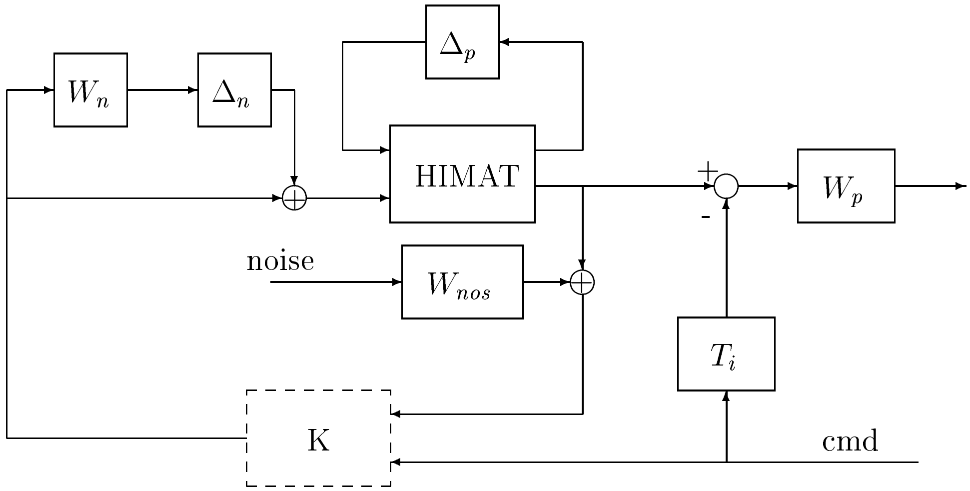


FIG. 4.1. *Interconnection structure for the model matching problem.*

**4.2. LPV Controller Design.** The control objective is to track a pitch angle command at actuator failure cases. A designed LPV controller should robustly stabilize the HiMAT vehicle over the failure parameter variations. The controller synthesis problem is formulated as a model matching problem in Figure 4.1.

The ideal response model $T_i$ of pitch angle is taken from the example in the $\mu$-synthesis Toolbox [3]. The performance weighting function $W_p$ and unmodeled dynamics $W_n$ are also taken from the example in the $\mu$-synthesis Toolbox [3]. The sensor noise is modeled as white noise with $0.6^\circ$ amplitude for angle of attack and pitch angle measurements. The weighting functions in Figure 4.1 are

$$\begin{aligned} T_i &= \frac{1}{\mathtt{s}/0.8+1}, \\ W_p &= 40\frac{\mathtt{s}/50+1}{\mathtt{s}/0.05+1}, \\ W_n &= 0.2\frac{\mathtt{s}/5+1}{\mathtt{s}/1000+1}I_{2\times 2}, \\ W_{nos} &= 0.01 I_{2\times 2}. \end{aligned}$$

The control synthesis problem of the HiMAT vehicle is formulated to minimize the induced-$\mathcal{L}_2$ norm of the augmented LPV system with the weighting functions.

To solve the control synthesis LMI optimization problem in equation (2.8), basis functions for $X(\bar{\rho})$ and $Y(\bar{\rho})$ are required since $X$ and $Y$ are assumed as functions of $\bar{\rho}$.

$$X(\bar{\rho}) = \sum_i f_i(\bar{\rho})X_i, \;\; X_i \in \mathcal{R}^{n\times n}, \quad Y(\bar{\rho}) = \sum_j g_j(\bar{\rho})Y_j, \;\; Y_j \in \mathcal{R}^{n\times n}. \tag{4.8}$$

where basis functions $f_i(\bar{\rho})$ and $g_j(\bar{\rho})$ are given before solving the LMI optimization in equation (2.8) over $X_i$ and $Y_j$. There is no analytic method to choose optimal basis functions for $X$ and $Y$ in general. The functions $f_i(\bar{\rho})$ and $g_j(\bar{\rho})$ are related with sensitivity of unknown matrices $X_i$ and $Y_i$, respectively. In this paper, the basis function set is defined as $\{1, 1/\bar{\rho}, \bar{\rho}\}$ for $X$ and $Y$ to help the LMI optimization for total failure cases ($\bar{\tau}_1 = 0, \bar{\tau}_2 = 0$). Note that it is not necessary to define that $g_j(\bar{\rho})$ is equal to $f_i(\bar{\rho})$. Since $X$ and $Y$ are functions of $\bar{\rho}$, the parameter rate bound is required to solve the LMI optimization in equation (2.8). Recall that the scheduling parameter is an actuator failure parameter. Thus, for example, the scheduling parameter can suddenly vary from 1 (no failure case) to 2 (total canard failure case). In this paper, the parameter rate bound is assumed as $|\dot{\bar{\rho}}| < 100$ to capture sudden variations of the scheduling parameter.

