

## CS 381 Final Exam

December, 2004

1. Find the contrapositive of each of the following statements in English: [9]

(a) Every picture is faded if it is old.

(b) Errors will be introduced only if the program is modified.

(c) Sufficient water is necessary for healthy plants.

2. Negate the following statements in English. Give a form other than simply putting "not" or "it is not the case that" in front: [9]

(a) Every picture is either old or faded.

(b) Every picture is faded if it is old.

(c) Some farmer grows only corn.

3. If  $|A|$  and  $|B|$  for sets  $A$  and  $B$ , what are the value of the following quantities ? [12]

(a)  $|A \times B|$

(b)  $|A^2|$

(c)  $\text{Max } |A \cap B|$

(d)  $\text{Min } |A \cup B|$

4. Translate the following sentences into wffs of predicate logic using the given predicates. The universe is the set of people.[9]

$J(x)$ :  $x$  is a judge.

$L(x)$ :  $x$  is a lawyer.

$W(x)$ :  $x$  is a woman.

$A(x, y)$ :  $x$  admires  $y$ .

(a) All judges admire only judges.

(b) All woman lawyers admire some judge.

(c) Some woman admires no lawyer.

5. Prove that if  $A \cup B = A - B$ , then  $B = \emptyset$ . [10]

6. Prove the following equalities by **induction**. [20]

(a)  $\sum_{i=0}^n (2i + 1) = (n + 1)^2$

(b)  $R^m R^n = R^{m+n}$ , where  $R^n$  is defined recursively as  
Basis Clause:  $R^0 = E$  (i.e. equality relation).  
Inductive Clause:  $R^{n+1} = R^n R$ .

7. Prove or disprove that  $R^2$  is transitive if  $R$  is transitive. [10]

8. Given the partition  $\{1, 2, 3\}$  and  $\{4, 5\}$  of the set  $\{1, 2, 3, 4, 5\}$ , answer the following questions: [6]

(a) List the ordered pairs in the equivalence relation defined by the partition.

(b) Find  $[1]$ .

9. For the poset  $\{1, 2, 3, 5, 6, 12\}$  with the relation  $x$  divides  $y$ , answer the following questions: [15]

(a) Draw the Hasse diagram

(b) Find maximal elements.

(c) Find the maximum element.

(d) Find minimal elements.

(e) Find the minimum element.