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One of the Grand Challenges of the Federal High Performance Computing and Communications (HPCC) Program is in remote exploration and experimentation (REE). The goal of the REE Project is to develop a space-borne computing technology base that will enable the next generation of missions to explore the Earth and the Solar System. This paper discusses an ongoing study that uses a recent development in communication control technology to implement hybrid hypercube structures. These architectures are similar to binary hypercubes, but they also provide added connectivity between the processors. This added connectivity increases communication reliability while decreasing the latency of interprocessor message passing. Because these factors directly determine the speed that can be obtained by multiprocessor systems, these architectures are attractive for applications such as REE, where high performance and ultrareliability are required. This paper describes and enumerates these architectures and discusses how they can be implemented with a modified version of the hyperswitch communication network (HCN). The HCN is analyzed because it has three attractive features that enable these architectures to be effective: speed, fault tolerance, and the ability to pass multiple messages simultaneously through the same hyperswitch controller.

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Abstract

One of the Grand Challenges of the Federal High Performance Computing and Communications (HPCC) Program is in remote exploration and experimentation (REE). The goal of the REE Project is to develop a space-borne computing technology base that will enable the next generation of missions to explore the Earth and the Solar System. This paper discusses an ongoing study that uses a recent development in communication control technology to implement hybrid hypercube structures. These architectures are similar to binary hypercubes, but they also provide added connectivity between the processors. This added connectivity increases communication reliability while decreasing the latency of interprocessor message passing. Because these factors directly determine the speed that can be obtained by multiprocessor systems, these architectures are attractive for applications such as REE, where high performance and ultrareliability are required. This paper describes and enumerates these architectures and discusses how they can be implemented with a modified version of the hyperswitch communication network (HCN). The HCN is analyzed because it has three attractive features that enable these architectures to be effective: speed, fault tolerance, and the ability to pass multiple messages simultaneously through the same hyperswitch controller.

1. Introduction

One of the Grand Challenges of the Federal High Performance Computing and Communications (HPCC) Program is in the area of remote exploration and experimentation (REE). The goal of the REE Project is to develop a space-borne computing technology base that will enable high-performance, fault-tolerant, adaptive space systems for a new generation of missions to explore the Earth and the Solar System. The specific objectives of the REE Project are to demonstrate that a thousandfold increase in performance is feasible and to identify a parallel, scalable architecture that can incorporate new technologies to meet a broad range of requirements. As described in The Remote Exploration and Experimentation Project Plan by the Jet Propulsion Laboratory, the architecture must also provide affordable fault tolerance and long-term reliability in an environment of limited power and weight, high radiation, and no maintainability. To meet these objectives, new architectures must be investigated with consideration given to REE-type applications.

This paper discusses an ongoing study that attempts to use a recent development in hypercube communications control technology, the hyperswitch communication network (HCN) chip set (ref. 1), to implement a variety of generalized and hybrid hypercube architectures. These architectures are similar to binary hypercubes; but they also provide added connectivity between the processors. This added connectivity increases communication reliability while decreasing the latency incurred when passing messages between processors. Because these factors directly determine the speed that can be obtained with multiprocessor systems, these architectures are attractive for applications such as REE, where high performance and ultrareliability are required.

This paper describes and enumerates these architectures and discusses how they can be implemented with a modified version of the HCN chip set developed at the Jet Propulsion Laboratory. The HCN chip set is analyzed here because it has three attractive features that enable these architectures to be effective: speed, fault tolerance, and ability to pass multiple messages simultaneously through the same hyperswitch controller.

This paper is organized as follows. Section 2 describes generalized interconnection networks: both their organization and their relation to binary hypercube implementations. Expressions are given for the number of links, the number of disjoint paths between nodes, and other characteristic indices. Section 3 describes the hyperswitch communication network chip set: both its capabilities and its limitations. Section 4 describes and enumerates the possible generalized hypercubes that become feasible when hyperswitch technology is used in the network input/output (I/O) elements. Section 5 describes how the HCN chips can be modified to implement these architectures. Section 6 presents the benefits of these networks when used for multiple instruction–multiple data (MIMD) architectures and how these networks can be used to increase system performance and reliability.

