# Testing

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1 The Process of Testing

Testing Process Components

- **Test case**: a general description of a required test
  - There may be many possible inputs that would serve
• **Regression Log**: Database of tests and expected outputs
  – Used during regression testing to quickly rerun old tests
• **Oracle**: The process, person, and/or program that determines if test output is correct
• **Failure**: An execution on which incorrect behavior occurs
• **Fault**: A defect in the code that (may) cause a failure
• **Error**: A human mistake that results in a fault

Why do we test?

• To show that the program works
  
  Dijkstra: “Testing can show the presence of errors, but never their absence.”

• To reveal faults
  
  – A test is “successful” only if it reveals a fault
  – A bit silly when large numbers of tests have executed without failure

• To gain confidence in the program
  
  – prior experience with similar testing methods
  – statistical confidence
Choosing Test Data
From these motives come different approaches to choosing test data

- Representative testing
  - tests designed to reflect the frequency of user inputs.
  - Used for reliability estimation.

- Directed testing
  - Tests designed to discover system defects.
  - A successful defect test is one which reveals the presence of defects in a system.

2 Representative Testing

Representative testing
Choose data that is representative of the way the end users will exercise the software.

- Advantages:
  - realism — catches the kinds of failures that the users would have encountered
  - relatively cheap to generate tests
  - time to failure during test reflects time to failure in operation
    * useful in statistical reliability modeling
Producing Representative Tests

- collected data from existing system
  - further selection may be needed
  - cannot accommodate new functionality

- random generators
  - non-uniform, to match desired distribution
  - can be hard to generate non-numeric ADT values

Representative Testing

- Requires *operational profile* for its definition
  - The operational profile defines the expected pattern of software usage

2.1 Operational Profile

Sample Op Profile

<table>
<thead>
<tr>
<th>Input Category</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transaction Proc.</td>
<td>85%</td>
</tr>
<tr>
<td>Balancing</td>
<td>14%</td>
</tr>
<tr>
<td>Year-end Report</td>
<td>1%</td>
</tr>
</tbody>
</table>
### Sample Op Profile

<table>
<thead>
<tr>
<th>Input Category</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transaction Proc.</td>
<td>85%</td>
</tr>
<tr>
<td>New account</td>
<td>7%</td>
</tr>
<tr>
<td>Close account</td>
<td>3%</td>
</tr>
<tr>
<td>Debits</td>
<td>70%</td>
</tr>
<tr>
<td>Credits</td>
<td>20%</td>
</tr>
<tr>
<td>Balancing</td>
<td>14%</td>
</tr>
<tr>
<td>Year-end Report</td>
<td>1%</td>
</tr>
</tbody>
</table>
### Testing

<table>
<thead>
<tr>
<th>Input Category</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transaction Proc.</td>
<td>85%</td>
</tr>
<tr>
<td>New account</td>
<td>7%</td>
</tr>
<tr>
<td>already exists</td>
<td>90%</td>
</tr>
<tr>
<td>Close account</td>
<td>15%</td>
</tr>
<tr>
<td>non-existant exists</td>
<td>15%</td>
</tr>
<tr>
<td>exists</td>
<td>85%</td>
</tr>
<tr>
<td>Debits</td>
<td>70%</td>
</tr>
<tr>
<td>non-existant exists</td>
<td>25%</td>
</tr>
<tr>
<td>exists</td>
<td>75%</td>
</tr>
<tr>
<td>Credits</td>
<td>20%</td>
</tr>
<tr>
<td>non-existant exists</td>
<td>25%</td>
</tr>
<tr>
<td>exists</td>
<td>75%</td>
</tr>
<tr>
<td>Balancing</td>
<td>14%</td>
</tr>
<tr>
<td>Year-end Report</td>
<td>1%</td>
</tr>
</tbody>
</table>

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**Representative testing difficulties**

- Uncertainty in the operational profile
  - This is a particular problem for new systems with no operational history. Less of a problem for replacement systems
- High costs of generating the operational profile
– Costs are very dependent on what usage information is collected by the organisation which requires the profile

• Statistical uncertainty
  – Difficult to estimate level of confidence in operational profile
  – Usage pattern of software may change with time

2.2 Reliability Growth Models

Reliability Growth Models

• Growth model is a mathematical model of the system reliability change as it is tested and faults are removed
• Used as a means of reliability prediction by extrapolating from current data

Reliability modeling procedure

• Determine operational profile of the software
• Generate a set of test data corresponding to the profile
• Apply tests, measuring amount of execution time between each failure
• After a statistically valid number of tests have been executed, reliability can be measured
Reliability metrics

• Probability of failure on demand
  – This is a measure of the likelihood that the system will fail when a service request is made
  – $POFOD = 0.001$ means 1 out of 1000 service requests result in failure
  – Relevant for safety-critical or non-stop systems

