CS 381 Test 1

February 17, 2004

1. Convert the following statements to **if_then form** in English:

(a) For the knight to win the fresh horse is necessary.

If the knight wins, the horse is fresh.

(b) You can have a healthy body only if your diet is healthy.

If you have a healthy body, your diet is healthy.

(c) The knight will win only if his armor is strong.

If the knight wins, his armor is strong.

(d) You can not cash a check which is void. [16]

If a check is void, you can not cash it.

2. Negate the following statements in English. Give a form other than simply putting "not" or "it is not the case that" in front:

(a) If the horse is fresh the knight will win.

The horse is fresh and the knight will not win.

(b) Someone visited every room in this building.

Everyone has not visited some room in the building.

(c) Either the horse is fresh or the armor is strong, and the knight wins.

Neither the horse is fresh nor the armor is strong, or the knight does not win.

(d) Tax will be reduced only if the economy remains strong. [16]

Tax will be reduced but the economy does not remain strong.

3. Find the converse **and** contrapositive of the following statement in English: [16]

(a) If it snows, I will stay home.

Converse: If I stay home, then it snows.
Contrapositive: If I don't stay home, then it does not snow.

(b) Either the horse is fresh or the armor is strong only if the knight wins.

Converse: If the knight wins, either the horse is fresh or the armor is strong.
Contrapositive: If the knight does not win, neither the horse is fresh nor the armor is strong.

4. Find the dual of $(True \vee (P \wedge \neg Q)) \wedge \neg(Q \vee False)$. [5]

$(False \wedge (P \vee \neg Q)) \vee \neg(Q \wedge True)$.

5 (a) Express the argument given below using the symbols suggested for each proposition. [8]
(b) Check whether or not the reasoning is correct using inference rules on the propositions in symbolic form. [12]

Argument:
It is not the case that that if electricity rates go up (R), then usage will go down (U), nor is it true that either new power plants will be built (P) or bills will not be late (B). Therefore, usage will not go down and bills will be late.

(a) $\neg(R \rightarrow U)$
$\neg(P \vee B)$

$\neg U \wedge \neg B$

Since $\neg(R \rightarrow U) \Leftrightarrow (R \wedge \neg U)$, and $(R \wedge \neg U) \Rightarrow \neg U$, $\neg U$ is obtained.
Also since $\neg(P \vee B) \Leftrightarrow (\neg P \wedge \neg B)$, and $(\neg P \wedge \neg B) \Rightarrow \neg B$, $\neg B$ is obtained.
From $\neg U$ and $\neg B$, $\neg U \wedge \neg B$ is obtained.
Hence the argument is correct.

6. Fill in the blanks with the **shortest string**: [15]

(a) $P \wedge \neg[\neg P \vee Q] \Leftrightarrow P \wedge [\boxed{\text{P}} \wedge \neg Q] \Leftrightarrow [P \wedge [\boxed{\text{P}}] \wedge \neg Q \Leftrightarrow \boxed{\text{P}} \wedge \neg Q \Rightarrow \boxed{\text{P}}$

(b) $[P \wedge Q] \vee [P \wedge R] \Leftrightarrow \boxed{\text{P}} \wedge [Q \vee R]$

(c) $[P \rightarrow Q] \wedge [P \rightarrow \neg Q] \Leftrightarrow [\boxed{\neg P} \vee Q] \wedge [\neg P \vee \boxed{\neg Q}] \Leftrightarrow \neg P \vee [Q \wedge \boxed{\neg Q}] \Leftrightarrow \neg P \vee [\boxed{\text{F}}]$
$\Leftrightarrow \boxed{\neg P}$

7. Translate the following wffs into English using the given predicates. The universe is the set of objects.[12]

$B(x)$: x is a bee.
$F(x)$: x is a flower.
$L(x, y)$: x loves y.

(a) $\forall x[B(x) \rightarrow \forall y[F(y) \rightarrow L(x, y)]]$

Every bee loves every flower.

(b) $\forall x[B(x) \rightarrow \exists y[F(y) \wedge L(x, y)]]$

Every bee loves some flower.

(c) $\neg\forall x\exists y[[\neg B(x) \vee [F(y) \wedge L(x, y)]]$

Some bee does not love any flower.