2. Generalized Hypercube Topologies

References 2 and 3 define and discuss generalized hypercubes. This class of architectures can best be described by the following definition. For any multi-dimensional grid with vertices at any point where two or more lines on the grid intersect, a generalized hypercube is defined by interconnecting, with point-to-point links, all vertices in the same dimension (fig. 1). The graph vertices represent the processing nodes of the hypercube, while the interconnection links represent bidirectional I/O channels between these nodes.

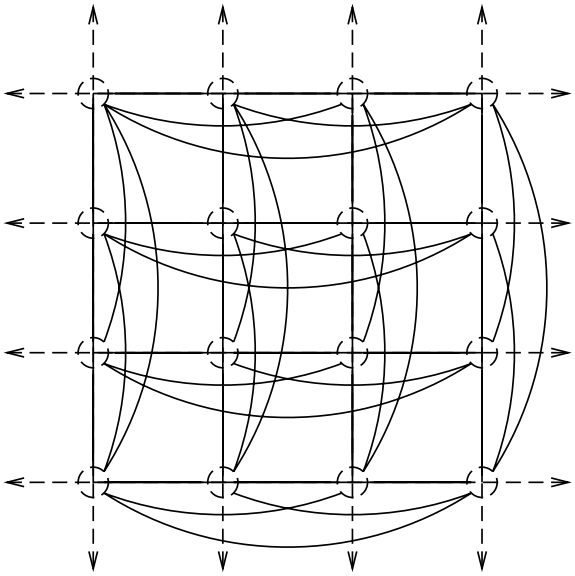

Figure 1. Two-dimensional generalized hypercube.

Most commercial hypercubes available today are binary hypercubes (e.g., iPSC/2, NCUBE, and Mark III). These systems represent a subset of generalized hypercubes where the number of nodes per dimension is limited to two. For generalized hypercubes, the number of nodes per dimension is limited only by the number of I/O ports per node, as subsequently discussed. The address and interconnection schema is characterized by the following. Let

$$(R_1, R_2, R_3, R_4, R_5, ..., R_d)$$

be the representation of a generalized hypercube configuration, where R_i represents the number of nodes in dimension i. The total number of nodes in the network is

$$N = \prod_{i=1}^{d} R_i \qquad (1)$$

The number of dimensions in the cube is d. Furthermore, node addresses are defined by

$$A_1 A_2 A_3 A_4 A_5, ..., A_d$$

where A_i is represented in base R_i. Interconnection links exist between any two nodes whose addresses differ in one digit position (i.e., any two nodes in the same dimension). For example, consider the configuration (2,3,4). For this generalized hypercube, $N = 24$ nodes exist in a $d =$ Three-dimensional space. The node addresses are shown in table 1.

Table 1. Node Addresses for (2,3,4) Generalized Hypercube

Node	Address	Node	Address	Node	Address	Node	Address
0	000	6	012	12	100	18	112
1	001	7	013	13	101	19	113
2	002	8	020	14	102	20	120
3	003	9	021	15	103	21	121
4	010	10	022	16	110	22	122
5	011	11	023	17	111	23	123

Figure 2 shows the architecture and node addresses of the (2,3,4) generalized hypercube. All nodes that are connected to each other have addresses that differ in only one digit position.

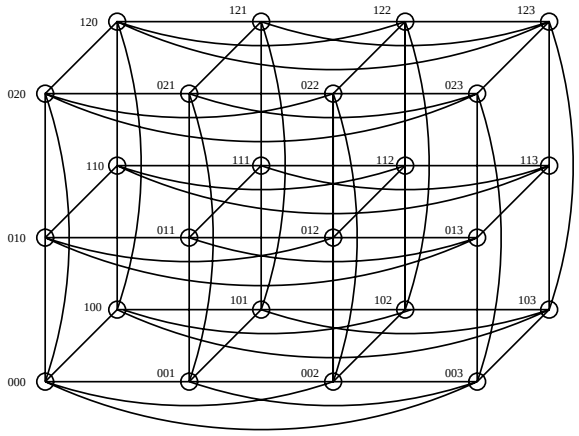

Figure 2. (2,3,4) Generalized hypercube.