To make the LMI optimization computationally tractable, the LMI constraints are evaluated at the following grid points:

$$\bar{\rho} \in \{\bar{\rho}|0.01,\ 0.1,\ 0.2,\ \cdots,\ 1.9,\ 2\}.$$

TABLE 4.1
*$\gamma$ values in the LMI optimization in equation (2.8)*

| Iteration | $\gamma$ | $S$ |
|---|---|---|
| 1 | 1.23 | diag([1,1,1,1]) |
| 2 | 0.71 | diag([0.497, 0.168, 1.186, 1.277]) |
| 3 | 0.60 | diag([1.404, 1.289, 1.801, 1.453]) |
| 4 | 0.54 | diag([1.580, 2.193, 2.100, 1.187]) |
| 5 | 0.85 | diag([2.007, 1.430, 2.506, 1.849]) |

Also, the same grid points are used to solve the LMI optimization in equation (2.15). To solve the LMI optimization in equation (2.15), the basis function set for $P$ is required. Since the matrix $P$ is related with $X$ and $Y$, the basis function set for $P$ is chosen as $\{1, 1/\bar{\rho}, \bar{\rho}\}$.

In this paper, the scaling factor $S$ is assumed as constant over all scheduling parameter variations. The $\gamma$ values and the scaling factor $S$ for each iteration are written in Table 4.2. The scaling factor $S$ is associated with the uncertainty block $\Delta$ which is

$$\Delta = diag([\delta_1,\ \delta_2,\ \delta_{ele},\ \delta_{can}]).$$

Recall that the real uncertainty parameters $\delta_1$ and $\delta_2$ are associated with elevon and canard actuator failure parameters, respectively. The multiplicative uncertainty parameters $\delta_{ele}$ and $\delta_{can}$ are also associated with elevon and canard control channels in Figure 4.1.

The iteration process is stopped at the $5^{th}$ iteration since the $\gamma$ value at the iteration is greater than the previous iteration. Recall that the iteration process is not guaranteed to be converged. However, the performance index $\gamma$ in the LMI optimization of equation (2.8) is significantly reduced from 1.23 to 0.54 by using the scaling factor $S$. In the remainder of this paper, the LPV controller for the HiMAT vehicle denotes the designed LPV controller at the $4^{th}$ iteration.

