# CS 381 Test

October, 2007

1. Fill in the blanks with the **SHORTEST** string of characters so that the resultant proposition is valid. [15]

(a) $(P \to Q) \wedge (P \to R)$
$\Leftrightarrow (\boxed{\neg P} \vee Q) \wedge (\neg P \boxed{\vee} R)$
$\Leftrightarrow \neg P \boxed{\vee} (Q \wedge R)$
$\Leftrightarrow P \boxed{\to} (Q \wedge R)$

(b) $(P \vee Q) \to R$
$\Leftrightarrow (R \vee \boxed{\neg} (P \boxed{\vee} Q)$
$\Leftrightarrow R \boxed{\vee} (\neg P \boxed{\wedge} \neg Q)$
$\Leftrightarrow (\boxed{\neg P} \vee R) \wedge (\neg Q \boxed{\vee} R)$
$\Leftrightarrow (P \boxed{\to} R) \wedge (\boxed{\text{Q}} \to R)$

(c) $\neg(P \to Q) \Leftrightarrow \neg(\neg P \boxed{\vee} \boxed{\text{Q}})$
$\Leftrightarrow P \boxed{\wedge} \boxed{\neg Q}$

2. State each of the following formulas **in English**, **if** it is a wff. If it is not a wff, then give a reason why it is not a wff. Here $C(x)$ means $x$ is a computer science student, $D(x)$ means $x$ takes the discrete structures course, $L(x)$ means $x$ likes the discrete structures course, and the universe is the set of all people in the world: [15]

(a) $\forall x[C(x) \to L(x)]$
Every CS student likes the discrete strudtures course.

(b) $\forall x[[D(x) \wedge L(x)] \to C(x)]$
Everyone who takes the discrete structures course and likes it is a CS student.

(c) $\exists x[C(x) \to L(x)]$
There is someone who likes the discrete structures course if he/she is a CS student.

(d) $\exists x[C(x) \wedge L(x)]$
There is someone who is a CS student and who likes the discrete structures course.

(e) $\forall x[C(x) \wedge L(x)]$
Everyone is a CS student and likes the discrete structures course.

3. Negate the following sentences in English. Change the quantifiers if there is any. DO NOT simply say "It is not the case that ..." or something similarly trivial. [15]

(a) Every person in every mathematics class understands logic.
Someone in some mathematics class does not understand logic.

(b) In every mathematics class there is some student who solves every problem in the textbook.
In some mathematics class every student does not solve some problem in the textbook.

(c) If you like mathematics, you are good at mathematical induction.
You like mathematics and (but) you are not good at mathematical induction.

(d) Someone likes mathematics.
Everyone does not like mathematics.

(e) Someone likes mathematics and someone likes mathematical induction.
Everyone does not like mathematics or everyone does not like mathematical induction.

4. Restate each of the following statements as an "If_then" statement: [15]

(a) A necessary condition for an integer to be greater than 1 is that its square is greater than 1.
If an integer is greater than 1 then its square is greater than 1.

(b) Whenever it rains our street gets flooded.
If it rains, our street gets flooded.

(c) For our street to get flooded it is necessary that it rains.
If our street gets flooded, it rains/it must rain/it must be raining.

(d) Either it does not rain or our street gets flooded.
If it rains, then our street gets flooded.

(e) For an integer to be greater than 1 it is sufficient that its square is greater than 1.
If the square of an integer is greater than 1, then the integer is greater than 1.

5. Express each of the assertions given below as a proposition of a predicate logic using the following predicates. The universe is the set of all people in the world.[15]

$C(x)$ means $x$ is a computer science student,
$D(x)$ means $x$ takes the discrete structures course, and
$L(x)$ means $x$ likes the discrete structures course.

(a) Not everyone likes the discrete structures course.
$\neg\forall x L(x)$

(b) Every computer science student takes the discrete structures course.
$\forall x[C(x) \rightarrow D(x)]$

(c) Not everyone person who takes the discrete structures course likes it.
$\neg\forall x[D(x) \rightarrow L(x)]$

(d) Only computer science students may like the discrete structures course.
$\forall x[L(x) \rightarrow C(x)]$

(e) Everyone is a computer science student and likes the discrete structures course.
$\forall x[C(x) \wedge L(x)]$

6. Indicate which of the following are true and which are false. [15]

(a) $\emptyset \subseteq \{\emptyset\}$ : True
(b) $\emptyset \in \{\{\emptyset\}\}$ : False
(c) $\emptyset \subseteq \{\emptyset\}$ :True
(d) $\{1\} \in \{1, 2\}$ :False
(e) $\{\{1\}\} \in \{\{1\}, \{2\}\}$ :False
(f) $\{1, 2, 1, 2\} \subseteq \{1, 2\}$ :True
(g) $\{1\} \in \{2^n : n \in N\}$, where N is the set of natural numbers :False
(h) $\{\emptyset\} \times \{\emptyset, \{1\}\} = \emptyset$ :False
(i) $\{1\} \subseteq \{\emptyset, \{1\}\} \times \{\emptyset\}$ :False
(j) $\{\emptyset\} \subseteq \{\emptyset, \{\{\emptyset\}\}\}$ :True

7 Prove that $\emptyset \subseteq A$ for an arbitrary set A. [15]
Hint: $\emptyset \subseteq A$ is equivalent to $\forall x[x \in \emptyset \rightarrow x \in A]$.

Proof by contradiction, that is assume that $\emptyset \not\subseteq A$ for some set A and try to derive some statement that is not true.
If $\emptyset \not\subseteq A$, then $\exists x[x \in \emptyset \wedge x \notin A]$.
Hence $\exists x x \in \emptyset$, which contradicts $\forall x x \notin \emptyset$, the definition of $\emptyset$.
Hence the assumption that $\emptyset \not\subseteq A$ for some set A is wrong.
Hence $\emptyset \subseteq A$ for an arbitrary set A.