Binary hypercubes are those in which $R_i = 2$ for all i in the generalized hypercube representation defined previously (e.g., (2,2,2,2) is a 16-node binary hypercube). Hyperrectangles can be formed by interconnecting only those nodes whose addresses differ in one digit position, and this digit differs only by one in the two addresses. Hybrid hypercubes can also be constructed by varying the degree of connectivity in each dimension. Examples of these architectures are shown in figures 3 and 4.

Important indices of hypercube architectures are the total number of links in the network, the hamming distance between any two nodes, and the number of disjoint paths between any two nodes in the network. The total number of links in the network has significant ramifications on the implementation cost. The hamming distance between any two nodes provides minimum message latency bounds, and the number of disjoint paths between any two nodes in the network helps characterize the fault tolerance capabilities of the communication mechanisms.

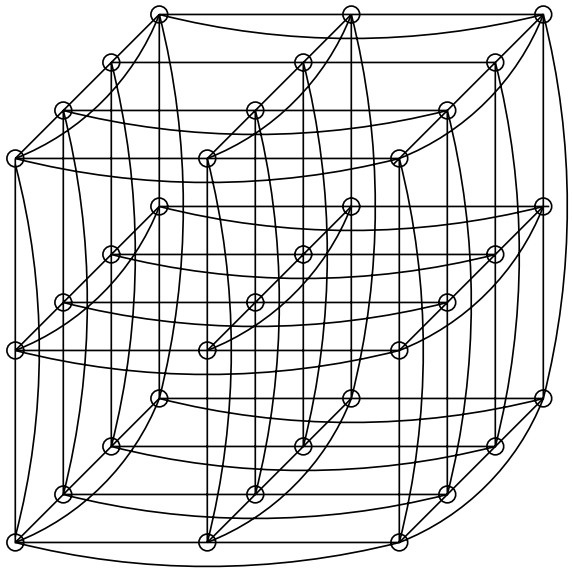

Figure 3. (4,3,3) hyperrectangle.

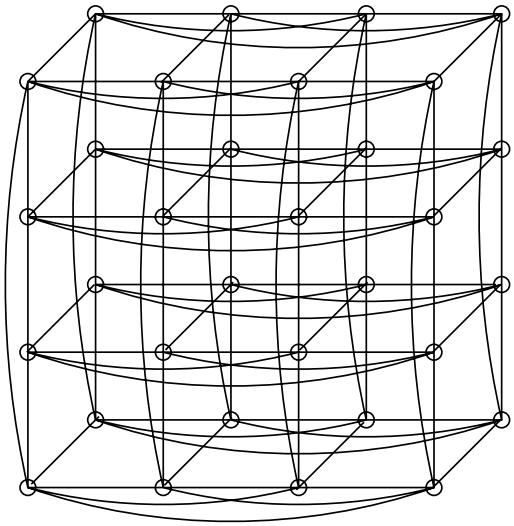

Figure 4. (2,4,4) hybrid hypercube.

For generalized hypercubes (i.e., full connectivity in every dimension) the number of bidirectional links L is

$$L = \frac{N}{2}\sum_{i=1}^{d} R_i - 1 \qquad (2)$$

The hamming distance $H_{x,y}$ is equal to the number of differing digits in the addresses of node x and node y. The number of disjoint paths $DP_{x,y}$ is the minimum number of I/O ports on node x or node y. The number of I/O ports required per node P is

$$P = \sum_{i=1}^{d} R_i - 1 \qquad (3)$$

For example, table 2 shows the indices of the (2,2,2,2,2,2) binary hypercube and the (4,4,4) generalized hypercube. Both have $N = 64$ processing elements.