• Rate of fault occurrence ($ROCOF$)
  – Frequency of occurrence of unexpected behaviour
  – $ROCOF$ of 0.02 means 2 failures are likely in each 100 operational time units
  – Relevant for operating systems, transaction processing systems

Reliability metrics

• Mean time to failure
  – Measure of the time between observed failures
  – $MTTF = 500$ means that the time between failures is 500 time units
  – Relevant for systems with long transactions e.g. CAD systems
Reliability metrics

- Availability
  - Measure of how likely the system is available for use. Takes repair/restart time into account
  - $Avail = 0.998$ means software is available for 998 out of 1000 time units
  - Relevant for continuously running systems e.g. telephone switching systems

Reliability measurement

- Measure the number of system failures for a given number of system inputs
  - Used to compute $POFOD$
- Measure the time (or number of transactions) between system failures
  - Used to compute $ROCOF$ and $MTTF$
- Measure the time to restart after failure
  - Used to compute $Avail$
Time units

- Time units in reliability measurement must be carefully selected. Not the same for all systems
- Raw execution time (for non-stop systems)
- Calendar time (for systems which have a regular usage pattern e.g. systems which are always run once per day)
- Number of transactions (for systems which are used on demand)

Jelinski-Moranda Model
Assumptions:

- Software contains $N$ faults ($N$ is unknown)
- Each fault *manifests* (causes a failure) at rate $\phi$
- Faults manifest independently
- Faults are fixed perfectly, without introducing new ones

If we have repaired $i$ faults, the program’s failure rate $\lambda$ is

$$\lambda_i = (N - i)\phi$$
**Observed reliability growth**

- Simple equal-step model but does not reflect reality
- Reliability does not necessarily increase with change as the change can introduce new faults
- The rate of reliability growth tends to slow down with time as frequently occurring faults are discovered and removed from the software

**Musa Logarithmic Poisson Model**

Assumptions:

- Software can never be completely free of faults.
- Faults manifest independently
- Faults are found in decreasing order of failure rate.
- The program failure rate before repairing any faults is $\lambda_0$
- Faults are fixed perfectly, without introducing new ones

If we have repaired $i$ faults, the program's failure rate $\lambda$ is

$$\lambda_i = \lambda_0 e^{-\theta i}$$
Fitting Example

![Graph showing Failure Rate vs. Failure Number for LSq Fit - Sequence 2. The graph plots failure rate on a logarithmic scale against failure number. The data points are represented by diamonds and are scattered across the graph, indicating a trend in failure rate over failure number.]

Fitting Example
3 Directed Testing

Directed testing
Choose tests designed to reveal

• many faults
Choosing Good Test Data

Techniques for selecting test data are generally termed either

• white-box, or
• black-box

White-Box Testing

White-box testing is the choice of test data based upon the details of the algorithms in the code under test.

• Formally coverage measures, E.g., statement coverage, branch coverage

For example, a common goal in white-box testing are to be sure that every code statement is executed at least once by some test. Another common goal is that, for every conditional branch (ifs, loops, etc.) in the code, that on at least one test that condition will have been true and on at least one test that condition will have been false.

The difficulty with such white-box testing is that it's almost impossible to be sure you have met one of these goals unless you have special testing software to track the execution of statements and branches over the whole history of testing. Lacking this software, it's easier to do black-box testing.

• Informally, design tests specifically to exercise "tricky" parts of the code

However, there is a kind of informal white-box testing that we can do. As programmers, we often write a bit of code and think to ourselves “I wonder if this is really going to work?” That's a good time to pause and design a test that will check exactly that bit of dicey code.
3.1 Black-Box Testing

Black-Box Testing

*Black-box* (a.k.a. *specification-based*) testing chooses tests without consulting the implementation. 

- based simply upon our understanding of what the unit is supposed to do.

One of the goals of black-box testing is to be sure that every distinct behavior expected of a unit has been triggered in at least one test. Another is to try to choose tests that are likely to cause trouble, no matter what the actual algorithm is.

Some of the best-known techniques for choosing black-box tests focus on the input values that will be supplied to the unit during testing.

**Functional Coverage**

a.k.a *Equivalence partitioning*

- Choose at least one test that covers each distinct "behavior" described in the requirements.
  - Different functions performed by the program
  - Different types or ranges of input that get treated or described separately.
  - Different types or ranges of output that get treated or described separately.

- Large, structured projects place emphasis on tracking requirements to functional test cases
Boundary Values Testing

a.k.a., Extremal Values Testing

• Choose as test data the largest and the smallest values for each input and for each "functional" case
  – Often misunderstood as simply choosing largest & smallest possible inputs
  – Assumes that we have started by attempting functional coverage

Special Values Testing

Choose as test data those certain data values that just tend to cause trouble. Programmers eventually develop a sense for these. They include

• For integers: -1, 0, 1
• For floating point numbers: -e, 0, +e, where "e" is a very small number
• For strings: the empty string, strings containing only blanks, strings containing no alphabetic characters

What is Special?

