

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{d}{dx} \sqrt{x} = \frac{d}{dx} x^{\frac{1}{2}}$$

$$= \frac{1}{2} x^{-\frac{1}{2}}$$

$$(\text{cond } f)(x) = \left| \frac{x f'(x)}{f(x)} \right|$$

general case

$$x \neq 0 \wedge y \neq 0$$

$$= \left| \frac{x^{\frac{1}{2}} \cdot x^{-\frac{1}{2}}}{x^{\frac{1}{2}}} \right|$$

$$= \frac{1}{2} \left| \frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} \right|$$

$$= \frac{1}{2}$$

$$\frac{1}{2} < 1$$

$\therefore$  well conditioned

Note

$$(\text{cond } f)(x) \geq 0 \quad \text{i.e., } (\text{cond } f)(x) \in \mathbb{R}^+ \cup \{0\}$$

Again... with a subtle trick!

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$\frac{f'(x)}{f(x)} = \frac{d}{dx} \ln(f(x))$$

$$\ln(x^{\frac{1}{2}}) = \frac{1}{2} \ln(x)$$

$$\frac{d}{dx} \left( \frac{1}{2} \ln(x) \right)$$

$$= \frac{1}{2} \frac{d}{dx} \ln(x)$$

$$= \frac{1}{2x}$$

$$(\text{cond } f)(x) = \left| \frac{x f'(x)}{f(x)} \right|$$

$$= \left| \frac{x}{2x} \right|$$

$$= \frac{1}{2}$$