Table 2. 64-Node Hypercube Comparison

Index	Binary	Generalized
L	192	288
P	6	9
$H_{x,y}(\max)$	6	3
$DP_{x,y}(\max)$	6	9

These values suggest that the link cost for the (4,4,4) generalized hypercube is 1.5 times more than for the (2,2,2,2,2,2) binary hypercube; however, the (4,4,4) generalized hypercube provides increased communication reliability because it has more paths from source to destination that can be explored. In fact, for a fixed N, the total number of paths (not just disjoint) from node x to node y always increases as the number of dimensions decreases. This observation implies that for generalized hypercubes, the smaller the diameter of the cube, the more reliably communication can be performed because as links fail or become busy in the network, more alternate routes exist for messages to take. Furthermore, the average message latency is decreased because of this increased number of message routes and the decreased average hamming distance that messages must travel.

3. Hyperswitch Communication Network

The hyperswitch architecture was designed as a message-passing communication architecture for fine-grain MIMD computation in concurrent computers (ref. 4). By attaching a hyperswitch to each node in hypercube-like systems and using it as the network I/O element, we remove the burden of interprocessor communication from the application processor. The hyperswitch is used because of three attractive

features: the ability to pass multiple messages simultaneously through the same hyperswitch (up to 11), the ability to reroute around busy channels and most importantly, the ability to reroute these messages quickly (less than 200 μsec for 512 byte messages).

The hyperswitch chip set (HSP) (fig. 5) consists of a custom hyperswitch (crossbar) element (HS), a hyperswitch I/O element (HSIO), and a message dispatch processor element (DP) (ref. 5). The HSP interfaces with other HSP's through 11 bidirectional channels (Ch0 to Ch10). These chips were designed specifically to provide fast dynamic circuit- and packet-switching capabilities in binary hypercube architectures.

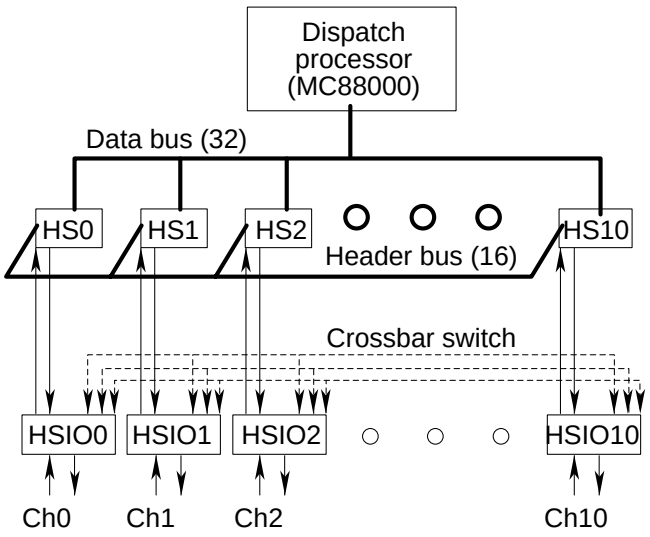

Figure 5. Hyperswitch processor.

In circuit-switching mode, the HSP establishes a path from source to destination before message transmission. This path is established by emitting a circuit probe (1 to 4 bytes) from the source node. The probe contains the destination node address, message length information, distance information, and some history information in case backtracking is required to establish the virtual link. The probe is then sent through intermediate nodes to the destination and the virtual link is established. At this time, the message itself can be transmitted across the virtual link at a rate equal to the link bandwidth. For circuit-switching mode, the message transmission latency T_{ckt} is

$$T_{\mathrm{ckt}} = (S_{\mathrm{probe}} H B_{\mathrm{link}}) + (S_{\mathrm{msg}} B_{\mathrm{link}}) \qquad (4)$$

where S_{probe} is the size of the probe, H is the number of hops in the virtual link, B_{link} is the bandwidth of the links, and S_{msg} is the size of the message.

In packet-switching mode, the HSP passes an entire message as a packet or set of packets, just as it passes a probe in circuit-switching mode. For packet-switching mode, the message transmission latency T_{pkt} is

$$T_{\mathrm{pkt}} = S_{\mathrm{pkt}} N H B_{\mathrm{link}} \qquad (5)$$

where S_{pkt} is the size of each packet, and N is the number of packets required to send the entire message.