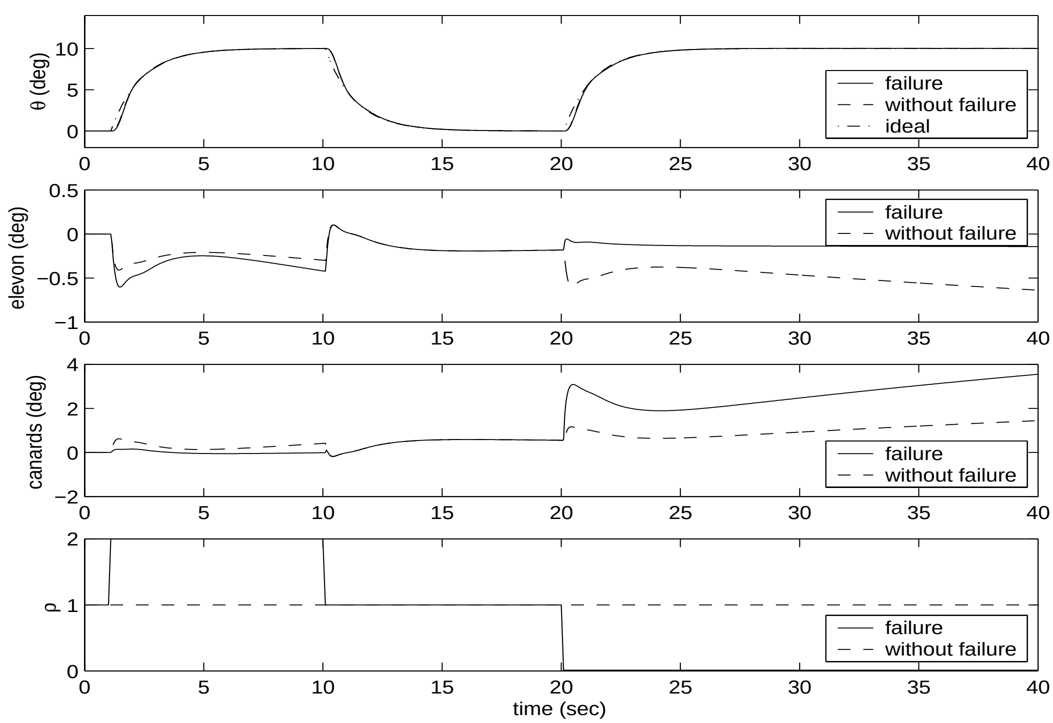


FIG. 5.1. *The LPV controller simulations with and without actuator failures.*

**5. Simulations.** In this section, the designed LPV controller is applied to control the HiMAT vehicle for pre-defined failure scenarios. One of the failure scenarios is that a total canard failure occurs 1 and 10 seconds and a total elevon failure occurs between 20 and 40 seconds. The scheduling parameter corresponding to the failure scenario is:

$$\rho = \begin{cases} 0.01, & 1 \le t < 10 \text{ sec}, \\ 1.00, & 0 \le t < 1 \text{ sec}, \qquad 10 \le t \le 20 \text{ sec} \\ 2.00, & 20 < t \le 40 \text{ sec}. \end{cases} \tag{5.1}$$

For the purpose of comparison, the LPV controller for the HiMAT vehicle is simulated both with and without actuator failures. The simulation results are shown in Figure 5.1. The pitch angle commands are given as 10° at 1 sec, 0° at 10 sec, and 10° at 20 sec, sequentially.

In this simulation, the scheduling parameter corresponding to the true failure parameters, as shown in the bottom of Figure 5.1 is fed into the LPV controller. It is observed that the LPV controller achieves the desired goal of tracking pitch commands in the presence of actuator failures. It can be seen from the second and the third plots in Figure 5.1 that the LPV controller always relies on the healthy actuator to track the pitch commands abandons the failed actuator. For example, the LPV controller keeps the elevon actuator signals close to zero at the elevon actuator failure case.

The LPV controller is also simulated for the same faulty system as described in equation (5.1) but with bounded real parameter perturbations: $\Delta_p = diag([\delta_1, \delta_2]), \quad ||\Delta_p|| \le 1$. Two examples of simulations with perturbations are shown in Figure 5.2. “pert1” and “pert2” in Figure 5.2 denote the cases $[\delta_1 \;\; \delta_2] = [1 \;\; 1]$ and

