6. For sets of states $S$ of an NFA-$A$, prove that $A(S)$ is a subset of $\bigcup_{p \in S} A\{p\}$, that is $A(S)$ is a subset of the union of $A\{p\}$ for all $p \in S$. [15 points]

To prove $A(S) \subseteq \bigcup_{p \in S} A\{p\}$

Definition of $A(S)$

Basis Clause: $S \subseteq A(S)$

Inductive Clause: If $q \in A(S)$ for any state $q$, then $\delta(q, A) \subseteq A(S)$

Proof. Basis Step: To prove $S \subseteq \bigcup_{p \in S} A\{p\}$

Let $p$ be an arbitrary state of $S$. Then $q \in A\{p\}$ by definition of $A\{p\}$. Hence $q \in \bigcup_{p \in S} A\{p\}$

Inductive Step: To prove that if for any $q \in A(S)$, $q \in \bigcup_{p \in S} A\{p\}$ holds, then $\delta(q, A) \subseteq \bigcup_{p \in S} A\{p\}$.

Proof: Since $q \in A\{p\}$, there is a state $r$ in $S$ such that $q \in A\{r\}$.

By definition of $A\{r\}$, if $q \in A\{r\}$, then $\delta(q, A) \subseteq A\{r\}$.

Hence $\delta(q, A) \subseteq \bigcup_{p \in S} A\{p\}$