A more complete Example

$f(x) = \sqrt{1+x} - \sqrt{x}$ for $x \geq 0$ $|x| \approx 0$

$$f'(x) = \frac{d}{dx}\left(\sqrt{1+x} - \sqrt{x}\right)$$

$$= \frac{d}{dx}\left(\sqrt{1+x}\right) - \frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{x}}$$

$$= \frac{\sqrt{x}}{2\sqrt{x}\sqrt{1+x}} - \frac{\sqrt{1+x}}{2\sqrt{x}\sqrt{1+x}}$$

$$= -\frac{\left(\sqrt{1+x} - \sqrt{x}\right)}{2\sqrt{x}\sqrt{1+x}}$$

$$(\text{cond} f)(x) = \left|\frac{x f'(x)}{f(x)}\right|$$

$$= \left|\frac{x(-1)\cancel{\left(\sqrt{1+x} - \sqrt{x}\right)}}{2\sqrt{x}\sqrt{1+x}} \cdot \frac{1}{\cancel{\sqrt{1+x} - \sqrt{x}}}\right|$$

$$= \left|\frac{-x}{2\sqrt{x}\sqrt{1+x}}\right| \qquad x \geq 0$$

$$= \left|\frac{-\sqrt{x}}{2\sqrt{1+x}}\right|$$

$$= \frac{\sqrt{x}}{2\sqrt{1+x}}$$

3 cases

$$(\text{Cond } f)(x) = \frac{1}{2} \cdot \frac{\sqrt{x}}{\sqrt{1+x}} \qquad x \geq 0 \quad x \in \mathbb{R}$$

<u>case $x >> 0$</u> $\quad (x \to \infty)$

$$\lim_{x \to \infty} \frac{1}{2} \frac{\sqrt{x}}{\sqrt{1+x}} \approx \frac{1}{2} \lim_{x \to \infty} \frac{\sqrt{x}}{\sqrt{x}} \approx \frac{1}{2}(1) = \frac{1}{2}$$

$(\text{cond } f)(x) \approx \frac{1}{2}$ for $x \to \infty$

<u>case $x \approx 0$</u>

$$(\text{cond } f)(x) = \frac{1}{2} \frac{\sqrt{x}}{\sqrt{1+x}} \approx \frac{\sqrt{x}}{2} \approx 0$$

<u>case $x = 0$</u>

$(\text{cond } f)(x) = 0$ $\qquad$ Verify