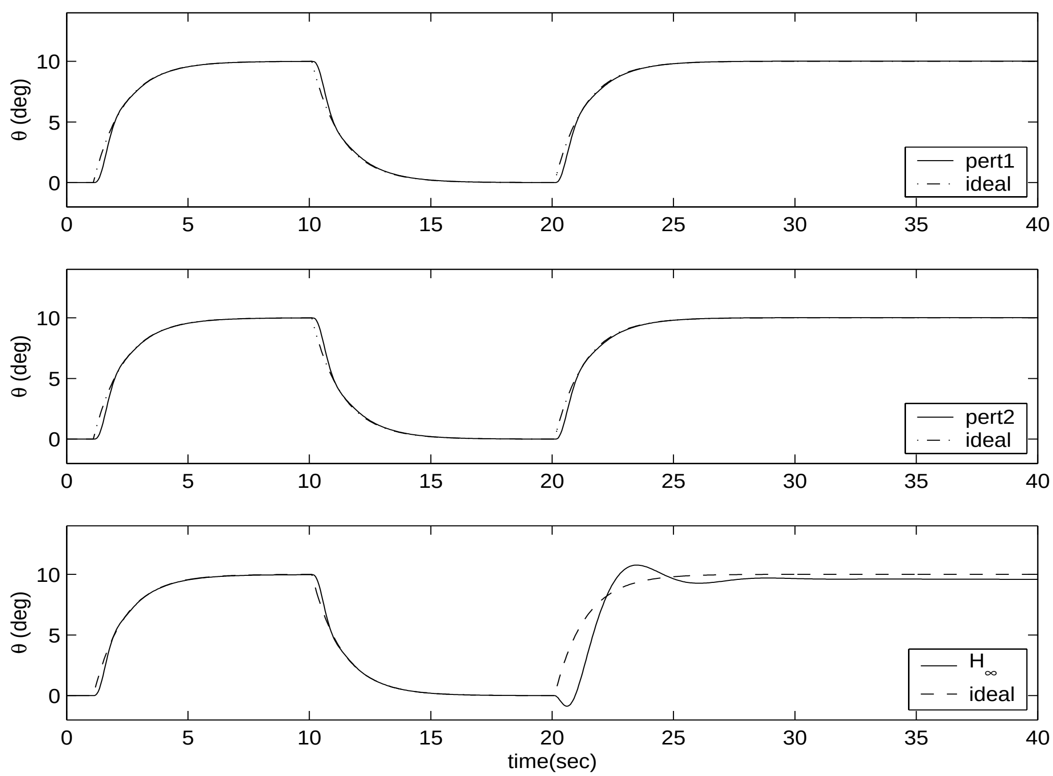


FIG. 5.2. *Time responses of pitch angle with LPV and $H_\infty$ controllers.*

$[\delta_1 \;\; \delta_2] = [-1 \;\; -1]$, respectively. The simulation results show that the LPV controller can robustly stabilize the perturbed system and achieve the desired performance level of tracking the pitch angle commands.

For the purpose of comparison, a fixed $H_\infty$ controller is designed at $\bar{\rho} = 1$ (without failures) using $\mu$-synthesis Toolbox [3] with the same weighting functions described in Section 4. The closed-loop response with this $H_\infty$ controller is also simulated for the actuator failure scenario of equation (5.1), and shown in the bottom plots of Figure 5.2. It can be seen that the $H_\infty$ controller can achieve the desired performance level when canard fails($1 \leq t < 10$ sec). The $H_\infty$ controller cannot, however, achieve the desired performance level at the elevon failure. This is consistent with the finding through reconfigurability calculation [10] that the canards are less effective in controlling the pitch movement than elevons, and that loss of eleovn effectiveness can significantly affect tracking the pitch commands.

The control signals of the LPV and $H_\infty$ controllers are plotted in Figure 5.3 for the same failure scenario. Since the control signals of the LPV controller are the same with or without perturbations, only simulation

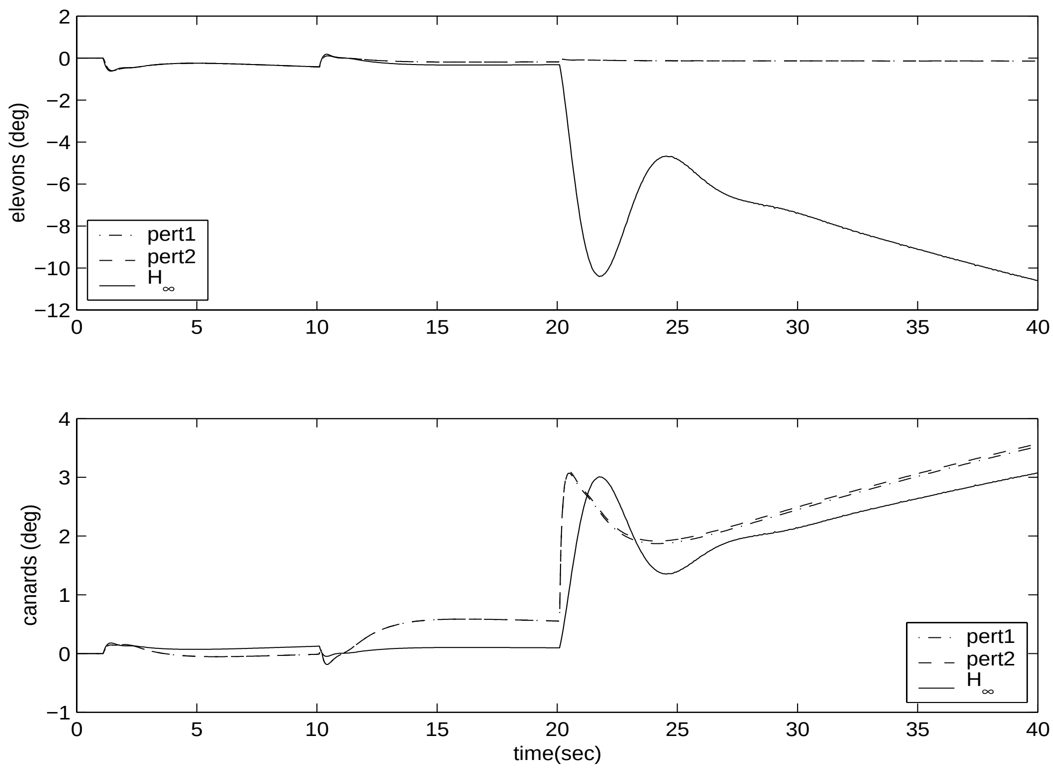


FIG. 5.3. *Control signals for each actuator with the LPV and $H_\infty$ controllers.*