In busy networks, both equations (4) and (5) must be appended to include the effects of encountering busy or failed links when establishing a path from source to destination. When a busy or failed link is encountered, one of three options is available: buffer the message until the link becomes available, drop the transaction and try again at a later time, or detour around the link. Each of these options increases the overall message latency.

Each HSP has 11 hyperswitch elements that act as the I/O ports for each node in the hypercube. Therefore, for binary hypercubes, the maximum number of nodes is 2^{11} (2048) because only one port is needed for each dimension. For nonbinary (e.g., generalized) hypercubes, a slightly different interpretation is discussed in section 4. For each hyperswitch, an HSIO performs the parallel-to-serial–serial-to-parallel conversion of the 8-bit data that travel between the hyperswitch and serial links that connect to neighboring HSP's (up to 11 serial links connect every node).

The DP is a Motorola MC88000 32-bit reduced instruction set computer (RISC), which can provide 17 million instructions per second. The DP performs transfers to and from system memory and acts as the interface between the HSP and the application processor. This processor also controls all crossbar settings in the hyperswitches of the HSP when establishing paths from source to destination during message transmission. The DP can act as the application processor as well.

Message routing latency is reduced with an adaptive backtracking algorithm implemented in the DP. This algorithm automatically avoids congested links based on its current knowledge of congestion in the network. When a message encounters a busy link, it does not wait for the link to become idle; instead, it tries to reach the destination by backtracking to the previous intermediate node and departing from another port. Virtual links between nodes are established by the switching elements in the HSP's of each node. This dynamic routing method has been shown to significantly reduce message routing overhead as well as increase the communication reliability

because of the ability to backtrack and avoid busy or faulty network links (ref. 4).

4. Generalized Structures and the HCN

Using an HSP as the I/O controller at each node of a generalized hypercube architecture allows a wide variety of configurations to be implemented. As discussed previously, each HSP has 11 I/O ports that can be used to interconnect a number of processing sites. The chip set specification denotes that one of these ports should be used for diagnostic purposes; that is, it should be connected to itself and periodically have test data run through the port. The other 10 ports are then free to be interconnected to the HSP's of other nodes in the system.

Therefore, we can now calculate the number of possible generalized hypercube architectures that can be constructed with a maximum of 10 ports per node. This number equals the number of unique integer partitions of 10 as well as any integer less than 10. An integer partition of an integer r is the division of r into a number of integers whose sum is r. Thus, the list of generalized hypercubes that can be implemented with the hyperswitch can be represented by any set of integers whose sum is less than or equal to 10. For example, the partition $\{2,2,3,3\}$ is an integer partition of 10. The corresponding four-dimensional generalized hypercube is a (3,3,4,4) configuration consisting of 144 nodes. The integers in the partition correspond to the number of ports required in each dimension.

From reference 6, the number of unique integer partitions of a number r is obtained from the coefficient of x^r in the following generating function:

$$G(x) = \prod_{m=1}^{\infty} \sum_{k=0}^{\infty} x^{km} \tag{6}$$

Specifically, for $r \leq 10$,

$$\begin{aligned} G(x) = {} & (1 + x + x^2 + ... + x^8 + x^9 + x^{10}) \\ & \times (1 + x^2 + x^4 + x^6 + x^8 + x^{10}) \\ & \times (1 + x^3 + x^6 + x^9)(1 + x^4 + x^8) \\ & \times (1 + x^5 + x^{10})(1 + x^6)(1 + x^7) \\ & \times (1 + x^8)(1 + x^9)(1 + x^{10}) \end{aligned} \tag{7}$$

or

$$\begin{aligned} G(x) = {} & 1 + x + 2x^2 + 3x^3 + 5x^4 + 7x^5 \\ & + 11x^6 + 15x^7 + 22x^8 + 30x^9 + 42x^{10} \end{aligned} \tag{8}$$

