## Interest Points and Features

## Finding Correspondence

Matching points, patches, edges, or regions


## Interest points

- Note: "interest points" = "keypoints", also sometimes called "features"


## Applications

- Keypoints are used for:
- Image alignment
- 3D reconstruction
- Motion tracking
- Robot navigation
- Indexing and database retrieval

- Object recognition



## Interest points

- Suppose you have to click on some point, go away and come back after I deform the image, and click on the same points again.
- Which points would you choose?



## Overview of Keypoint Matching



1. Find a set of distinctive keypoints
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

## Goals for Keypoints



Detect points that are repeatable and distinctive

## Key trade-offs



## Detection of interest points



Robust detection
Precise localization

More Points
Robust to occlusion
Works with less texture

## Description of patches

More Distinctive
Minimize wrong matches

More Flexible
Robust to expected variations Maximize correct matches

## Invariant Local Features

- Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters


Features Descriptors

## Choosing interest points

## Where would you tell your friend to meet you?



## Choosing interest points

Where would you tell your friend to meet you?


## Many Existing Detectors Available

Hessian \& Harris
Laplacian, DoG
Harris-/Hessian-Laplace Harris-/Hessian-Affine EBR and IBR
MSER
Salient Regions Others...
[Beaudet '78], [Harris '88]
[Lindeberg '98], [Lowe 1999]
[Mikolajczyk \& Schmid ‘01]
[Mikolajczyk \& Schmid ‘04]
[Tuytelaars \& Van Gool '04]
[Matas '02]
[Kadir \& Brady '01]


- What points would you choose?


## Corner Detection: Basic Idea

- We should easily recognize the point by looking through a small window
- Shifting a window in any direction should give a large change in intensity

"flat" region:
no change in all directions

"edge":
no change along
the edge direction

"corner":
significant change in all directions


## Corner Detection: Mathematics

Change in appearance of window $w(x, y)$ for the shift $[u, v]$ :

$$
E(u, v)=\sum_{x, y} w(w, y)[I(x+u, y+v)-I(x, y)]^{2}
$$



$$
E(u, v)
$$



## Corner Detection: Mathematics

Change in appearance of window $w(x, y)$ for the shift $[u, v]$ :

$$
E(u, v)=\sum_{x, y} w(w, y)[I(x+u, y+v)-I(x, y)]^{2}
$$

We want to find out how this function behaves for small shifts

$$
E(u, v)
$$

## Taylor series approx to shifted image

$$
\begin{aligned}
E(u, v) & \approx \sum_{x, y} w(x, y)\left[I(x, y)+u I_{x}+v I_{y}-I(x, y)\right]^{2} \\
& =\sum_{x, y} w(x, y)\left[u I_{x}+v I_{y}\right]^{2} \\
& =\sum_{x, y} w(x, y)(u \quad v)\left[\begin{array}{ll}
I_{x} I_{x} & I_{x} I_{y} \\
I_{x} I_{y} & I_{y} I_{y}
\end{array}\right]\binom{u}{v}
\end{aligned}
$$

## Corner Detection: Mathematics

The quadratic approximation simplifies to

$$
E(u, v)=\left[\begin{array}{ll}
u & v
\end{array}\right] M\left[\begin{array}{l}
u \\
v
\end{array}\right]
$$

where $M$ is a second moment matrix computed from image derivatives:

$$
\begin{gathered}
M=\sum_{x, y} w(x, y)\left[\begin{array}{cc}
I_{x}^{2} & I_{x} I_{y} \\
I_{x} I_{y} & I_{y}^{2}
\end{array}\right] \\
M=\left[\begin{array}{cc}
\sum I_{x} I_{x} & \sum I_{x} I_{y} \\
\sum I_{x} I_{y} & \sum I_{y} I_{y}
\end{array}\right]=\sum\left[\begin{array}{c}
I_{x} \\
I_{y}
\end{array}\right]\left[I_{x} I_{y}\right]=\sum \nabla I(\nabla I)^{T}
\end{gathered}
$$

## Corners as Distinctive Interest Points

$$
M=\sum_{x, y} w(x, y)\left[\begin{array}{cc}
I_{x}^{2} & I_{x} I_{y} \\
I_{x} I_{y} & I_{y}^{2}
\end{array}\right]
$$

$2 \times 2$ matrix of image derivatives (averaged in neighborhood of a point).


Notation:


$$
I_{x}=\frac{\partial I}{\partial x}
$$



$$
I_{y}=\frac{\partial I}{\partial y}
$$

$I_{x} I_{y}=\frac{\partial I}{\partial x} \frac{\partial I}{\partial y}$

## Harris Detector: Mathematics

Intensity change in shifting window: eigenvalue analysis

$$
E(u, v)=\left[\begin{array}{cc}
u & v
\end{array}\right] M\left[\begin{array}{l}
u \\
v
\end{array}\right] \quad \lambda_{1}, \lambda_{2}-\text { eigenvalues of } M
$$



## Statistics of $x$ and $y$ derivatives



## Covariance matrix: ellipse containing data



## Selecting Good Features



$\lambda_{1}$ and $\lambda_{2}$ are large

## Selecting Good Features



large $\lambda_{1}$, small $\lambda_{2}$

## Selecting Good Features



small $\lambda_{1}$, small $\lambda_{2}$

## Interpreting the eigenvalues

Classification of image points using eigenvalues of $M$ :