results with "pert1" and "pert2" are plotted in Figure 5.3. It can be observed from Figure 5.3 that the elevon signals of the $H_\infty$ controller are significant despite the failure of the elevons, while the elevon signals from the LPV controller are insignificant and the canard signals are to compensate the elevon failure in Figure 5.3.

Now, the fault parameters are estimated with an on-line estimator that is integrated into the LPV controller as shown in Figure 5.4. The on-line estimator in Figure 5.4 has two parts: one is a two-stage discrete Kalman filter and the other carries out a simple logic of equation (4.7) that converts the bias estimates to the corresponding scheduling parameter estimate. The following set of parameter values are used in the the two-stage Kalman filter. Sampling time is set at 0.01 sec to capture the response details of the open-loop dynamics of the vehicle. The covariance matrices $Q_k^x$, $Q_k^\gamma$ and $R_k^\gamma$ described in Section 3 are set as constant matrices with values:

$$
\begin{aligned}
Q_k^x &= 3diag([1,\ \ 0.01^2,\ \ 0.01^2,\ \ 0.01^2]),\\
Q_k^\gamma &= 3diag([0.05^2,\ \ 0.05^2]),\\
R_k^\gamma &= 3diag([0.01^2,\ \ 0.01^2]).
\end{aligned}
$$

The covariance matrices affect the convergence of the estimator and the noise level considered in the control synthesis in Section 4. The initial values of estimated state $\hat{x}_{0|0}$ and biases $\hat{\gamma}_{0|0}$ are set as $[0\ \ 0\ \ 0\ \ 0]^T$ and $[0\ \ 0]^T$. The initial covariance matrices $P_{0|0}^\gamma$ and $\tilde{P}_{0|0}^x$ are set at $10I_2$ and $10I_4$. It is found that the estimates

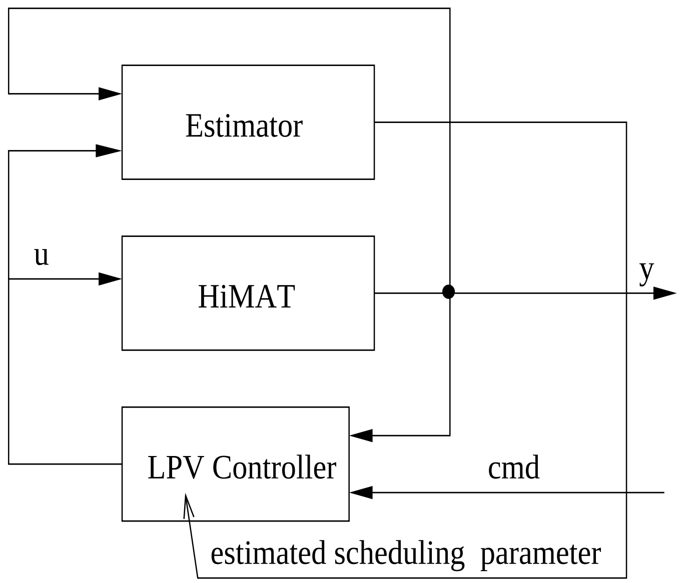


FIG. 5.4. *Simulation block diagram of a closed-loop system with an on-line estimator.*

TABLE 5.1
*Different sets of the varying forgetting factors*

| Case | $\lambda_0$ | $\alpha_{min}$ | $\alpha_{max}$ |
|---|---|---|---|
| 1 | 0.90 | 10 | 100 |
| 2 | 0.95 | 10 | 100 |
| 3 | 1.00 | 10 | 100 |
| 4 | 1.00 | $10^6$ | $10^7$ |

are sensitive to the selection to its initial values, but insensitive to the selection of its initial error covariances.