• As we move to other data structures, we may develop a suspicion of other special values, e.g.,
  – For times of the day: midnight, noon
  – For containers of data: an empty container
Testing

- Special values and Boundary values often overlap
- (F14 example)

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3.1.1 Illegal versus Unusual Values

Should You Test With Illegal Values?
Many people advocate testing programs with illegal inputs. For example, if a program is supposed to accept a positive integer as its input, they would suggest running it on a zero and a negative input value. This is nonsense.

You Can't Test an Illegal Input
A test is only useful if it is possible for the program to either pass or fail the test. So just what is a program supposed to do on an illegal input?

1. Should it abort/crash/stop running?

   - If so, how should it inform you that it has stopped?
     - Write a message to standard error (cerr)?
     - Write a message to a log file?
     - Pop up a dialog box on the screen?
     - Send a "failed" status code to the operating system?

   - Perhaps even more importantly, how can you possibly test for this behavior if you don’t know what the program is supposed to do?
2. Should it continue running?

- If so, how should it recover from the illegal state that it will find itself in?

There's a real danger in trying to second-guess ways to recover from erroneous inputs that you will wind up generating bogus output that someone will nonetheless assume is real (e.g., the rather common-place news stories about people who receive utility bills for millions of dollars).

Illegal versus unusual Inputs

There's a big difference between the following two specifications:

**Negatives are Illegal**

The function $\sqrt{x}$ should be run on non-negative $x$. It returns a number $z$ such that $z \times z$ is approximately equal to $x$.

**Negatives are Legal, but Unusual**

The function $\sqrt{x}$, when run on non-negative $x$ should return a number $z$ such that $z \times z$ is approximately equal to $x$.

When run on a negative $x$ it should abort execution with status code 144.

You can argue about which specification is better. But once you have been given the specification, you have to accept what it tells you about testing.

Illegal Inputs

The function $\sqrt{x}$ should be run on non-negative $x$. It returns a number $z$ such that $z \times z$ is approximately equal to $x$.

- In this spec, a negative input is illegal.
  - It's the programmer's responsibility to never do this.
– We can’t use a negative input as a test because, no matter what the function does, it’s *correct* by definition.

Unusual Inputs

The function \( \text{sqrt}(x) \), when run on non-negative \( x \) should return a number \( z \) such that \( z \times z \) is approximately equal to \( x \).
When run on a negative \( x \) it should abort execution with status code 144.

• In this spec, a negative input is legal. It is probably intended to be an unusual case.
  – Programmers are allowed to supply this input if they are ready to deal with the specified “error” handling condition.
  – We can (and should) test the function with a negative input, because we know exactly what the desired behavior is supposed to be.

3.2 White-Box Testing

White-Box Testing

*White-Box* (a.k.a. *Implementation-based* testing) uses information from the implementation to choose tests.

• Structural Testing (a.k.a., “path testing” (not per your text)
  Designate a set of paths through the program that must be exercised during testing.
  – Statement Coverage
  – Branch Coverage
– Cyclomatic coverage ("independent path testing")
– Data-flow Coverage

• Mutation testing

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**Statement Coverage**

Require that every statement in the code be executed at least once during testing. Special programs ("software tools") will monitor this requirement for you.

• **gprof** in GNU gcc suite

• Java?

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**Example**

```cpp
cin >> x >> y;
while (x > y)
{
    if (x > 0)
        cout << x;
    x = f(x, y);
}
cout << x;
```

What kinds of tests are required for statement coverage?

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**Branch Coverage**

Requires that every "branch" in the flowchart be tested at least once

- Equivalent to saying that each conditional stmt must be tested as both true and false
- Branch coverage implies Statement Coverage, but not vice versa

```plaintext
if X < 0 then
    X := -X;
    Y := sqrt(X);
```

```
cin >> x >> y;
while (x > y)
{
    if (x > 0)
    cout << x;
    x = f(x, y);
}
cout << x;
```

What kinds of tests are required for branch coverage?
Cyclomatic Coverage
(a.k.a “independent path coverage”, “path testing”)

- The latter term (used in your text) should be discouraged as it is both vague and means something entirely different to most of the testing community

- Each independent path must be tested
  - An independent path is one that includes a branch not previously taken.

