5 (a) Let $L$ be an arbitrary language over the alphabet $\{a, b\}$. Define $L^*$ and $L^+$ recursively. [5 points]

$L^*$

- **Basis Clause**: $\lambda \in L^*$
- **Inductive Clause**: if $x \in L^*$ and $y \in L$ for any strings $x, y$, then $xy \in L^*$
- **Extremal Clause**: As usual

$L^+$

- **Basis Clause**: $\lambda \in L^+$
- **Inductive Clause**: if $x \in L^+$ and $y \in L$ for any strings $x, y$, then $xy \in L^+$
- **Extremal Clause**: As usual

(b) Using your definition of $L^*$ and $L^+$ of (a), prove by general induction (a.k.a. structural induction) that $L^+$ is a subset of $L^*$. [15 points]

**Basis Step**: To prove $x \in L \Rightarrow x \in L^+$.

**Proof**: Since $x \in L$, by inductive clause of definition of $L^*$, for any $y \in L$, $xy \in L^*$.

**Inductive Step**: To prove that if $x \in L^+$ and $x \in L^+$, then for any $y \in L$, $xy \in L^+$.

**Proof**: Since $x \in L^+$, by inductive clause of definition of $L^*$, for any $y \in L$, $xy \in L^*$. 
