

CS 381 Final Exam

December, 2011

1. State each of the following formulas **in English**, if it is a wff. If it is not a wff, then give a reason why it is not a wff. Here $B(x)$ means x is a book, $P(x)$ means x is a person, $L(x, y)$ means x likes y , and the universe is the set of all objects in the world: [15]

(a) $\forall x \exists y [P(x) \rightarrow [B(y) \wedge L(x, y)]]$

(b) $\exists x \forall y [P(x) \wedge [B(y) \rightarrow L(x, y)]]$

(c) $\exists x \forall y [P(x) \rightarrow [B(y) \rightarrow L(x, y)]]$

(d) $\forall x \forall y [[P(x) \wedge B(y)] \rightarrow L(x, y)]$

(e) $\exists x \exists y [P(x) \wedge B(y) \wedge L(x, y)]$

2. Recursively define the following sets: [10 each]

- (a) The set of natural numbers that are divisible by 3.

(b) The relation R on the set of natural numbers such that $\langle x, y \rangle \in R$ if and only if $y \leq 2x$.

3. Prove the following by mathematical induction [10 each] :

(a) $\sum_{i=1}^n (2i - 1) = n^2$

(b) $n^3 + 5n$ is divisible by 6 for every natural number n .

4 (a) Represent the \geq relation (denote it R) on $\{0, 1, 2, 3\}$ as a set of ordered pairs. [6]

(b) Find R^2 on $\{0, 1, 2, 3\}$. [6]

(c) Find R^2 on the set of natural numbers N . Justify your answer. [8]

5. Indicate which of the following are true and which are false. [15]

(a) $10x + 1000 = O(x^2)$.

(b) $-100x^3 + 500x^2 + 300x - 10000 = O(x^2)$.

(c) $2^x - 100x^{50} = O(x^{100})$.

(d) $f(S) \cap f(T) \subseteq f(S \cap T)$ for a function f and subsets S and T of the domain of f .

(e) $1/(n^2 + 1)$ is a one-to-one function from the set of reals onto $[0, 1]$.

(f) The range of the function $1/(n + 1)^2$ on $[0, \infty)$ is $(0, 1]$.

(g) A partition of a set determines an equivalence relation over the set.

(h) If a binary relation R is symmetric then R^2 is symmetric.

(i) The domain of the function that assigns to each pair of positive integers the maximum of these two integers is the Cartesian product of the set of nonnegative integers.

(j) If $f(x) = x^2 + 2$ and $g(x) = 3x - 1$, then $f(g(x)) = 9x^2 - 6x + 2$.

6. Prove that the cardinality (size) of the power set of a set A of cardinality n is 2^n . [10]