CS 381 Test 1

October 16, 2004

1. Fill in the blanks with the **shortest** string of characters so that the resultant proposition is valid. [15]

$[[P \to Q] \wedge [Q \to R]] \to [P \to R]$

$\Leftrightarrow \neg[[P \to Q] \wedge [Q \to R]] \ \boxed{\quad \vee \quad} \ \neg P \ \boxed{\quad \vee \quad} \ R$

$\Leftrightarrow \neg[P \to Q] \ \boxed{\quad \vee \quad} \ \neg[Q \ \boxed{\quad \to \quad} \ R] \vee \neg P \vee R$

$\Leftrightarrow \neg[\neg P \ \boxed{\quad \vee \quad} \ Q] \vee \neg[\ \boxed{\quad \neg Q \quad} \ \vee R] \vee \neg P \vee R$

$\Leftrightarrow [P \wedge \ \boxed{\quad \neg Q \quad} \ ] \vee [Q \ \boxed{\quad \wedge \quad} \ \neg R] \vee \neg P \vee R$

$\Leftrightarrow [P \wedge \neg Q] \ \boxed{\quad \vee \quad} \ \neg P \vee [Q \ \boxed{\quad \wedge \quad} \ \neg R] \vee R$

$\Leftrightarrow [[P \ \boxed{\quad \vee \quad} \ \neg P] \wedge [\neg Q \vee \neg P]] \vee [[Q \vee R] \wedge [\neg R \ \boxed{\quad \vee \quad} \ R]]$

$\Leftrightarrow [\ \boxed{\quad \mathrm{T} \quad} \ \wedge [\neg Q \vee \neg P]] \vee [[Q \vee R] \wedge \ \boxed{\quad \mathrm{T} \quad} \ ]$

$\Leftrightarrow \ \boxed{\quad \mathrm{T} \quad}$

2 (a) Express the argument given below using the symbols suggested for each proposition. [5]
(b) Find where the glasses are using inference rules on the propositions in symbolic form. [12]

Argument:
If my glasses are on the kitchen table(T), then I saw them at breakfast(B). I was reading the newspaper in the living room (L)or in the kitchen(K). If I was reading the newspaper in the living room, then my glasses are on the coffee table(C). I did not see my glasses at breakfast. If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.

(a)
$T \to B$
$L \vee K$
$L \to C$
$\neg B$
$K \to T$

(b)
$T \to B$
$\neg B$
———————-
$\neg T$
$K \to T$

$\neg K$
$L \vee K$

L
$L \rightarrow C$

C

Hence my glasses are on the coffee table(C).

3. Negate the following statements in English. Give a form other than simply putting "not" or "it is not the case that" in front:

(a) If it is sunny, I will go to the beach.

It is sunny but (and) I will not go to the beach.

(b) Someone has read every book in the library on this subject.

Everyone has not read some book in the library on this subject.

(c) Either it is raining, or it is snowing and the sun is shining.

It is not raining, and either it is not snowing or the sun is not shining.

(d) It is raining but the sun is shining. [16]

It is not raining or the sun is not shining.

4. Translate the following wffs into English using the given predicates. The universe is the set of objects.[12]

$H(x)$: x is healthy.
$P(x)$: x is happy.
$L(x, y)$: x likes y.

(a) $\forall x H(x)$

Everything is healthy.

(b) $\neg \exists x[H(x) \wedge P(x)]$

There is nothing that is healthy and happy.

(c) $\forall x[[H(x) \wedge P(x)] \rightarrow \exists y L(y, x)]$

For any object x, if x is healthy and happy, then there is something that likes x. In other words, something likes anything(everything) that is healthy and happy.

5. Express the assertions given below as a proposition of a predicate logic using the following predicates. The universe is the set of objects.[16]

$L(x, y)$: x likes y.
$F(x)$: x is a flower.
$P(x)$: x is a person.
$R(x)$: x is red.

(a) Not everyone likes a (any) flower.
$\neg\forall x[P(x) \rightarrow \forall y[F(y) \rightarrow L(x, y)]]$
OR
$\neg\forall x\forall y[P(x) \rightarrow [F(y) \rightarrow L(x, y)]]$
OR
$\exists x\exists y[P(x) \wedge F(y) \wedge \neg L(x, y)]$

(b) Everyone likes a (any) red flower.
$\forall x[P(x) \rightarrow \forall y[[F(y) \wedge R(y)] \rightarrow L(x, y)]]$
OR
$\forall x\forall y[P(x) \rightarrow [[F(y) \wedge R(y)] \rightarrow L(x, y)]]$

(c) Mary likes a (any) flower if it is red.
$\forall x[[F(x) \wedge R(x)] \rightarrow L(Mary, x)]$

(d) Some person likes a (any) flower only if it is red.
$\exists x[P(x) \wedge \forall y[[F(y) \wedge L(x, y)] \rightarrow R(y)]]$
OR
$\exists x\forall y[P(x) \wedge [[F(y) \wedge L(x, y)] \rightarrow R(y)]]$

6. Find the power set of each of the following sets: [9]

(a) $\{3, 5\}$
$\{\emptyset, \{3\}, \{5\}, \{3, 5\}\}$

(b) $\{\emptyset\}$
$\{\emptyset, \{\emptyset\}\}$

(c) $\{\{1\}\ ,\ \emptyset\}$
$\{\emptyset, \{\emptyset\}, \{\{1\}\}, \{\{1\}, \emptyset\}\}$

7. Indicate which of the following are true and which are false. [15]

(a) $\{x\} \in \{x\}$ False
(b) $\{x\} \subseteq \{\{x\}\}$ False
(c) $\emptyset \subseteq \{\emptyset\}$ True
(d) $\emptyset \in \{\emptyset\}$ True
(e) $\{x\} \in \{x, \{x\}\}$ True