The most delicate part of the bias estimation lies with the selection of values for $\lambda_0$, $\alpha_{min}$ and $\alpha_{max}$ in equation (3.7). This is done by experiments and with little theoretical guidance. Different sets of values in Table 5 have been attempted. The fault parameter on-line estimate results are shown in Figure 5.5. It is obvious that bias estimation results vary in different cases studied. From the top plots of Figure 5.5, it is noticed that the value of $\lambda_o$ affect on convergence rate of estimation. When $\lambda_o$ is set as 1 at Case 3, the bias estimate is not convergent at canard failure situation. For this case, the two fault parameter estimates are strongly coupled that can cause false identification of faults. In Cases 1, 2 and 4, initial transients in estimates are visible, since the canards are less effective in controlling pitch angle. It is unknown how the control surface effectiveness is directly related with the transient behavior of the parameter estimator of the two-stage Kalman filter. For Case 4, the covariance matrix $P^\gamma$ is immediately high value after one step integration since $\alpha_{min}$ is defined as $10^6$. It is founded from the results of Case 4 that the high value of the covariance matrix $P_k^\gamma$ leads to good estimate of the scheduling parameter.

Simulation results at Case 1 are shown in Figure 5.6. “TV” and “VFF’ in Figure 5.6 denote that the LPV controller is evaluated at the true values (TV) of the failure parameters and at the estimated failure parameters with a variable forgetting factor (VFF), respectively. It is important to note that the LPV controller evaluated at the estimated parameter can achieve the desired performance of tracking the pitch commands. The difference in tracking performance between using the estimated parameter and the true parameter is very small at the steady state. The time delay and transient in the estimate has not formalized in the LPV control synthesis process. We currently rely on the robustness of the LPV controller.

The delay, though undesirable, is helpful in satisfying the rate bounds on the scheduling parameter, which is one of the assumptions of the LPV control synthesis methodology. Large delays in fault parameter estimates can be detrimental to the stability of a closed-loop system. This is a subject of future study.

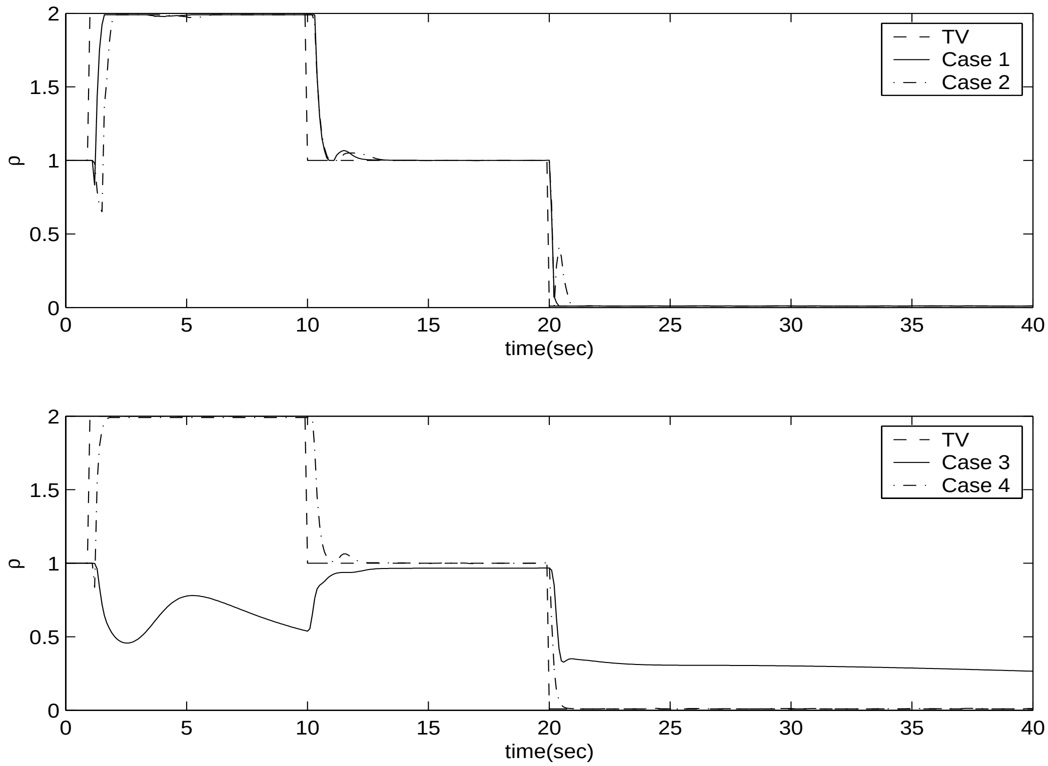


FIG. 5.5. *Scheduling parameter estimates for the cases in Table 5.*

**6. Conclusion.** In this paper, the LPV controller is designed based on the estimated scheduling parameter. The system variations due to actuator failures are modeled as functions of estimated parameters and bounded parameter estimation errors. An LPV controller synthesis problem with bounded parameter errors is formulated into two LMI optimizations: one is an LPV control synthesis problem with fixing a scaling factor on a uncertainty block and the other is to find a scaling factor given a control law. The optimization problem is solved by an iteration method. The performance level of the closed-loop system with the designed LPV controller is reduced by the iteration method.