## Harris Detector: Mathematics

Measure of corner response:

$$
\begin{aligned}
R= & \operatorname{det}(M)-k(\operatorname{trace}(M))^{2} \\
& \operatorname{det}(M)=\lambda_{1} \lambda_{2} \\
& \operatorname{trace}(M)=\lambda_{1}+\lambda_{2}
\end{aligned}
$$

This expression does not requires computing the eigenvalues.

$$
(k-\text { empirical constant, } k=0.04-0.06)
$$

## Corner response function

$$
R=\operatorname{det}(M)-k(\operatorname{trace}(M))^{2}
$$

$\alpha$ : constant (0.04 to 0.06)


## Interest operator values



## Harris corner detector

1) Compute $M$ matrix for each image window to get their cornerness scores.
2) Find points whose surrounding window gave large corner response ( $f>$ threshold)
3) Take the points of local maxima, i.e., perform non-maximum suppression
[^0]Harris Detector: Steps


Harris Detector: Steps
Compute corner response $R$


## Harris Detector: Steps

Find points with large corner response: $R>$ threshold


## Harris Detector: Steps

Take only the points of local maxima of $R$

Harris Detector: Steps


## Invariance and covariance

- We want corner locations to be invariant to photometric transformations and covariant to geometric transformations
- Invariance: image is transformed and corner locations do not change
- Covariance: if we have two transformed versions of the same image, features should be detected in corresponding locations



## Affine intensity change

$$
\square \leadsto \quad \square \rightarrow a I+b
$$

- Only derivatives are used => invariance to intensity shift $I \rightarrow I+b$
- Intensity scaling: $I \rightarrow a I$



Partially invariant to affine intensity change

## Image translation



- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

## Image rotation



Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

## Scaling



Corner location is not covariant to scaling!

## Automatic Scale Selection



How to find corresponding patch sizes?

## Automatic Scale Selection

- Function responses for increasing scale (scale signature)



## Automatic Scale Selection

- Function responses for increasing scale (scale signature)



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Difference-of-Gaussian (DoG)


## Scale Invariant Detectors

- Harris-Laplacian ${ }^{1}$

Find local maximum of:

- Harris corner detector in space (image coordinates)
- Laplacian in scale

${ }^{1}$ K.Mikolajczyk, C.Schmid. "Indexing Based on Scale Invariant Interest Points". ICCV 2001
${ }^{2}$ D.Lowe. "Distinctive Image Features from Scale-Invariant Keypoints". IJCV 2004


## DoG - Efficient Computation

- Computation in Gaussian scale pyramid



# Find local maxima in position-scale space of Difference-of-Gaussian 



$$
L_{x x}(\sigma)+L_{y y}(\sigma) \rightarrow \sigma^{3}
$$


$\Rightarrow$ List of ( $x, y, s$ )

## Results: Difference-of-Gaussian



## Orientation Normalization

- Compute orientation histogram [Lowe, SIFT, 1999]
- Select dominant orientation
- Normalize: rotate to fixed orientation




## Image representations

- Templates
- Intensity, gradients, etc.

- Histograms
- Color, texture, SIFT descriptors, etc.


## Image Representations: Histograms



## Global histogram

- Represent distribution of features
- Color, texture, depth, ...


## What kind of things do we compute histograms of?



L*a*b* color space


HSV color space

- Texture (filter banks or HOG over regions)


## What kind of things do we compute histograms of?

Histograms of oriented gradients


SIFT - Lowe IJCV 2004

## SIFT vector formation

- Computed on rotated and scaled version of window according to computed orientation \& scale
- resample the window
- Based on gradients weighted by a Gaussian of variance half the window (for smooth falloff)



## SIFT vector formation

- $4 \times 4$ array of gradient orientation histogram weighted by magnitude
- 8 orientations $\times 4 \times 4$ array $=128$ dimensions
- Motivation: some sensitivity to spatial layout, but not too much.

showing only $2 \times 2$ here but is $4 \times 4$


## Ensure Smoothness

- Gaussian weight
- Trilinear interpolation
- a given gradient contributes to 8 bins:

4 in space times 2 in orientation



Keypoint descriptor

## Reduce Effect of Illumination

- 128-dim vector normalized to 1
- Threshold gradient magnitudes to avoid excessive influence of high gradients
- after normalization, clamp gradients $>0.2$
- renormalize



## Local Descriptors

- Most features can be thought of as templates, histograms (counts), or combinations
- The ideal descriptor should be
- Robust
- Distinctive
- Compact
- Efficient
- Most available descriptors focus on edge/gradient information
- Capture texture information
- Color rarely used


## Things to remember

- Keypoint detection: repeatable and distinctive
- Corners, blobs, stable regions
- Harris, DoG

- Descriptors: robust and selective
- spatial histograms of orientation
- SIFT



[^0]:    C.Harris and M.Stephens. "A Combined Corner and Edge Detector." Proceedings of the 4th Alvey Vision Conference: pages 147-151, 1988.