Where in equations (7) and (8), all terms with powers larger than 10 have been eliminated, because 10 is the maximum r we are interested in for this example. Furthermore, the generating function in equation (8) indicates the number of possible architectures with respect to the number of ports required per node (table 3). Finally, we can calculate the total number of generalized hypercube architectures possible by simply adding the coefficients of equation (8) as follows:

$$1 + 1 + 2 + 3 + 5 + 7 + 11 + 15 + 22 + 30 + 42 = 139$$

Table 3. Possible Generalized Hypercubes

Number of ports/node	0	1	2	3	4	5	6	7	8	9	10
Number of architectures	1	1	2	3	5	7	11	15	22	30	42

These architectures are listed in the appendix (with the exception of the trivial architecture that has 0 ports per node) and grouped according to the number of dimensions. The one-dimensional architectures in the appendix represent the fully connected systems that can be implemented. In addition to the list in the appendix, a large number of hyperrectangular and hybrid hypercubes can be constructed. Again, the only constraint imposed is the number of I/O ports required per node.

Architectures can now be chosen based on the characteristics of the application. For example, consider an application with three distinct distributed components: A, B, and C. Each component has increasing levels of communication bandwidth requirements. Choose a three-dimensional architecture with the processors in dimension 1 connected in a ring, processors in dimension 2 connected in a mesh, and processors in dimension 3 fully connected. Finally, map component A onto the processors in dimension 1, component B onto the processors in dimension 2, and component C onto the processors in dimension 3. Choosing the number of processors in each dimension now depends on the amount of parallelism inherent in the corresponding distributed components of the application.

5. Modifying HSP Element

To implement generalized hypercubes with the hyperswitch network element (fig. 5), two issues must be addressed. The first issue relates to the header information within the probes and message packets. The second issue requires changes in the coding of the DP as well as any hardwired functions pertaining to the architecture being configured (neighbor addresses) and the routing algorithm used.

Depending on which of seven modes of operation are enabled, the header (probe) word format consists of one to four 32-bit words. Reference 5 describes each of these modes in detail. The basic format consists of 32 bits (b_0, ..., b_{31}). Bits b_0 and b_{16} decode the mode; bits b_{12}, ..., b_{15} indicate the distance between the source and destination or further decode the mode; bits b_1, ..., b_{11} designate the identification number (ID) or the pseudo-ID number of the destination node; bits b_{17}, ..., b_{27} can be the destination node ID; and bits b_{28}, ..., b_{31} can be the length of the message or the communication channel number. Again, the format depends on the mode.

Thus, the only necessary change in the header is any additional bits that might be required to represent destination addresses in the generalized hypercube architectures. The number of bits B_b required to represent binary hypercube node addresses is

$$B_b = \lceil \log_2(N) \rceil \tag{9}$$

where N is the number of nodes in the network. With 11 bits allotted for the destination address, the current header format allows up to 2^{11} (2048) addressable nodes.

The number of bits B_g required to represent generalized hypercube node addresses is

$$B_g = \sum_{i=1}^{d} \lceil \log_2(R_i) \rceil \tag{10}$$

or, in terms of the number of I/O ports in each dimension P_d

$$B_g = \sum_{i=1}^{d} \lceil \log_2(P_d + 1) \rceil \tag{11}$$

Therefore, to maintain an identical header format (i.e., 11 bits represent the destination address), the total number of ports used per node is limited. The appendix lists B_g for each possible configuration. Because B_g never exceeds 10, no additional bits are needed in the header.

The four distance bits b_{12}, ..., b_{15} can represent a maximum hamming distance of 16 hops between processors. The generalized hypercubes in the appendix each have a maximum hamming distance equal to the number of dimensions d which is less than 11 for all configurations. Thus, no additional bits are required to represent distance.

To take full advantage of the additional connectivity provided by generalized hypercubes, modifications to the coding (both code and hardwired functions) of the hyperswitch are required to change the routing protocol. These modifications reflect the addressing schema used by the generalized hypercube, the mapping of I/O ports to neighboring nodes in the architecture, and the routing decisions made as messages pass through intermediate hyperswitches.