The iteration approach of the LPV synthesis methodology is applied to control of the vehicle at actuator failure cases. It is assumed that the actuators are failed one at a time. Actuator failure parameters of the vehicle are estimated as biases using an augmented Kalman filter. The LPV controller evaluated at the estimated failure parameter is simulated with the plant model which varies as true values of the failure parameter. The simulation results show that the LPV controller achieves the desired performance level of tracking pitch angle commands for actuator failure cases and robustly stabilizes the vehicle.

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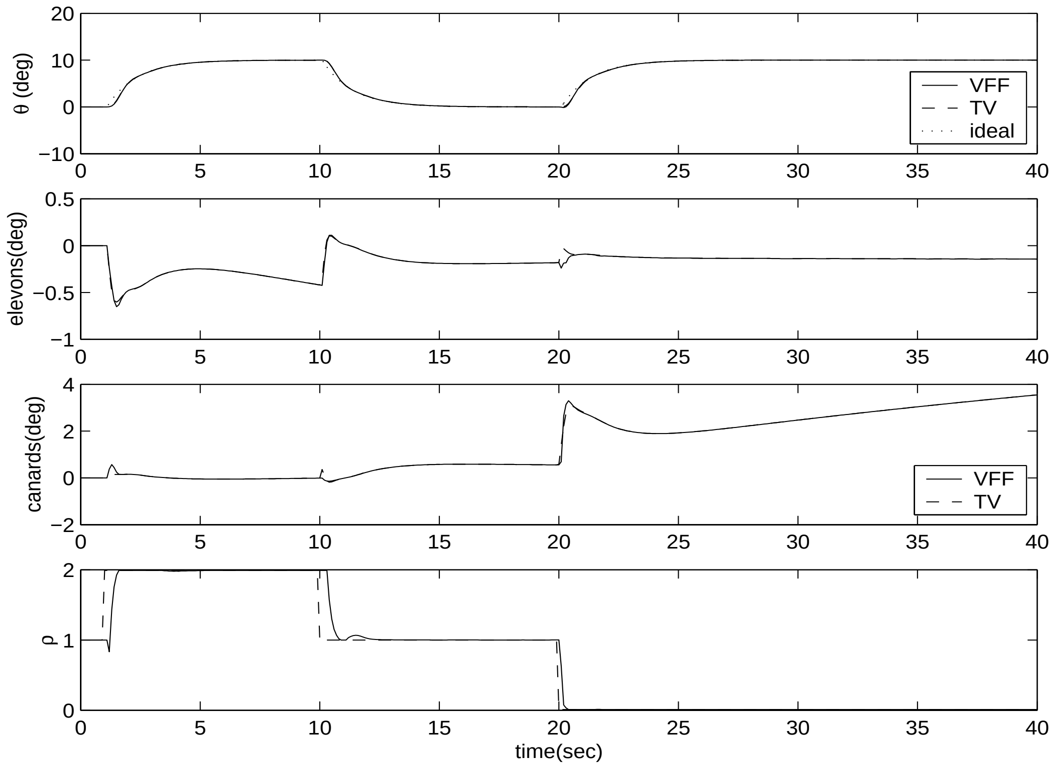


FIG. 5.6. *Simulations with the on-line estimator.*


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**13. ABSTRACT** *(Maximum 200 words)*
A robust linear parameter varying (LPV) control synthesis is carried out for an HiMAT vehicle subject to loss of control effectiveness. The scheduling parameter is selected to be a function of the estimates of the control effectiveness factors. The estimates are provided on-line by a two-stage Kalman estimator. The inherent conservatism of the LPV design is reducing through the use of a scaling factor on the uncertainty block that represents the estimation errors of the effectiveness factors. Simulations of the controlled system with the on-line estimator show that a superior fault-tolerance can be achieved.

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