A Control Flow Graph
What are the independent paths?
1, 2, 3, 4, 12, 13
1, 2, 3, 5, 6, 11, 2, 12, 13
1, 2, 3, 5, 7, 8, 10, 11, 2, 12, 13
1, 2, 3, 5, 7, 9, 10, 11, 2, 12, 13

Cyclomatic Complexity

the number of independent paths in a program can be discovered by computing the cyclomatic complexity (McCabe, 1976) …

\[
CC(G) = \text{Number(}\text{edges}\text{)} - \text{Number(}\text{nodes}\text{)} + 1
\]

• This is a popular metric for module complexity.
• Actually pretty trivial: for structured programs with only binary decision constructs, equals number of conditional statements +1
• relation to testing is dubious
  – simply branch coverage hidden behind smoke and mirrors

Uniqueness
Sets of independent paths are not unique, nor is their size:

1,2,3,5,6,11,
2,3,5,7,8,
10,11,2,3,
5,7,9,10,11,
2,12,13

1,2,3,4,12,13

...............................

**Data-Flow Coverage**
Attempts to test significant combinations of branches.

- Any stmt \(i\) where a variable \(X\) may be assigned a new value is called a *definition* of \(X\) at \(i\): \(\text{def}(X,i)\)

- Any stmt \(i\) where a variable \(X\) may be used/retrieved is called a *reference* or *use* of \(X\) at \(i\): \(\text{ref}(X,i)\)

...............................

**def-clear**

- A path from stmt \(i\) to stmt \(j\) is *def-clear with respect to \(X\)* if it contains no definitions of \(X\) except possibly at the beginning (\(i\)) and end (\(j\))

...............................

**all-defs**

The *all-defs* criterion requires that each definition \(\text{def}(X,i)\) be tested some def-clear path to some reference \(\text{ref}(X,j)\).
What kinds of tests are required for all-defs coverage?

all-uses

The **all-uses** criterion requires that each pair \((\text{def}(X, i), \text{ref}(X, j))\) be tested using some def-clear path from \(i\) to \(j\).
**Mutation Testing**

Given a program $P$,

- Form a set of *mutant* programs that differ from $P$ by some single change

- These changes (called *mutation operators*) include:
  - exchanging one variable name by another
  - altering a numeric constant by some small amount
  - exchanging one arithmetic operator by another
  - exchanging one relational operator by another
  - deleting an entire statement
  - replacing an entire statement by an *abort()* call

**Mutation Testing (cont.)**

- Run $P$ and each mutant $P_i$ on a previously chosen set of tests

- Compare the output of each $P_i$ to that of $P$
  - If the outputs differ on any test, $P_i$ is *killed* and removed from the set of mutant programs
  - If the outputs are the same on all tests, $P_i$ is still considered *alive.*
Mutation Testing (cont.)

A set of test data is considered *inadequate* if it cannot distinguish between the program as written \((P)\) and programs that differ from it by only a simple change.

- So if any mutants are still alive after running a set of tests, we augment the tests until we can kill all the mutants.

Mutation Testing Problems

- Even simple programs yield tens of thousands of mutants. Executing these is time-consuming.
  - But most are killed on first few tests
  - And the process is automated

- Some mutants are actually *equivalent* to the original program:
  \[
  X = Y; \quad X = Y; \\
  \text{if} (X > 0) \text{ then} \quad \text{if} (Y > 0) \text{ then} \\
  \vdots \quad \vdots
  \]
  - Identifying these can be difficult (and cannot be automated)

Perturbation Testing

Perturbation testing (Zeil) treats each arithmetic expression \(f(\vec{x})\) in the code as if it had been modified by the addition of an error term \(f(\vec{v}) + e(\vec{v})\), where \(v\) are the program variables and \(e\) can be a polynomial of arbitrarily high degree (can approximate almost any error)

- Monitor the variable values actually encountered during testing
Testing

• Solve for the possible \( e \) functions that would have escaped detection on the tests done so far
  
  – If there are none, we're done.
  
  – If some exist, can generally be expressed as “coincidental” relations between variables
    
    * e.g., \( x \) and \( y \) have always been equal in all tests, so substitutions of one for the other could yet not have been detected

Reliability Modeling with Directed Tests
Most literature on reliability models assumes that it can only be done with representative testing, because

• Directed tests’ time-to-failure is unrelated to operational time-to-failure

• Directed tests may find faults “out of order”

Order Statistic Model
Zeil & Mitchell (1996) presented a model for reliability growth under either representative or directed testing. Assumptions:

• Software contains \( N \) faults, whose failure rates are described by a distribution \( F \).

• Faults manifest independently

• The test process is biased towards finding faults with higher failure rates.
Measurement Process

- Fault failure rates are measured when the fault has been identified and corrected.
- Fault failure rate data is then sorted by failure rate.
- The sorted data is fitted to an order statistic distribution.
  
  - Order statistics is the study of the probability of selecting certain values when the selection process is biased.

Fitting Example
Failure Rate vs. Failure Number

LSq Fit - Sequence 2

Representative
Directed
OS
JM
ML