It is apparent that for generalized hypercube implementations with 2^n nodes, the hyperswitch can be used with no modifications to the current routing protocols (i.e., the $k(k$ - 1) family of backtracking protocols); however, as is, this protocol does not take advantage of all redundant paths provided by the generalized structure because it only tries one output port per node before backtracking to the previous node in the path (ref. 4). This observation suggests that a more adaptive protocol can be encoded that accesses these redundant paths to ensure minimum transfer latency and maximum communication reliability. Reference 7 describes three such adaptive protocols and presents performance data that reveal their effectiveness.

6. Concluding Remarks

This paper shows the feasibility of implementing a variety of generalized hypercube architectures with a modified version of the hyperswitch as the network I/O controller. By partitioning the 10 available I/O ports on the hyperswitch, a multidimensional architecture can be constructed. Depending on the amount of communication bandwidth required by the nodes in the dimension, the connectivity within each dimension can vary from fully connected (generalized) to a ring (hyperrectangular). The number of conceivable architectures that can be constructed depends on this intradimensional connectivity as well as the number of dimensions in the architecture.

Modifications to the hyperswitch are required to define the set of neighboring nodes and the output ports associated with them. Also, the hyperswitch should be reprogrammed to implement a fully adaptive routing protocol that can take full advantage of the additional connectivity provided by the generalized structure. Defining such protocols is an area of ongoing work.

This study will continue by developing simulation models of these architectures to fully characterize their behavior under various loads similar to those envisioned under the Remote Exploration and Experimentation (REE) Project. These models include remote sensing, data compression, and real-time autonomous control. Also, reliability analysis of these structures has begun to determine the benefit

of added connectivity in multidimensional MIMD machines.

Interprocessor communication delay and reliability is directly proportional to the speed that can be obtained in parallel systems. Thus, these structures provide a viable alternative to current binary hypercube implementations through not only their increased connectivity but also their ability to use this connectivity to maintain communication in busy networks by rerouting around busy or faulty network links.

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Appendix

Generalized Hypercubes With the HCN

Tables A1 to A10 list the generalized hypercubes that can be implemented with a modified version of the hyperswitch communication network (HCN). Architectures are described by the generalized hypercube representation (which conveys the number of nodes in each dimension and the number of dimensions d), the number of I/O ports required for each node P, the number of bits required to represent the node addresses B_g, and the total number of nodes in the topology N.

Table A1. Ten-Dimensional Generalized Hypercubes

Configuration	P	B_g	N
2,2,2,2,2,2,2,2,2,2	10	10	1024

Table A2. Nine-Dimensional Generalized Hypercubes

Configuration	P	B_g	N
2,2,2,2,2,2,2,2,2	9	9	512
2,2,2,2,2,2,2,2,3	10	10	768

Table A3. Eight-Dimensional Generalized Hypercubes

Configuration	P	B_g	N
2,2,2,2,2,2,2,2	8	8	256
2,2,2,2,2,2,2,3	9	9	384
2,2,2,2,2,2,3,3	10	10	576
2,2,2,2,2,2,2,4	10	10	512

Table A4. Seven-Dimensional Generalized Hypercubes

Configuration	P	B_g	N
2,2,2,2,2,2,2	7	7	128
2,2,2,2,2,2,3	8	8	192
2,2,2,2,2,3,3	9	9	288
2,2,2,2,2,2,4	9	8	256
2,2,2,2,3,3,3	10	10	432
2,2,2,2,2,3,4	10	9	384
2,2,2,2,2,2,5	10	9	320

Table A5. Six-Dimensional Generalized Hypercubes

Configuration	P	B_g	N
2,2,2,2,2,2	6	6	64
2,2,2,2,2,3	7	7	96
2,2,2,2,3,3	8	8	144
2,2,2,2,2,4	8	7	128
2,2,2,3,3,3	9	9	216
2,2,2,2,3,4	9	8	192
2,2,2,2,2,5	9	8	160
2,2,3,3,3,3	10	10	324
2,2,2,3,3,4	10	9	288
2,2,2,2,4,4	10	8	256
2,2,2,2,3,5	10	9	240
2,2,2,2,2,6	10	8	192

