CS 381 Test

October, 2007

1. Fill in the blanks with the **SHORTEST** string of characters so that the resultant proposition is valid. [15]

(a) $(P \to Q) \wedge (P \to R)$
$\Leftrightarrow (\boxed{\quad ? \quad} \vee Q) \wedge (\neg P \boxed{\quad ? \quad} R)$
$\Leftrightarrow \neg P \boxed{\quad ? \quad} (Q \wedge R)$
$\Leftrightarrow P \boxed{\quad ? \quad} (Q \wedge R)$

(b) $(P \vee Q) \to R$
$\Leftrightarrow (R \vee \boxed{\quad ? \quad} (P \boxed{\quad ? \quad} Q)$
$\Leftrightarrow R \boxed{\quad ? \quad} (\neg P \boxed{\quad ? \quad} \neg Q)$
$\Leftrightarrow (\boxed{\quad ? \quad} \vee R) \wedge (\neg Q \boxed{\quad ? \quad} R)$
$\Leftrightarrow (P \boxed{\quad ? \quad} R) \wedge (\boxed{\quad ? \quad} \to R)$

(c) $\neg(P \to Q) \Leftrightarrow \neg(\neg P \boxed{\quad ? \quad} \boxed{\quad ? \quad})$
$\Leftrightarrow P \boxed{\quad ? \quad} \boxed{\quad ? \quad}$

2. State each of the following formulas **in English**, **if** it is a wff. If it is not a wff, then give a reason why it is not a wff. Here $C(x)$ means x is a computer science student, $D(x)$ means x takes the discrete structures course, $L(x)$ means x likes the discrete structures course, and the universe is the set of all people in the world: [15]

(a) $\forall x[C(x) \to L(x)]$

(b) $\forall x[[D(x) \wedge L(x)] \to C(x)]$

(c) $\exists x[C(x) \to L(x)]$

(d) $\exists x[C(x) \wedge L(x)]$

(e) $\forall x[C(x) \wedge L(x)]$

3. Negate the following sentences in English. Change the quantifiers if there is any. DO NOT simply say "It is not the case that ..." or something similarly trivial. [15]

(a) Every person in every mathematics class understands logic.

(b) In every mathematics class there is some student who solves every problem in the textbook.

(c) If you like mathematics, you are good at mathematical induction.

(d) Someone likes mathematics.

(e) Someone likes mathematics and someone likes mathematical induction.

4. Restate each of the following statements as an "If_then" statement: [15]

(a) A necessary condition for an integer to be greater than 1 is that its square is greater than 1.

(b) Whenever it rains our street gets flooded.

(c) For our street to get flooded it is necessary that it rains.

(d) Either it does not rain or our street gets flooded.

(e) For an integer to be greater than 1 it is sufficient that its square is greater than 1.

5. Express each of the assertions given below as a proposition of a predicate logic using the following predicates. The universe is the set of all people in the world.[15]

$C(x)$ means x is a computer science student,
$D(x)$ means x takes the discrete structures course, and
$L(x)$ means x likes the discrete structures course.

(a) Not everyone likes the discrete structures course.

(b) Every computer science student takes the discrete structures course.

(c) Not everyone person who takes the discrete structures course likes it.

(d) Only computer science students may like the discrete structures course.

(e) Everyone is a computer science student and likes the discrete structures course.

6. Indicate which of the following are true and which are false. [15]

(a) $\emptyset \subseteq \{\emptyset\}$
(b) $\emptyset \in \{\{\emptyset\}\}$
(c) $\emptyset \subseteq \{\emptyset\}$
(d) $\{1\} \in \{1, 2\}$
(e) $\{\{1\}\} \in \{\{1\}, \{2\}\}$
(f) $\{1, 2, 1, 2\} \subseteq \{1, 2\}$
(g) $\{1\} \in \{2^n : n \in N\}$, where N is the set of natural numbers
(h) $\{\emptyset\} \times \{\emptyset, \{1\}\} = \emptyset$
(i) $\{1\} \subseteq \{\emptyset, \{1\}\} \times \{\emptyset\}$
(j) $\{\emptyset\} \subseteq \{\emptyset, \{\{\emptyset\}\}\}$

7 Prove that $\emptyset \subseteq A$ for an arbitrary set A. [15]
Hint: $\emptyset \subseteq A$ is equivalent to $\forall x[x \in \emptyset \rightarrow x \in A]$.