Table A6. Five-Dimensional Generalized Hypercubes

Configuration	P	B_g	N
2,2,2,2,2	5	5	32
2,2,2,2,3	6	6	48
2,2,2,3,3	7	7	72
2,2,2,2,4	7	6	64
2,2,3,3,3	8	8	108
2,2,2,3,4	8	7	96
2,2,2,2,5	8	7	80
2,3,3,3,3	9	9	162
2,2,3,3,4	9	8	144
2,2,2,4,4	9	7	128
2,2,2,3,5	9	8	120
2,2,2,2,6	9	7	96
3,3,3,3,3	10	10	243
2,3,3,3,4	10	9	216
2,2,3,4,4	10	8	192
2,2,3,3,5	10	9	180
2,2,2,4,5	10	8	160
2,2,2,3,6	10	8	144
2,2,2,2,7	10	7	112

Table A7. Four-Dimensional Generalized Hypercubes

Configuration	P	B_g	N
2,2,2,2	4	4	16
2,2,2,3	5	5	24
2,2,3,3	6	6	36
2,2,2,4	6	5	32
2,3,3,3	7	7	54
2,2,3,4	7	6	48
2,2,2,5	7	6	40
3,3,3,3	8	8	81
2,3,3,4	8	7	72
2,2,4,4	8	6	64
2,2,3,5	8	7	60
2,2,4,5	9	7	80
2,2,3,6	9	7	72
2,2,2,7	9	6	56
3,3,4,4	10	8	144
3,3,3,5	10	9	135
2,4,4,4	10	7	128
2,3,4,5	10	8	120
2,3,3,6	10	8	108
2,2,5,5	10	8	100
2,2,4,6	10	7	96
2,2,3,7	10	7	84
2,2,2,8	10	6	64

Table A8. Three-Dimensional Generalized Hypercubes

Configuration	P	B_g	N
2,2,2	3	3	8
2,2,3	4	4	12
2,3,3	5	5	18
2,2,4	5	4	16
3,3,3	6	6	27
2,3,4	6	5	24
2,2,5	6	5	20
3,3,4	7	6	36
2,4,4	7	5	32
2,3,5	7	6	30
2,2,6	7	5	24
3,4,4	8	6	48
3,3,5	8	7	45
2,4,5	8	6	40
2,3,6	8	6	36
2,2,7	8	5	28
4,4,4	9	6	64
3,4,5	9	7	60
3,3,6	9	7	54
2,5,5	9	7	50
2,4,6	9	6	48
2,3,7	9	6	42
2,2,8	9	5	32
4,4,5	10	7	80
3,5,5	10	7	75
3,4,6	10	7	72
3,3,7	10	7	63
2,5,6	10	7	60
2,4,7	10	6	56
2,3,8	10	6	48
2,2,9	10	6	36

Table A9. Two-Dimensional Generalized Hypercubes

Configuration	P	B_g	N
2,2	2	2	4
2,3	3	3	6
3,3	4	4	9
2,4	4	3	8
3,4	5	4	12
2,5	5	4	10
4,4	6	4	16
3,5	6	5	15
2,6	6	4	12
4,5	7	5	20
3,6	7	5	18
2,7	7	4	14
5,5	8	6	25
4,6	8	5	24
3,7	8	5	21
2,8	8	4	16
5,6	9	6	30
4,7	9	5	28
3,8	9	5	24
2,9	9	5	18
6,6	10	6	36
5,7	10	6	35
4,8	10	5	32
3,9	10	6	27
2,10	10	5	20

Table A10. One-Dimensional Generalized Hypercubes

Configuration	P	B_g	N
11	10	4	11
10	9	4	10
9	8	4	9
8	7	3	8
7	6	3	7
6	5	3	6
5	4	3	5
4	3	2	4
3	2	2	3
2	1	1	2

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